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In 2007 all ECB publications feature a motif taken from the €20 banknote.

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CONTENTS

Abstract

This paper studies optimal monetary policy responses in an economy featuring sectorial heterogeneity in the frequency of price adjustments. It shows that a central bank facing heterogeneous nominal rigidities is more likely to behave less aggressively than in a fully sticky economy. Hence, the supposedly excessive caution in the conduct of monetary policy shown by central banks could be partly explained by the existence of a relevant sectorial dispersion in the frequency of price adjustments.

JEL Classification: E43, E52, E58

Keywords: core inflation, elasticity of intertemporal substitution, heterogeneity, nominal rigidity.

Non technical summary

Sectorial heterogeneity in price setting is a well established fact. Among others, Bils and Klenow (2004) and Dhyne et al. (2004) investigate the issue of the frequency of price adjustments for CPI data in several sectors of the United States and the euro area. While, on average, about 15% of prices in the euro area and 25% in the United States are adjusted each month, dispersion across sectors is most relevant. In the euro area and the United States, energy and unprocessed food prices display the highest frequency of price adjustment, while service prices are the stickiest.

Another fairly well established fact is the puzzling "caution" that central banks, and the Federal Reserve in particular, seem to adopt in the conduct of monetary policy. Rudebusch and Sevnnsson (1999), for example, estimate a small-scale model of the United States economy and, assuming a commonly accepted loss function for the central bank, show that the optimal Taylor rule in their setting has much larger coefficients on inflation and the output gap than the actual (estimated) Taylor rule of the Federal Reserve. Sack (2000) provides more evidence by performing a similar exercise and showing that the response of the federal funds rate to five identified economic shocks should be stronger than what is actually observed.

This paper argues that the two stylized facts above may be linked. In fact, both the economic literature and the practice of policy-making assign a relevant role to core inflation indices. These indices filter out high frequency fluctuations from prices in order to improve the understanding of medium-term inflationary pressures on the economy (for a survey of methods see Cristadoro et al., 2005). In particular, simple core inflation indices are derived by eliminating the most volatile components (usually, unprocessed food and energy prices) from the aggregate price index. How does this practice of central banks affect their behavior? In particular, do central banks behave more or less aggressively by using core inflation rather than overall inflation as a measure to assess inflationary pressures?

Aoki (2001) studies an economy featuring a continuum of sticky price goods and one flexible price good and shows that, in this economy, an optimizing central bank should fully stabilize inflation in the sticky price sector rather than overall inflation. Sticky price inflation, responding to smoothed expectations of output gaps and relative price changes, is defined as core inflation to the extent that it captures a persistent component of inflation. Both the form of nominal rigidities in Aoki (2001) and the definition of core inflation are maintained in this paper.

However, in contrast to Aoki (2001), this paper assumes the existence of non-negligible transaction frictions. This creates a trade-off between macroeconomic stabilization (that is, stabilization of inflation and the output gap) and interest rate stabilization, a feature that is rather plausible empirically. As a result of this assumption, full stabilization of core inflation is no longer the only desired target of the optimizing central bank. Then, the equilibrium solution for inflation, aggregate activity and the interest rate conditional to the policy rule followed by the central bank are derived, based on two assumptions.

First, it is assumed that the central bank can only implement an optimal non-inertial plan as in Woodford (1999): the policy instrument is assumed to be a linear function of only present and future values of policy relevant variables. Conditional to the optimal non-inertial plan, the dynamics of the economy can be derived analytically in order to identify the drivers of the aggressiveness in the reaction of the interest rate to shocks. Comparing the outcomes in the economy with heterogeneous price setters (heterogeneous economy) with those in an economy with 100% of sticky price goods (hereafter, baseline New Keynesian economy, described in Woodford, 1999), it turns out that the optimal non-inertial plan in the heterogeneous economy may generate less aggressive responses of the interest rate to supply and demand shocks than in the baseline New Keynesian (NK) economy. A sufficient condition for the latter is that output is rather interest sensitive, that is the transmission of the monetary impulse to output is rather strong. In fact, the presence of sectorial heterogeneity in nominal rigidities makes aggregate activity less interest-sensitive than in the baseline fully sticky price economy. This effect is more pronounced the stronger the transmission mechanism of monetary policy to output. Moreover, inflation sensitivity to output is "small" when output is strongly interest-sensitive. These two features reduce the willingness of monetary policy-makers to engage in macroeconomic stabilization increasing their willingness to preserve interest rate stability. However, if the sufficient condition above is not satisfied, other structural features of the economy shape the aggressiveness of monetary policy.

Finally, the assumption of forward-looking policy is relaxed and it is shown that the unconstrained optimal and time consistent policy rule of the central bank features history-dependence. Conditional to this policy, the equilibrium path for inflation, output gap and the interest rate can no longer be analytically derived. Numerical outcomes show that, conditional to the fully optimal policy and to a wide range of parameters, the reaction of the interest rate to supply and demand shocks is less aggressive in the heterogeneous economy compared with the fully sticky economy.

Hence, a central bank in an economy featuring heterogeneity in the mechanisms of price adjustments is more likely to behave less aggressively than in a fully sticky economy.

1 Introduction

Sectorial heterogeneity in price setting is a well established fact. Among others, Bils and Klenow (2004) and Dhyne et al. (2004) investigate the issue of the frequency of price adjustments for CPI data in several sectors of the United States and the euro area. While, on average, about 15% of prices in the euro area and 25% in the United States are adjusted each month, dispersion across sectors is most relevant. In the euro area and the United States, energy and unprocessed food prices display the highest frequency of price adjustment, while service prices are the stickiest.

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This paper argues that the two stylized facts above may be linked. In fact, both the economic literature and the practice of policy-making assign a relevant role to core inflation indices. These indices filter out high frequency fluctuations from prices in order to improve the understanding of medium-term inflationary pressures on the economy (for a survey of methods see Cristadoro et al., 2005). In particular, simple core inflation indices are derived by eliminating the most volatile components (usually, unprocessed food and energy prices) from the aggregate price index. How does this practice of central banks affect their behavior? In particular, do central banks behave more or less aggressively by using core inflation rather than overall inflation as a measure to assess inflationary pressures?

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Finally, the assumption of forward-looking policy is relaxed and it is shown that the unconstrained optimal and time consistent policy rule of the central bank features history-dependence. Conditional to this policy, the equilibrium path for inflation, output gap and the interest rate can no longer be analytically derived. Numerical outcomes show that, conditional to the fully optimal policy and to a wide range of parameters, the reaction of the interest rate to supply and demand shocks is less aggressive in the heterogeneous economy compared with the fully sticky economy.

Hence, a central bank in an economy featuring heterogeneity in the mechanisms of price adjustments is more likely to behave less aggressively than in a fully sticky economy.

The structure of the paper is as follows: section 2 presents the model economy and its loglinear approximation; section 3 describes the optimal monetary policy problem; section 4 solves the optimal monetary problems and provides an interpretation for the results; section 5 concludes.

2 The Model

Heterogeneity in the degree of nominal rigidities across sectors is a relevant empirical feature in the United States and the euro area. Consequently, a model that describes the optimal behavior of the monetary policy-maker should take this feature into account. To this end, this paper follows Aoki (2001) by adopting a framework in which there is a good that has a flexible price[2] and a continuum of differentiated goods whose price

[2]Benigno (2003) and Woodford (2003) specify two sectors (or countries) models with different degrees of price stickiness and imperfect competition. However, the simplified framework of Aoki (2001) is adopted here in order to account for the extremely skewed distribution of frequency of adjustment in goods and services prices.

is sticky[3].

The economy is populated by a continuum $(0-1)$ of infinitely lived households. Each household can work at the production of only one single good. A fraction of size γ of these households supply their labor for the production of a continuum $0-\gamma$ of differentiated sticky price goods. The remaining households (of measure $1-\gamma$) supply their services for the production of a flexible price good. Households receive utility directly from holding real balances. The expected utility function of household i is

$$E_0 \sum_{T=0}^{\infty} \beta^T \left[U(C^i_{t+T}, B_{t+T}) + K(\frac{M^i_{t+T}}{P_{t+s}}, D_{t+T}) - V(h^i_{j,t+T}, E_{j,t+T}) \right] \qquad (2.1)$$

where β is the constant subjective discount factor, C^i_t total consumption expenditure of household i, M^i_t nominal balances, P_t the aggregate price index and $h^i_{j,t}$ hours worked into the production of the good i $(i = 0-\gamma; f)$ in sector j $(j = s, f)$. B_t, D_t and $E_{j,t}$ are preference shocks with B_t and D_t common to all the agents while $E_{j,t}$ is allowed to vary across the two sectors. The utility function is concave and additive separable in its arguments and there is a positive level of real balances at which agents are satiated. C^i_t is a CES aggregator

$$C^i_t = \frac{C^{i\gamma}_{st} C^{i1-\gamma}_{ft}}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \qquad (2.2)$$

where C^i_{st} and C^i_{ft} are, respectively, aggregate household i consumption expenditure in sticky price and flexible price goods. C^i_{st} is a CES aggregator of the single differentiated sticky price goods

$$C^i_{st} = \left[\frac{1}{\gamma}^{\frac{1}{\vartheta}} \int_0^\gamma c_{s,t}(i)^{\frac{\vartheta-1}{\vartheta}} di \right]^{\frac{\vartheta}{\vartheta-1}} \qquad (2.3)$$

where ϑ represents the constant degree of substitutability across goods and it is assumed to be bigger than 1. As in Woodford (2003), the flow budget constraint of household i is given by

$$\frac{i_t - i^m_t}{1+i_t} M^i_t + E_t Q_{t,t+1} W^i_{t+1} \leq W^i_t - T_t - P_t C^i_t + w^i_{j,t} h^i_t + \int_0^1 \Pi_t(z) dz \qquad (2.4)$$

where W^i_{t+1} represents the amount of financial wealth carried over to the next period by household i, $w^i_{j,t}$ is the wage earned by household i for the production of good i in sector j, $\Pi_t(z)$ is the amount of profits stemming from the sale of good z accruing to household i, i_t is the risk-free interest rate, i^m_t is the interest rate earned on nominal

[3]Here, flexible and sticky sectors are defined as the set of firms producing the flexible price goods and the sticky price goods, respectively

balances[4] and $Q_{t,t+1}$ is the unique stochastic discount factor or asset-pricing kernel in this economy. Since i_t is the risk - free interest rate, it must hold that

$$\frac{1}{1+i_t} = E_t Q_{t,t+1} \tag{2.5}$$

The aggregate price index is given by

$$P_t = P_{st}^{\gamma} P_{ft}^{1-\gamma} \tag{2.6}$$

where P_{ft} is the price of the unique flexible price good, while P_{st} is the index that aggregates all the price $p_{s,t}(i)$ of the sticky price goods and is defined as

$$P_{st} = \left[\frac{1}{\gamma} \int_0^{\gamma} p_{s,t}(i)^{1-\vartheta} di \right]^{\frac{1}{1-\vartheta}} \tag{2.7}$$

Household i chooses its optimal contingent plan for consumption expenditure, financial wealth, hours worked and real balances taking good prices, financial prices and wages as given. The consumption-saving trade-off is regulated by the Euler Equation

$$\frac{1}{1+i_t} = \beta E_t \left[\frac{U_c(C_{t+1}, B_{t+1})}{U_c(C_t, B_t)} \frac{P_t}{P_{t+1}} \right] \tag{2.8}$$

which links the expected evolution of the consumption path to the expected real interest rate. Higher expected real interest rates induce households to save more and then to a steeper positively sloped or a flatter negatively sloped consumption path. Notice that consumption expenditure is not indexed in (2.8) since, in equilibrium, consumers share risk perfectly. Money demand is the same for all agents: it is positively affected by the level of transactions and negatively affected by the opportunity cost of holding wealth in the form of money, the risk-free interest rate. In fact,

$$\frac{K_m(m_t, D_t)}{U_c(C_t, B_t)} = \frac{i_t}{1+i_t} \tag{2.9}$$

Overall consumption expenditure is split across sticky and flexible price goods according to the following relationships

$$\mathbb{C}_{st} = \gamma(\frac{P_{st}}{P_t})^{-1} C_t \tag{2.10}$$

and

$$\mathbb{C}_{ft} = (1-\gamma)(\frac{P_{ft}}{P_t})^{-1} C_t \tag{2.11}$$

that is, aggregate consumption of the goods in the two sectors depends negatively on the relative prices of the sectorial consumption aggregates with respect to the aggregate

[4] Here, $i_t^m = 0$ which turns out to represent the institutional arrangements of actual central banks, like the Federal Reserve.

consumption price index. The overall expenditure devoted to sticky price goods is further split across the single differentiated goods according to

$$c_{st}(i) = \frac{1}{\gamma}\left(\frac{P_{st}(i)}{P_{st}}\right)^{-\vartheta} C_{st} \tag{2.12}$$

Finally, households in the two sectors choose hours worked according to the labor supply relationships

$$\frac{V_{h_s}(h^i_{s,t}, E_{s,t})}{U_c(C_t, B_t)} = \frac{w^i_{s,t}}{P_t} \tag{2.13}$$

and

$$\frac{V_{h_f}(h^l_{f,t}, E_{f,t})}{U_c(C_t, B_t)} = \frac{w^l_{f,t}}{P_t} \tag{2.14}$$

that is, they equal the marginal rate of substitution between hours worked and consumption to the real wage they earn. Firms set prices optimally by taking the demand for their goods as given. For each good i, the following market clearing conditions hold

$$y_{s,t}(i) = c_{s,t}(i)$$

and

$$y_{f,t} = c_{f,t}$$

that, on aggregate, imply

$$Y_{st} = C_{st}, Y_{ft} = C_{ft}, Y_t = C_t$$

Moreover, the production function of each of the firms takes the simple linear form

$$y_{j,t}(i) = A_{j,t}h_{j,t}(i) \tag{2.15}$$

where there is only a production input, labor, whose marginal productivity is affected by a stochastically evolving and possibly sector-specific technological factor. Consequently, total variable costs for producer i in sector j are represented by

$$VC^i_{j,t} = w^i_{j,t}h^i_{j,t} = w^i_{j,t}\frac{y_{j,t}(i)}{A_{j,t}}$$

which implies the following marginal costs

$$MC^i_{j,t} = \frac{w^i_{j,t}}{A_{j,t}}$$

Since firms in the flexible price sector act in a regime of perfect competition, their optimal price equals marginal costs

$$P_{f,t} = \frac{w_{f,t}}{A_{f,t}}$$

which, in general equilibrium, becomes

$$P_{f,t} = P_t \frac{V_{h_f}(h_{f,t}, E_{f,t})}{U_c(C_t, B_t)} \frac{1}{A_{f,t}} \tag{2.16}$$

The sticky price sector, on the other hand, features both real and nominal rigidities. Goods are differentiated and firms act in a regime of monopolistic competition. Moreover, in each period, only a fraction $1-\xi$ of randomly chosen firms can optimally reset their price at the value $P^*_{st}(i)$[5]. ξ can be alternatively interpreted as the probability for each firm in the sticky price sector not to be drawn among the price setters at each period. Equation (2.7) then implies

$$P_{st}^{1-\vartheta} = \xi P_{st-1}^{1-\vartheta} + (1-\xi) P^*_{st}(i)^{1-\vartheta} \tag{2.17}$$

As for $P^*_{st}(i)$, every firm entitled to reset its price at time t realizes that with a constant probability at each time and independently of when it last changed its price, it will not be able to reset its price. Moreover, such a firm takes into account a demand curve for its good of the form (2.12). Finally, as in Rotemberg and Woodford (1997), the production in the sticky price sector is subsidized at a rate τ such that the effects of markup pricing in monopolistic competition are completely offset. Consequently, firm i solves the following problem[6]

$$\max_{P^*_t(i)} E_t \sum_{T=0}^{\infty} \xi^T Q_{t,t+T} \left[(1+\tau) P^*_{st}(i) y_{t+T}(i) - w^i_{s,t+T} h^i_{s,t+T} \right] \tag{2.18}$$

s.t.

$$y_{t+T}(i) = \left(\frac{P_{st+T}}{P_{t+T}} \right)^{-1} \left(\frac{P_{st}(i)}{P_{st+T}} \right)^{-\vartheta} Y_t$$

$$y_{t+T}(i) = A_{st+T} h^i_{s,t+T}$$

whose first order condition is given by

$$E_t \sum_{T=0}^{\infty} \xi^T Q_{t,t+T} \left[P^*_{st}(i) - \frac{1}{1+\tau} \frac{\vartheta}{\vartheta+1} \frac{w^i_{s,t+T}}{A_{st+T}} \right] y_{t+T}(i) = 0$$

implying that producers in the sticky sector optimally reset their price at a present discounted value of their current and expected future marginal costs. Assuming that $\tau = \frac{1}{\vartheta - 1}$ and replacing the second term in the square parenthesis by equation (2.13) we obtain

$$E_t \sum_{T=0}^{\infty} \xi^T Q_{t,t+T} \left[P^*_{st}(i) - \frac{P_{t+T}}{A_{st+T}} \frac{V_{h_s}(h^i_{s,t+T}, E_{s,t+T})}{U_c(C_{t+T}, B_{t+T})} \right] y_{t+T}(i) = 0 \tag{2.19}$$

[5]This form of nominal rigidities is proposed by Calvo (1983).

[6]E_t denotes conditional expectation based on the information set available at time t.

In order to solve the model, the first order conditions are loglinearized around a deterministic steady state with zero inflation. The loglinear version of the model is specified in terms of relative prices, defined as follows

$$X_{ft} = \frac{P_{ft}}{P_t}, X_{st} = \frac{P_{st}}{P_t}, X_t^* = \frac{P_{st}^*(i)}{P_t}$$

Moreover, the sectorial inflation rates are defined as

$$\pi_{j,t} = \log \frac{P_{jt}}{P_{jt-1}}, \text{j=s,f}$$

and the overall inflation rate as

$$\pi_t = \log \frac{P_t}{P_{t-1}}$$

Hereafter, small cases indicate logarithmic deviations from steady state. By manipulating and loglinearizing equation (2.6), one obtains

$$\pi_t = \gamma \pi_{st} + (1-\gamma)\pi_{ft} \tag{2.20}$$

$$x_{st} = -\frac{1-\gamma}{\gamma} x_{ft} \tag{2.21}$$

and[7]

$$\pi_t = \pi_{st} - \Delta x_{st} = \pi_{st} + \frac{1-\gamma}{\gamma}\Delta x_{ft} \tag{2.22}$$

As for aggregate demand, loglinearization of the Euler equation (8) provides

$$y_t - b_t = E_t(y_{t+1} - b_{t+1}) - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) \tag{2.23}$$

which is usually defined as the New IS equation[8]. As its non-linear counterpart, this equation states that the expected slope of the temporal path of aggregate demand depends on the expected real interest rate $i_t - E_t\pi_{t+1}$. However, the sensitivity of the path to the riskless interest rate (and consequently to monetary policy) is affected by the (steady state) value of the elasticity of intertemporal substitution $\frac{1}{\sigma} = -\frac{U_C}{U_{CC}C}$. The more agents are willing to substitute intertemporally (that is, the bigger the elasticity of intertemporal substitution), the more aggregate demand is sensitive to the interest rate and hence to monetary policy. The equations (2.11) and (2.12), relating to the individual demand for goods, can be aggregated and loglinearized providing the sectorial demand curves

$$y_{st} = y_t - x_{st} \tag{2.24}$$

[7] The symbol Δ indicates first differencing

[8] Throughout this paper, all the stochastic processes of the shocks are assumed to be stationary and normalized to 1 in steady state. Consequently, $b_t = -\frac{U_{CB}}{U_{CC}C}B_t$.

$$y_{ft} = y_t - x_{ft} \tag{2.25}$$

which are downward sloping in the corresponding prices relative to the overall index. Importantly, sectorial outputs may be affected by the other sector dynamics through their dependence on aggregate output and the linkages in relative prices x_{ft} and x_{st}, highlighted in equation (2.21). The first order condition for real balances (2.9) gives

$$m_t = \eta_y y_t - \eta_i i_t + \varepsilon_t \tag{2.26}$$

where η_y and η_i are two positive coefficients, while ε_t is a composite shock that involves both B_t and D_t[9]. This equation is a stochastic version of the traditional LM equation where money demand depends positively on the level of transactions and negatively on the opportunity cost of holding wealth in a monetary form. Natural (or potential) output[10] can be derived by setting prices equal to current marginal cost in both sectors. By loglinearizing the perfect competition/flexible price allocations in the two sectors, one obtains

$$y_{jt}^n = \frac{1+\omega}{\omega+\sigma} a_{jt} - \frac{\sigma}{\omega+\sigma} b_t - \frac{\eta}{\omega+\sigma} e_{jt} \text{ with j=s,f} \tag{2.27}$$

where $\omega = \frac{V_{hh}}{V_h} h$ (and is assumed equal in both sectors) and $\eta = \frac{V_{he}}{V_h}$. It is worth noticing that, if the labor supply shock and the technology shock are common to the whole economy, natural output in the sticky price sector and in the flexible price sector are the same. Aggregate output gap g_t is defined as

$$g_t = \gamma \left(y_t - y_{st}^n \right) + (1-\gamma) \left(y_t - y_{ft}^n \right) \tag{2.28}$$

which, in the case of common technology and labor supply shocks, simplifies to[11]

$$g_t = y_t - y_{st}^n = y_t - y_{ft}^n$$

Core inflation depends only on the prices of the firms in the sticky price sector that can reset the price. In fact, by loglinearizing equation (2.17), one obtains

$$\pi_{st} = \frac{1-\xi}{\xi} p_{st}^* \tag{2.29}$$

where in p_{st}^*, the index i is dropped since each firm sets it at the same value, in equilibrium. The optimal pricing equation in the sticky price sector (2.19) may be loglinearized along the lines of Aoki (2001) and Woodford (2003) to derive a stochastic difference equation in p_{st}^*. Putting this equation together with equation (2.29) and manipulating slightly, one finds

[9] In particular, $\eta_y = \frac{\sigma}{\sigma^m}$ where σ has previously been defined and $\sigma^m = -\frac{K_{MM}}{K_M} M$ and $\eta_i = \frac{U_C}{K_M} \frac{1}{(1+i)\sigma^m}$, while $\varepsilon_t = \frac{1}{\sigma^m} \left[\frac{K_{MD}}{K_M} D_t - \frac{U_{CB}}{U_C} B_t \right]$.

[10] Defined as output in the fully flexible price economy.

[11] In other words, when all the shocks in the economy are aggregate, aggregate output gap is equal to the sectorial output gaps $g_{st} = y_t - y_{st}^n$ and $g_{ft} = y_t - y_{ft}^n$

$$\pi_{st} = \beta E_t \pi_{st+1} + \frac{(\omega+\sigma)(1-\xi\beta)(1-\xi)}{\xi(1+\vartheta\omega)\gamma} g_t \tag{2.30}$$

Core inflation is forward-looking because firms forecast the future evolution of their marginal costs when they are allowed to reset the price, as they might not be able to adjust it in the future. For the sake of simplicity, the elasticity of inflation to current output gap is defined as

$$k = \frac{(\omega+\sigma)(1-\xi\beta)(1-\xi)}{\xi(1+\vartheta\omega)\gamma} \tag{2.31}$$

Finally, it is worth noting that by substituting the definition of aggregate output gap (2.28) in equation (2.23), one obtains

$$g_t = E_t(g_{t+1}) - \frac{1}{\sigma}\left(i_t - E_t\pi_{t+1} - r_t^n\right) \tag{2.32}$$

where

$$r_t^n = \sigma\left[b_t - E_t b_{t+1} + \gamma(E_t y_{st+1}^n - y_{st}^n) + (1-\gamma)(E_t y_{ft+1}^n - y_{ft}^n)\right] \tag{2.33}$$

is the so called "natural interest rate": the real interest rate in a fully flexible price economy.

3 The policy problem

This section describes the objectives of monetary policy and the constraints that the structure of the economy imposes on the central bank in setting the desired contingent path for the welfare relevant variables.

The quadratic loss function[12] of the central bank may be derived by means of a second order Taylor approximation[13] to the utility function of the agents that gives

$$L_t = E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_{st}^2 + \lambda_i (i_t - i^*)^2 + \lambda_g g_t^2\right] \tag{3.34}$$

where

$$\lambda_i = \frac{v^{-1}\eta_i(1-\xi)(1-\xi\beta)}{\xi\gamma\vartheta(1+\omega\vartheta)} \tag{3.35}$$

and

[12] For a derivation, see appendix 1.

[13] In deriving the approximation, the transaction frictions described in the previous section are assumed to be "small" in the sense of Woodford (2003). Moreover, it is also assumed that the government subsidizes production in the sticky price sector in order to eliminate the effects of monopolistic pricing, as in Rotemberg and Woodford (1997). On the consequences of different assumptions (small or deliberately large monopolistic distortions and distortionary taxation), see Woodford (2003) and Benigno and Woodford (2004).

$$\lambda_g = \frac{(\omega + \sigma)(1 - \xi)(1 - \xi\beta)}{\xi\gamma^2\vartheta(1 + \omega\vartheta)} \tag{3.36}$$

represent, respectively, the weights of interest rate and output gap stabilization relative to core inflation stabilization[14] and $i^* = \ln \frac{1}{1+i^{ss}}$ with i^{ss} equal to the steady state value of the nominal interest rate. As in Aoki (2001), the central bank in this economy must stabilize core inflation (π_{st}) and aggregate output gap (g_t). Output gap appears in the loss function because the level of aggregate output in the economy is inefficient, due to nominal rigidities rather than imperfect competition, whose effect on output is eliminated by a government subsidy. However, eliminating the output gap is not enough to lead the economy to an efficient allocation of resources. In fact, price staggering implies an inefficient dispersion of prices. Such a distortion is eliminated by setting inflation to zero, which explains why inflation in the sticky price sector (core inflation) appears as an argument in the loss function. The relative price of the flexible good with respect to the aggregate is set at a sub-optimal level from a social welfare point of view only because of the linkages between the two sectors highlighted in (2.21). Consequently, eliminating distortions in the sticky price sector is enough to restore the efficient allocation in the flexible price sector and in the aggregate, as well. This is why no variables relative to the flexible price sector appear explicitly in the loss function. Output gap and core inflation stabilization will be jointly referred to as "macroeconomic stabilization" from now on.

On the other hand, the central bank in this model should also dampen fluctuations in its operating target, the risk-free interest rate, differently from Aoki (2001). In fact, money provides services to the agents and is included in their utility functions. Consequently, the central bank should give the agents as much money as needed to satiate them, driving the nominal interest rate to the return earned by money[15]. Hence, every variation of the interest rate from the return to money[16] is penalized in the loss function of the central bank. Hereafter, this objective will be defined as "operating target stabilization".

Operating target stabilization generally runs counter to macroeconomic stabilization. In fact, in (2.32), macroeconomic stabilization implies that the real interest rate equals the natural interest rate r_t^n, while operating target stabilization requires the nominal interest rate to be constantly equal to zero. The consequences of this trade-off will be explored at length in the next section.

The structure of the economy acts as a constraint in the problem of the central bank. Usually, one must consider the equations defining the dynamics of aggregate activity (2.32) and inflation in the sticky price goods (2.30). On the other hand, notice that, from equation (2.22)

$$\pi_t = \pi_{st} + \frac{1-\gamma}{\gamma}\Delta x_{ft}$$

[14] The weights depend on the deep parameters coming from the microfoundations of the model. The only parameter that has not been defined before is v, which represents steady state "money velocity".

[15] This fact is commonly defined as the Friedman Rule.

[16] Assumed to be zero in this paper.

implying that in order to solve for inflation and aggregate activity, the evolution of the relative price in the flexible price sector is to be taken into account. To this end, by loglinearizing (2.16) we get

$$x_{ft} = c(y_t - y_{ft}^n) \tag{3.37}$$

where

$$c = \frac{\omega + \sigma}{1 + \omega}$$

By replacing aggregate inflation in (2.32) with (2.22), the system of equations defined by (2.30) and (2.32) includes the three variables π_{st}, g_t and x_{ft}, as well as the monetary policy instrument i_t. Furthermore, (3.37) provides an expression for x_{ft} that depends only on the sectorial output gap in the flexible price sector. Assuming that all the shocks in the economy are common, sectorial output gaps become equal to aggregate output gap. This implies

$$x_{ft} = cg_t \tag{3.38}$$

that can be used to eliminate x_{ft} giving

$$g_t = E_t(g_{t+1}) - \frac{1}{S}\left(i_t - E_t\pi_{st+1} - r_t^n\right) \tag{3.39}$$

where

$$S = \sigma + \frac{1-\gamma}{\gamma}c > \sigma \tag{3.40}$$

In other words, the central bank is only subject to the two constraints (2.30) and (3.39) in its minimization problem. Notice that this problem is isomorphic to the one in Giannoni (2002) and Woodford (1999) "baseline New Keynesian model". However, inflation is replaced by core inflation and the parameters are different[17]. As the size of the sticky price sector (γ) approaches 1, core inflation approaches overall inflation and the problem here converges to the one in the baseline New Keynesian model. Then, the outocmes for the heterogeneous ($\gamma < 1$) economy of this paper are easily compared to those of the baseline model retrieved by setting $\gamma = 1$.

Assuming, instead, that the technological and/or labor supply shocks are sector specific, x_{ft} depends on the relative sectorial output gap rather than the aggregate output gap. At any rate, a structural constraint in the same form as (3.39) can be easily derived for this case as well. In fact, by defining

$$x_{ft}^n = c(y_t^n - y_{ft}^n) \tag{3.41}$$

[17]Giannoni (2002) and Woodford (1999) specify the same loss function as in this paper but with arbitrary weights on the different objectives. Instead, here the weights come from the microfoundations of the model. This makes it possible to measure the impact of a different γ (smaller than 1 as in the current model, rather than equal to 1 as in the baseline New Keynesian) on the relative importance of the macroeconomic and operating target stabilization.

as the natural relative flexible price[18], it can be shown that

$$g_t = E_t(g_{t+1}) - \frac{1}{S}\left(i_t - E_t\pi_{st+1} - u_t\right) \tag{3.42}$$

where, again

$$S = \sigma + \frac{1-\gamma}{\gamma}c > \sigma$$

and

$$u_t = r_t^n - E_t\Delta x_{ft+1} \tag{3.43}$$

Hence, assuming sectorial technology and labor supply shocks, the optimal problem of the central bank remains virtually the same. The only difference in (3.42) with respect to (3.39) is in the interpretation of the composite shock. For this reason, this paper only deals with the case of common shocks.

Summing up, in the economy outlined in the previous section, the central bank aims to minimize a loss function that embeds quadratic terms in core inflation, aggregate output gap and the interest rate. Both in the case of common and sectorial technology and labor supply shocks, the central bank is subject to two structural constraints (2.30 and 3.39 or 3.42) on the evolution of aggregate output and sticky price inflation. What changes on the basis of the assumption on the shocks, is the stochastic process (and interpretation) of the composite shocks appearing in the equations that define the evolution of aggregate activity (3.39 and 3.42). Finally, the results obtained in this paper are easily compared to those in the baseline model solved by Woodford (1999) and Giannoni (2002) retrieved by setting $\gamma = 1$ in the results for the model with heterogeneity.

4 Optimal monetary policy

This section derives the optimal monetary policy followed by the central bank in heterogeneous and baseline economies and studies the consequences (in terms of reaction of the interest rate to shocks) of their implementation.

In particular, in the first sub-section the central bank is assumed to follow the optimal policy from a timeless perspective in the class of the forward-looking policies (optimal non-inertial plan). While this policy is sub-optimal because of the constraints imposed by the forward-looking assumption, it permits to solve analytically for the path of the interest rate contingent to shocks and provides intuition on the mechanisms explaining the different reactions of a central bank in heterogeneous and baseline economies.

The second sub-section, on the other hand, derives the monetary policy that is fully optimal. This policy is not in the class of forward-looking policies since it features.

[18] This is the (logarithmic) relative price in the flexible sector that would prevail were the whole economy to feature flexible prices. This variable is different from zero only if, as in this case, the two sectors can be affected by different shocks.

However, results from the forward-looking case can be used in order to interpret the dynamics of baseline and heterogeneous economies under such a policy.

4.1 The optimal non-inertial plan

In this sub-section, it is assumed that the central bank implements a purely forward-looking policy: it sets its policy rate at each date depending only on the set of developments for the target variables that are possible from that period onwards. In particular, it follows a simple rule of the form[19]

$$i_t = i^{ss} + \varphi_\pi(\pi_{st} - \pi^{ss}) \tag{4.44}$$

Among the forward-looking plans, the central bank is assumed to follow the optimal non-inertial plan[20], that is the forward-looking policy minimizing the unconditional expectation of the loss function L over the stationary distribution of the possible initial exogenous states r_0^n. This implies that the optimal plan does not depend on the state of the economy at the time when the commitment is made, so that the central bank policy is time consistent as in Woodford (1999, 2003). Moreover, φ_π is assumed to imply determinacy of the rational expectation equilibrium. As a consequence of the determinacy and the AR(1) assumption for the shocks, the path followed by the interest rate, output gap and core inflation under the optimal non-inertial plan is

$$i_t = i^{ss} + f_i r_t^n,\ g_t = g^{ss} + f_g r_t^n,\ \pi_{st} = \pi^{ss} + f_\pi r_t^n \tag{4.45}$$

where i^{ss}, g^{ss} and π^{ss} are the steady state values for the nominal interest rate, the output gap and core inflation under the optimal policy defined in (4.45). Replacing (4.45) in (3.39), (3.34) and (2.30) and taking the unconditional expectation, the optimization problem of the entral Bank becomes

$$\min_{f_i, f_g, f_\pi, i^{ss}, g^{ss}, \pi^{ss}} \left[\pi^{ss2} + \lambda_i(i^{ss} - i^*)^2 + \lambda_g g^{ss2}\right] + (f_\pi^2 + \lambda_i f_i^2 + \lambda_g f_g^2)Var(r_t^n) \tag{4.46}$$

subject to

$$f_\pi(1-\beta\rho) = k f_g$$

$$f_g(1-\rho) = \frac{1}{S}(1 + f_\pi\rho - f_i)$$

and

$$\pi^{ss}(1-\beta) = k g^{ss}$$

[19] As for the specific form of the rule, notice that a slightly more complicated rule that also includes the output gap would not improve on this simple one. In fact, since the relevant stochastic term for the problem of the central bank is only the exogenous natural interest rate r_t^n or the composite shock u_t, a single variable rule is able to implement the non-inertial plan.

[20] See Woodford (2003) for an application.

$$i^{ss} = \pi^{ss}$$

This optimization problem may be split into two independent parts. On the one hand, the steady state component (choosing i^{ss}, g^{ss}, π^{ss}) and, on the other hand, the choice of f_i, f_g and f_π for stabilization purposes. The focus of this paper is exclusively on the impulse response function of the interest rate to shocks, so we only deal with the solutions for f_i, f_g and f_π[21]. The first order conditions for the stabilization problem are

$$f_\pi - \mu\rho\frac{1}{S} + \phi(1-\beta\rho) = 0 \tag{4.47}$$

$$\lambda_g f_g + \mu(1-\rho) - \phi k = 0 \tag{4.48}$$

$$\lambda_i f_i + \mu\frac{1}{S} = 0 \tag{4.49}$$

where μ and ϕ are the lagrangian multipliers associated with the structural constraints.

Together with the constraints, the first order conditions define a system of five linear equations whose solution[22] is

$$\begin{aligned} f_i &= \frac{\lambda_g(1-\beta\rho)^2 + k^2}{\lambda_g(1-\beta\rho)^2 + k^2 + \lambda_i\left[S(1-\rho)(1-\beta\rho) - k\rho\right]^2} \\ f_\pi &= \frac{k\lambda_i(S(1-\rho)(1-\beta\rho) - k\rho)}{\lambda_g(1-\beta\rho)^2 + k^2 + \lambda_i\left[S(1-\rho)(1-\beta\rho) - k\rho\right]^2} \\ f_g &= \frac{\lambda_i(1-\beta\rho)\left[S(1-\rho)(1-\beta\rho) - k\rho\right]}{\lambda_g(1-\beta\rho)^2 + k^2 + \lambda_i\left[S(1-\rho)(1-\beta\rho) - k\rho\right]^2} \end{aligned} \tag{4.50}$$

The coefficient φ_π that implements the desired equilibrium is easily derived since from (4.44) it holds

$$f_i = \varphi_\pi f_\pi$$

which implies

$$\varphi_\pi = \frac{\lambda_g(1-\beta\rho)^2 + k^2}{k\lambda_i(S(1-\rho)(1-\beta\rho) - k\rho)} \tag{4.51}$$

The analysis is restricted to positive values of φ_π and φ_π has to be bigger than one to ensure determinacy. This implies

$$0 < S(1-\rho)(1-\beta\rho) - k\rho < \frac{\lambda_g(1-\beta\rho)^2 + k^2}{k\lambda_i} \tag{4.52}$$

[21]The interested reader is referred to Woodford (1999, 2003) for the steady state solution in this context.

[22]From the solution for π_{st}, g_t and i_t, one can derive the solution for all the other variables in the economy.

$\frac{\lambda_g(1-\beta\rho)^2+k^2}{k\lambda_i}$ is positive, implying that the set of parameters identified in (4.52) is non-empty.

The impulse response function of the interest rate to a natural rate shock is characterized by the coefficient f_i, as in Giannoni (2002). It must also be noted that f_i is always positive and can take values between 0 and 1. In the policy problem defined in Aoki (2001), f_i is equal to 1, due to the lack of stabilization trade-offs for the central bank. Hence, f_i is independent of the size of the flexible price sector in that economy. However, when the central bank faces a trade-off between macroeconomic stabilization and operating target stabilization it may not be optimal to set f_i to 1. Operating target stabilization would require f_i to be as small as possible, closer to 0 than 1 and the central bank eventually sets f_i at the intermediate level reflecting the right balance among conflicting objectives. Interestingly, the optimal f_i when there is a trade-off does depend on γ, the size of the sticky price sector.

Studying the way in which f_i depends on the size of the sticky price sector sheds light on the question if, under certain circumstances, the optimal non-inertial plan in the heterogeneous economy leads to a less aggressive (a smaller f_i) reaction of the interest rate than in the baseline New Keynesian economy. The difference in the aggressiveness of the central bank in the two economies can be analyzed by comparing the f_i in (4.50) with f_i^* , the counterpart to f_i in the baseline case $\gamma = 1$. Notice that the value of γ affects both the structural equations and the weights of the three economic variables in the loss function. In fact, besides (3.40), the following relationships hold between coefficients in the heterogeneous and in the baseline models[23]

$$k = \frac{k^*}{\gamma} \tag{4.53}$$

$$\lambda_i = \frac{\lambda_i^*}{\gamma} \tag{4.54}$$

$$\lambda_g = \frac{\lambda_g^*}{\gamma^2} \tag{4.55}$$

The higher weights to interest and output gap stabilization with respect to inflation in the heterogeneous economy arise because in this type of economy, the sticky price distortion affects a smaller fraction of the firms than in the baseline economy. Moreover, sticky price inflation is more sensitive to the level of the output gap in the model with heterogeneity than in the one without ($k > k^*$) and then the output gap weight is further magnified in the model with heterogeneity even relative to the interest rate stabilization objective.

Setting $\gamma = 1$, we can derive the optimal reaction of the interest rate to a shock in the baseline case, that is

$$f_i^* = \frac{\lambda_g^*(1-\beta\rho)^2 + k^{*2}}{\lambda_g^*(1-\beta\rho)^2 + k^{*2} + \lambda_i^*\left[\sigma(1-\rho)(1-\beta\rho) - k^*\rho\right]^2} \tag{4.56}$$

[23]Coefficients of the baseline model are denoted by a star.

In order to facilitate the comparison between f_i and f_i^*, we can use equations from (4.53) to (4.55) and (3.40) to rearrange terms in f_i such that

$$f_i = \frac{\lambda_g^*(1-\beta\rho)^2 + k^{*2}}{\lambda_g^*(1-\beta\rho)^2 + k^{*2} + \frac{\lambda_i^*}{\gamma}\left\{\left[\gamma\sigma + (1-\gamma)c\right](1-\rho)(1-\beta\rho) - k^*\rho^2\right\}^2} \tag{4.57}$$

and consequently

$$f_i < f_i^* \Leftrightarrow \frac{1}{\gamma}\left\{\left[\gamma\sigma + (1-\gamma)c\right](1-\rho)(1-\beta\rho) - k^*\rho\right\}^2 > \left[\sigma(1-\rho)(1-\beta\rho) - k^*\rho\right]^2 \tag{4.58}$$

The inequality in (4.58) holds if σ is smaller than or equal to 1[24]. In fact, in that case σ is smaller than or equal to c and the left hand side of the equality in (4.58) is bigger than or equal to the right hand side. However, when σ is big enough, f_i can be either bigger than, smaller than or equal to f_i^* depending on other features of the economy.

The result can be interpreted by looking at equation (3.39) and (3.40), which allow us to recast the monetary policy problem in the model with heterogeneity in a format that only includes the policy relevant variables (which are those that explicitly appear in the loss function of the central bank) and is directly comparable to the traditional problem in the baseline New Keynesian model. These equations imply that the central bank in the model with nominal heterogeneity behaves like a central bank in a model with a representative sector but with output less interest-sensitive than in the baseline case ($S > \sigma$). Then, in the heterogeneous model, the monetary policy-maker has less incentive to use monetary policy for macroeconomic stabilization than in the baseline case since she acts as though monetary policy were less effective at dampening the fluctuations in aggregate activity and consequently in inflation. This explains why she rather engages in more interest rate stabilization and then the smaller reaction of the interest rate to economic shocks. However, when σ is big, the result is not so clear cut and, depending on the values taken by other structural parameters, both more or less aggressive behavior can stem from the optimal plan in the heterogeneous economy relative to the baseline. The ratio

$$\frac{S}{\sigma} = 1 + \frac{1-\gamma}{\gamma(1+\omega)} + \frac{\omega(1-\gamma)}{(1+\omega)\gamma}\frac{1}{\sigma} \tag{4.59}$$

may help to understand why. S is always bigger than σ implying that the policymaker in the core model always behaves as if output were less sensitive than in the baseline case, but this ratio decreases with the increase in σ, stabilizing around $1 + \frac{1-\gamma}{\gamma(1+\omega)}$. Hence, the difference in the strength of the effects of monetary policy on output tends to decrease when σ increases, making output less interest-sensitive. Moreover, an increase

[24]Notice that the positivity constraint on φ_π implies that $\left[\gamma\sigma + (1-\gamma)c\right](1-\rho)(1-\beta\rho) - k^*\rho^2$ is positive. The inequality also holds for some values of σ bigger than 1 since the result can be extended to those values by continuity. However, deriving the exact upper bound for σ is cumbersome and does not add much content to the main message of the paper.

in σ increases k more than k^* as shown by (4.53) and (2.31). Sticky price inflation is still more sensitive to aggregate activity in the core model than in the baseline model, but this difference is magnified by the increase in σ. These two factors imply that macroeconomic stabilization becomes more attractive relative to operating target stabilization for the central bank in the core model when σ increases. Eventually, macroeconomic stabilization may become more important in the heterogeneous model than in the baseline model relative to operating target stabilization, explaining why monetary policy might turn out to be more aggressive in the former than in the latter economy.

4.2 The unconstrained optimal interest rule

Although it allows useful insights, in general the optimal non-inertial plan represents a sub-optimal choice and also one that is not fully supported by the empirical analysis on the behavior of central banks. Indeed, with forward-looking private agents the central bank can improve on the non-inertial plan by committing to some form of inertial behavior that could shape expectations towards a stronger stabilization of the welfare relevant variables. Moreover, lagged interest rate terms are shown to be empirically relevant for the analysis of the behavior of central banks.

The fully optimal and time consistent interest rule of the central bank solves the following problem[25]:

$$min_{\pi_{st},g_t,i_t} L_t = E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_{st}^2 + \lambda_i (i_t - i^*)^2 + \lambda_g g_t^2 \right]$$

s.t.

$$\pi_{st} = \beta E_t \pi_{st+1} + k g_t$$

$$g_t = E_t(g_{t+1}) - \frac{1}{S}(i_t - E_t \pi_{st+1} - u_t)$$

Setting up the Lagrangian of this problem, taking the first order conditions and replacing the Lagrange multipliers, it can be shown that the solution of this problem from period 2 onwards is:

$$i_t = -\frac{k}{S\beta} i^* + \frac{k + \beta S}{\beta S} i_{t-1} + \frac{1}{\beta} \Delta i_{t-1} + \frac{k}{S\lambda_i} \pi_{st} + \frac{\lambda_g}{S\lambda_i} \Delta g_t$$

Demanding that this policy is already in place at time 0 and 1 allows the central bank to achieve time consistency (that is, it implements the policy that is optimal from a timeless perspective). The optimal rule has some notable features already investigated by Giannoni and Woodford (2002b) for the baseline New Keynesian economy[26]:

[25] See Giannoni and Woodford (2002a)

[26] Notice that the optimal rule in the baseline New Keynesian model can be retrieved once again by setting $\gamma = 1$

- it is history-dependent in that the optimal interest rates depends on its own two lags and on the first lag of the output gap;

- it resembles a Taylor rule in that the optimal interest rate is a linear function of inflation and the output gap.

Next, in order to derive the dynamics of this economy and, in particular, of the interest rate in response to shocks, the following system of three stochastic difference equations has to be solved:

$$\pi_{st} = \beta E_t \pi_{st+1} + k g_t \tag{4.60}$$

$$g_t = E_t(g_{t+1}) - \frac{1}{S}\left(i_t - E_t \pi_{st+1} - u_t\right) \tag{4.61}$$

$$i_t = -\frac{k}{S\beta} i^* + \frac{k+\beta S}{\beta S} i_{t-1} + \frac{1}{\beta}\Delta i_{t-1} + \frac{k}{S\lambda_i}\pi_{st} + \frac{\lambda_g}{S\lambda_i}\Delta g_t \tag{4.62}$$

It can be proved[27] that, under this policy, the economy can follow only one contingent path (in other words, the solution is determinate) for any value of the parameters. However, the solution for this system is to be derived numerically[28] due to the presence of three more state variables (the lags of the interest rate and output gap) than in the optimal non-inertial plan.

As in the previous section, the focus here is on the reaction of the nominal interest rate to a unitary shock in r_t^n, comparing the outcomes in the heterogeneous to those in the baseline New Keynesian economy. In order to solve the model, numerical values have to be assigned to the parameters in equations (4.60) to (4.62). The baseline calibration takes the values of the parameters estimated in Rotemberg and Woodford (1997), with some exceptions. In particular, ξ is set to 0.75, a value that is plausible empirically and that implies that firms reset their price on average once a year (with quarterly data). More importantly, the model in Rotemberg and Woodford (1997), does not feature sectoral heterogeneity in the price adjustment mechanisms. Hence, in order to calibrate the parameter γ (the percentage of firms with a sticky price adjustment), we follow Bils and Klenow (2004) and set it to 0.75. These and the other values of the structural parameters in this economy are reported in Table 1.

Table 1: Baseline Calibration

Model	β	σ	γ	ω	ξ	ϑ	ρ	ν	η	k	S	λ_i	λ_g
Heterogeneous economy	0.99	0.16	0.75	0.5	0.75	7	0.6	1	1	0.018	0.35	0.004	0.004
Baseline economy	0.99	0.16	1	0.5	0.75	7	0.6	1	1	0.013	0.16	0.003	0.002

[27] For a proof in the case of the baseline New Keynesian model, see Giannoni and Woodford (2002b). The case of the heterogeneous economy does not require a separate proof due to the isomorphism of the two minimization problems

[28] The numerical solutions are derived by using the AIM algorithm of Anderson and Moore.

Notice that the values of the first nine parameters (from β to η) are assumed, while the remaining four parameters are non-linear functions of the previous nine. The only difference in the values of the first nine parameters between the heterogeneous and baseline New Keynesian economies lies in the percentage of sticky price firms (γ) which is 100% in the baseline economy. Given these values of the parameters, figure 1 plots the reaction of the nominal interest rate in the heterogeneous (solid line) and baseline (dashed line) economies.

Figure 1: Reaction of nominal interest to a unitary shock in the natural real interest rate

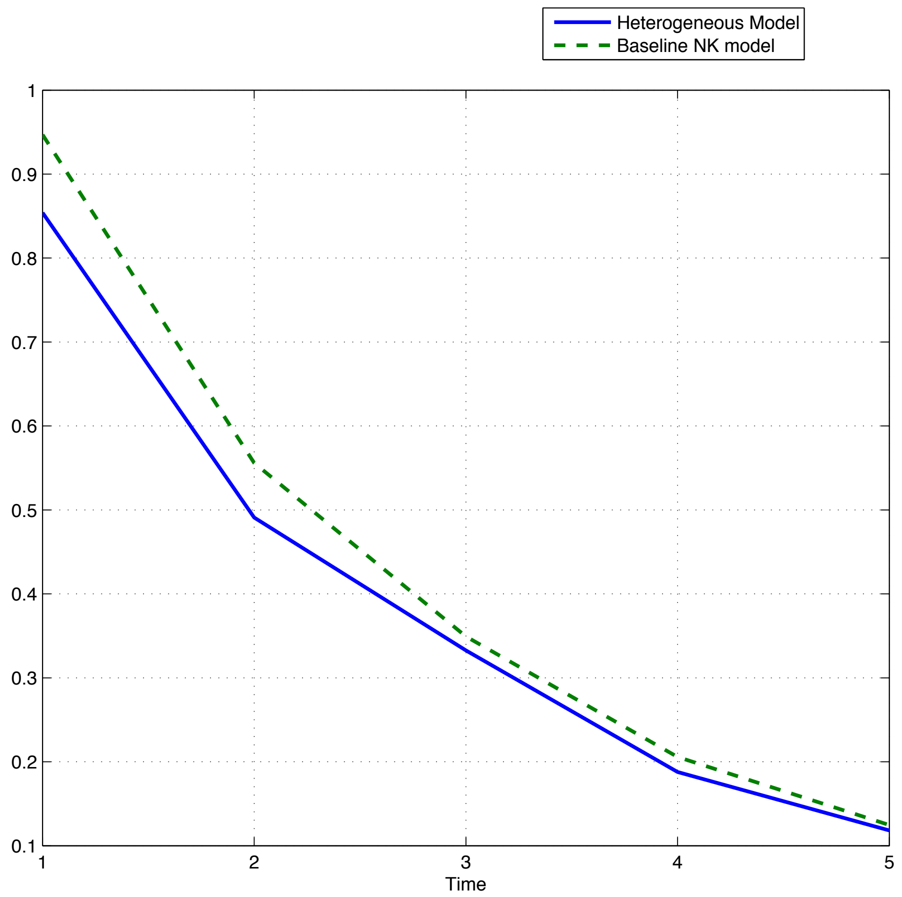

Figure 1 shows that, given the values of the parameters assumed in the baseline calibration, the reaction of the interest rate in the heterogeneous economy is less aggressive than in the baseline economy. In other words, the concern for interest rate stabilization is more relevant in the heterogeneous economy than in the fully sticky economy.

However, in the previous section, we saw that higher values of σ imply that this result can be reversed, with the reaction of the interest rate being less aggressive in the baseline than in the heterogeneous economy. Hence, figure 2 reports results on the reaction of the nominal interest rate in the heterogeneous and baseline New Keynesian economies under different values of the parameter σ. In particular, figure 2 reports

$$Imp_1 = \frac{i_1^{Het}}{i_1^{NK}}$$

which is the ratio of the reaction of the interest rate in the heterogenous economy relative to the corresponding reaction in the baseline New Keynesian economy at the time when the shock is realized (t=1). A value smaller than 1 in this ratio indicates that, on impact, monetary policy is less aggressive in the heterogeneous than in the baseline economy.

Figure 2: Ratio impact response interest rate in heterogeneous versus baseline NK model

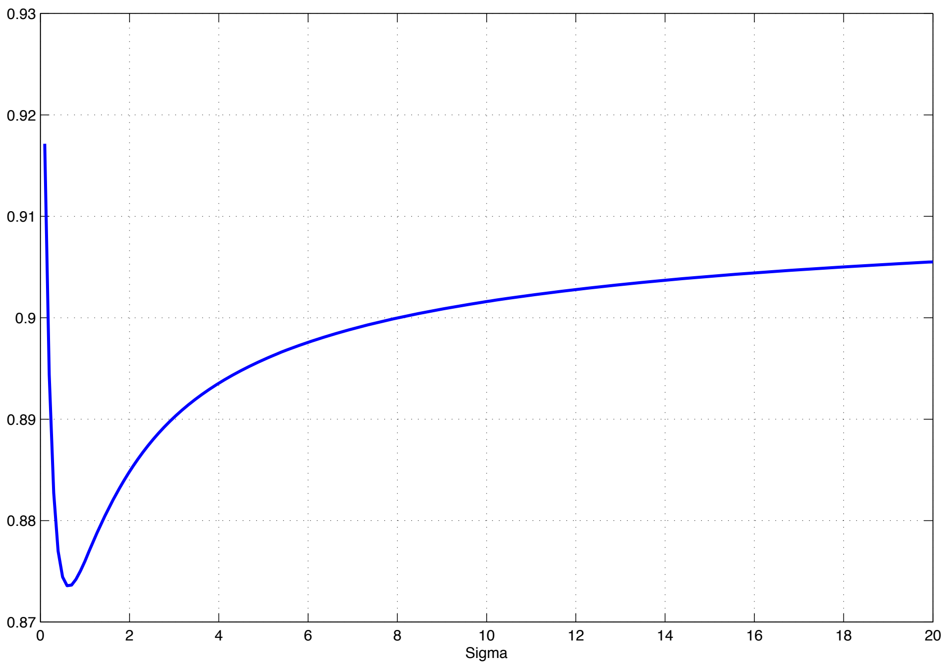

Figure 2 shows that the reaction of the interest rate in the model with heterogeneity is always less aggressive than in the case where all prices are sticky. This also happens for very high values of σ, although the ratio is monotonically increasing for values of σ higher than one. This result partly mirrors those obtained with the non-inertial plan: for high values of sigma, the relative importance of macroeconomic stabilization with respect to interest rate stabilization increases more in the model with heterogeneous price setters than in the baseline New Keynesian model. However, the numerical simulations show that, conditional to the values assigned to the other parameters of the models in the calibration, the result of a less aggressive reaction of the interest rate in the heterogeneous economy remains robust for higher values of σ.

Appendix 2 reports results for the ratio when several other parameters of the model are assumed to vary from the values assigned in the baseline calibration. The outcome of these simulations is that the reaction of the nominal interest rate in the heterogeneous economy remains robustly less aggressive than in the baseline New Keynesian economy.

In conclusion, the result of the attenuation in the reaction of the interest rate to shocks in the model with heterogeneous price setters is quite robust. This supports the idea that the puzzling caution in the conduct of monetary policy by actual central banks may be partly explained by the fact that they take into account the sectorial heterogeneity in the frequency of price adjustment.

5 Conclusion

This paper analyzes the optimal monetary policy problem of a central bank facing sectorial heterogeneity in price setting. The focus is on the reaction of the interest rate to structural shocks. Comparing this reaction in an economy with heterogeneity with one in which all goods have sticky prices, it is found that a central bank is much more likely to react less aggressively to shocks in the heterogeneous economy. Since actual central banks take into account the sectorial heterogeneity in price setting mechanisms in that they derive and monitor core inflation indices excluding the most volatile components of consumer prices, this finding might help to explain why their reaction to shocks is considered too cautious.

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Appendix 1: Derivation of the welfare criterion

This appendix expounds the steps taken to derive the loss function of the central bank by means of a second order approximation to social welfare, defined by aggregating the utility of the agents:

$$E_0\sum_{T=0}^{\infty}\beta^T\left[U(C_{t+T},B_{t+T})+K(\frac{M_{t+T}}{P_{t+s}},D_{t+T})-\int_0^{\gamma}V(H^i_{s,t+T}(i),E_{s,t+T})di+-(1-\gamma)V(H_{f,t+T},E_{f,t+T})\right] \tag{A.63}$$

For simplicity, define for the general variable A_t,

$$\breve{a}_t = A_t - A$$

where A is the steady state of A_t, and

$$a_t = \ln(\frac{A_t}{A})$$

Then, the following useful relationship holds

$$a_t = A(a_t + \frac{1}{2}a_t^2) + O(3) \tag{A.64}$$

Approximation of $U(C_t, B_t) = U(Y_t, B_t)$

$$\breve{U}(Y_t, B_t) = U_C\breve{Y}_t + \frac{1}{2}U_{CC}\breve{Y}_t^2 + U_{CB}\breve{Y}_t\breve{B}_t + t.i.p. + O(3)$$

where t.i.p. stands for terms independent of policy. Exploiting (4.61) and noticing that $b_t = -\frac{U_{CB}}{U_{CC}C}B_t$ and $\sigma = -\frac{U_{CC}C}{U_C}$, we get

$$\breve{U}(Y_t, B_t) = \bar{Y}U_c\left[y_t + \frac{1}{2}(1-\sigma)y_t^2 + \sigma b_t y_t\right] + t.i.p. + O(3) \tag{A.65}$$

Approximation of $K(\frac{M_t}{P_t}, D_t)$

Assume, ss in Woodford (2003), that the transaction technology implies that there is satiation in real money balances at a finite positive level. Moreover, all the conditions on the partial derivatives of the utility function with respect to consumption and real balances invoked in Woodford (2003), pg 422 are verified. Besides that, assume that $\Delta = \frac{I}{1+I}$ is small and can be treated as an expansion parameter. This leads to the conclusion that the economy is well approximated in a neighborhood of the steady state close to the point in which agents are satiated in real balances.

$$\breve{K}(M_t/P_t, D_t) = K_M\breve{m}_t + \frac{1}{2}K_{MM}\breve{m}_t^2 + K_{MD}\breve{m}_t\breve{d}_t + t.i.p. + O(3)$$

which gives

$$\breve{K}(M_t/P_t, D_t) = K_M M\left[m_t + \frac{1}{2}m_t^2\right] + \frac{1}{2}K_{MM}M^2m_t^2 + K_{MD}m_td_t + t.i.p. + O(3) \quad (A.66)$$

Putting (4.62) and (A.63) together we get

$$\breve{U}(Y_t, B_t) + \breve{K}(M_t/P_t, D_t) =$$
$$] \; YU_C\left[y_t + \frac{1}{2}(1-\sigma)y_t^2 + \sigma b_t y_t + \frac{MK_M}{YU_C}\left(m_t + \frac{1}{2}(1-\sigma_m)m_t^2 + \frac{K_{MD}}{K_M}m_td_t\right)\right] + t.i.p. + O(3) \quad (A.67)$$

where $\sigma_m = -\frac{K_{MM}M}{K_M}$. Defining $S_m = \frac{MK_M}{YU_C}$ and $\varepsilon_t = \frac{1}{\sigma^m}\left[\frac{K_{MD}}{K_M}D_t - \frac{U_{CB}}{U_C}B_t\right]$, this expression can be rewritten as

$$\breve{U}(Y_t, B_t) + \breve{K}(M_t/P_t, D_t) =$$
$$YU_C\left[y_t + \frac{1}{2}(1-\sigma)y_t^2 + \sigma b_t y_t + S_m m_t + \frac{1}{2}S_m(1-\sigma_m)m_t^2 + S_m\sigma_m\varepsilon_t m_t + S_m\sigma b_t m_t\right] + t.i.p. + O(3) \quad (A.68)$$

Now, notice that from (2.9), $S_m = O(\Delta)$. Moreover, $S_m\sigma_m = -\frac{K_{MM}m^2}{U_CY}$. Hence

$$S_m m_t + \frac{1}{2}S_m(1-\sigma_m)m_t^2 + S_m\sigma_m\varepsilon_t m_t + S_m\sigma b_t m_t = S_m m_t - \frac{1}{2}S_m\sigma_m m_t^2 + S_m\sigma_m\varepsilon_t m_t + O(3)$$

which, after some tedious calculations, gives

$$S_m m_t - \frac{1}{2}S_m\sigma_m m_t^2 + S_m\sigma_m\varepsilon_t m_t + O(3) = S_m m_t - \frac{1}{2}S_m\sigma_m(m_t - \varepsilon_t)^2 + t.i.p. + O(3) \quad (A.69)$$

If (steady state) money velocity is defined as $v = \frac{Y}{M}$, then

$$S_m\sigma_m = -\frac{K_{MM}M^2}{U_CY} = \frac{K_M}{U_C}v^{-1}\left(-\frac{K_{MM}M}{K_M}\right)$$

that can be used together with $\eta_i = \frac{U_C}{K_M}\frac{1}{(1+I)\sigma^m}$ to show that

$$\eta_i v = (S_m\sigma_m)^{-1} + O(\Delta)$$

This relationship, coupled with equation (A.66) implies that

$$\breve{U}(Y_t, B_t) + \breve{K}(M_t/P_t, D_t) =$$
$$YU_C\left[y_t + \frac{1}{2}(1-\sigma)y_t^2 + \sigma b_t y_t + S_m m_t - \frac{1}{2}(\eta_i v)^{-1}(m_t - \varepsilon_t)^2\right] + t.i.p. + O(3) \quad (A.70)$$

Finally,

$$m_t = \eta_y y_t - \eta_i i_t + \varepsilon_t$$

where $\eta_y{=}\frac{\sigma}{\sigma^m}$, and η_y and η_i are $O(\Delta)$. Hence,

$$\begin{aligned} &\breve{U}(Y_t, B_t) + \breve{K}(M_t/P_t, D_t) = \\ &YU_C\left[y_t + \frac{1}{2}\left(1-\sigma\right)y_t^2 + \sigma b_t y_t - S_m \eta_i i_t - \frac{1}{2}\eta_i v^{-1} i_t^2\right] + t.i.p. + O(3) \end{aligned} \tag{A.71}$$

Approximation of $-(1-\gamma)V(H_{f,t}, E_{f,t})$

$$\begin{aligned} &\breve{V}(H_{f,t}, E_{f,t}) = \\ &V_{Y_f} Y_f \left(y_{f,t} + \frac{1}{2} y_{f,t}^2\right) + Y_f V_{Y_f A_f} y_{f,t} a_{f,t} + Y_f V_{Y_f E_f} y_{f,t} e_{f,t} + \frac{1}{2} Y_f^2 V_{Y_f Y_f} y_{f,t}^2 + t.i.p. + O(3) \end{aligned} \tag{A.72}$$

Notice that for $j = s, f$

$$V_{Y_j} = V_{H_j}\frac{1}{A} = V_{H_j} \tag{A.73}$$

$$V_{Y_j Y_j} = V_{H_j H_j}\frac{1}{A^2} = V_{H_j H_j} \tag{A.74}$$

$$V_{Y_j A_j} = -(V_{H_j H_j} H_j + V_{H_j}) \tag{A.75}$$

Moreover,

$$\frac{V_{H_j H_j}}{V_{H_j}} H_j = \omega \text{ and } \frac{V_{H_j E_j}}{V_{H_j}} = \eta \tag{A.76}$$

which are equal in both sectors and

$$\frac{V_{H_s H_s}}{V_{H_s}} H_s = \frac{V_{H_f H_f}}{V_{H_j}} H_f = \frac{V_{HH}}{V_H} H \tag{A.77}$$

Then

$$\breve{V}(H_{f,t}, E_{f,t}) = V_H H\left[y_{f,t} + \frac{1}{2}(1+\omega) y_{f,t}^2 - (1+\omega) y_{f,t} a_{f,t} + \eta y_{f,t} e_{f,t}\right] + t.i.p. + O(3) \tag{A.78}$$

Approximation of $-\int_0^\gamma V(H^i_{s,t}(i), E_{s,t})di$

Before working out the approximation of this term, notice that

$$y_{s,t} = E_i\left(y_{s,t}(i)\right) + \frac{1}{2}\frac{\vartheta - 1}{\vartheta} Var_i\left(y_{s,t}(i)\right) \tag{A.79}$$

where E_i and Var_i are, respectively, the mean and the variance of $y_{s,t}(i)$ with respect to the cross section of sticky price firms. Then,

$$- \int_0^\gamma \breve{V}(H^i_{s,t}(i), E_{s,t})di = -\gamma\Big[V_H H E_i\left(y_{s,t}(i)\right) +$$
$$+\frac{1}{2}(V_H H + V_{HH}H^2)E_i^2 - V_H H E_i\left(y_{s,t}(i)\right) A_{s,t} + V_H H \eta E_i\left(y_{s,t}(i)\right) e_{s,t} + t.i.p. + O(3)$$

which, by using (A.76) in order to eliminate the expectation term $E_i\left(y_{s,t}(i)\right)$, becomes

$$\begin{aligned} &- \int_0^\gamma \breve{V}(H^i_{s,t}(i), E_{s,t})di = -\gamma V_H H\Big\{y_{s,t} - \frac{1}{2}\frac{\vartheta - 1}{\vartheta} Var_i\left(y_{s,t}(i)\right) \\ &+ \frac{1}{2}(1+\omega)E_i\left[\left(y_{s,t}(i)\right)^2\right] - (1+\omega)y_{s,t}a_{s,t} + \eta y_{s,t}e_{s,t}\Big\} + t.i.p. + O(3) \end{aligned} \tag{A.80}$$

Exploiting the definition of variance and again (A.76) in order to eliminate negligible terms, $E_i\left[\left(y_{s,t}(i)\right)^2\right]$ can be further simplified to obtain

$$\begin{aligned} &- \int_0^\gamma \breve{V}(H^i_{s,t}(i), E_{s,t})di = -\gamma V_h h\Big\{y_{s,t} - \frac{1}{2}\frac{1 + \vartheta\omega}{\vartheta} Var_i\left(y(i)_{s,t}\right) \\ &+ \frac{1}{2}(1+\omega)y^2_{s,t} - (1+\omega)y_{s,t}a_{s,t} + \eta y_{s,t}e_{s,t}\Big\} + t.i.p. + O(3) \end{aligned} \tag{A.81}$$

Approximation of flow utility function

$$\begin{aligned} &\breve{U}\ (C_t, B_t) + \breve{K}(\frac{M_t}{P_t}, D_t) - \int_0^\gamma \breve{V}(H^i_{s,t}(i), E_{s,t})di - (1-\gamma)\breve{V}(H_{f,t}, E_{f,t}) = \\ &YU_C\left[y_t + \frac{1}{2}\left(1-\sigma\right)y_t^2 + \sigma b_t y_t - S_m \eta_i i_t - \frac{1}{2}\eta_i v^{-1} i_t^2\right] \\ &-\gamma V_h h\Big\{y_{s,t} - \frac{1}{2}\frac{1+\vartheta\omega}{\vartheta}Var_i\left(y(i)_{s,t}\right) + \frac{1}{2}(1+\omega)y^2_{s,t} - (1+\omega)y_{s,t}a_{s,t} + \eta y_{s,t}e_{s,t}\Big\} \\ &- (1-\gamma)V_H H\left[y_{f,t} + \frac{1}{2}(1+\omega)y^2_{f,t} - (1+\omega)y_{f,t}a_{f,t} + \eta y_{f,t}e_{f,t}\right] \end{aligned} \tag{A.82}$$

$Var_i\left(y_{s,t}(i)\right)$

From

$$Y_{st}(i) = \frac{1}{\gamma}\left(\frac{P_{st}(i)}{P_{st}}\right)^{-\vartheta} Y_{st}$$

one can show that

$$Var_i\left(y_{s,t}(i)\right) = \vartheta^2 Var_i\left(p_{s,t}(i)\right)$$

The term on the right hand side represents the dispersion of prices in the sticky price sector due to the staggered price setting mechanism described by Calvo. Intuitively, this term depends on inflation in the sticky price sector, since if inflation in the sticky price sector is zero at each time, no dispersion would be observed among relative prices set by sticky price setters. Define

$$\bar{p}_{s,t} = E_i \ln p_{s,t}(i)$$

and notice that

$$\begin{aligned} \bar{p}_{s,t} - \bar{p}_{s,t-1} &= E_i\left[\ln p_{s,t}(i) - \bar{p}_{s,t-1}\right] \\ &= \xi E_i\left[\ln p_{s,t-1}(i) - \bar{p}_{s,t-1}\right] + (1-\xi)\left[p^*_{s,t} - \bar{p}_{s,t-1}\right] \\ &= (1-\xi)\left[p^*_{s,t} - \bar{p}_{s,t-1}\right] \end{aligned} \tag{A.83}$$

The term we are interested in is

$$\Psi_t = Var_i\left(lnP_{s,t}(i) - \bar{p}_{s,t-1}\right) \tag{A.84}$$

Notice that

$$\Psi_t = \xi E_i\left[\left(\ln P_{s,t-1}(i) - \bar{p}_{s,t-1}\right)^2\right] + (1-\xi)\left[\left(\ln P^*_t - \bar{p}_{s,t-1}\right)^2\right] - \left(\bar{p}_{s,t} - \bar{p}_{s,t-1}\right)^2 \tag{A.85}$$

which reduces to

$$\Psi_t = \xi\Psi_{t-1} + \frac{\xi}{1-\xi}\left(\bar{p}_{s,t} - \bar{p}_{s,t-1}\right)^2 \tag{A.86}$$

Now, notice also that

$$P_{s,t} = \left[\frac{1}{\gamma}\int_0^\gamma P_{s,t}(i)^{1-\vartheta} di\right]^{\frac{1}{1-\vartheta}}$$

implies

$$\ln P_{s,t} = \frac{1}{1-\vartheta}\ln\left[E_i\left(p_{s,t}(i)^{1-\vartheta}\right)\right] \tag{A.87}$$

which, in turn, implies that

$$p_{s,t} = \bar{p}_{s,t} + O(2) \tag{A.88}$$

Hence,

$$\Psi_t = \xi \Psi_{t-1} + \frac{\xi}{1-\xi} \pi_{s,t}^2 + O(3)$$

which, by backward iteration, gives

$$\Psi_t = \xi^{t+1} \Psi_{-1} + \frac{\xi}{1-\xi} \sum_{k=0}^{t} \xi^{t-k} \pi_{s,k}^2 + O(3)$$

and, finally,

$$\sum_{t=0}^{\infty} \Psi_t = \frac{\xi}{(1-\xi)(1-\xi\beta)} \sum_{t=0}^{\infty} \beta^t \pi_{s,t}^2 + t.i.p. + O(3) \tag{A.89}$$

Final Form of the loss function

Notice that, from the first order conditions

$$V_H = U_C \Rightarrow V_H H = U_C Y$$

Moreover,

$$y_t = \gamma y_{s,t} + (1-\gamma) y_{f,t}$$

$$y_t^n = \gamma y_{s,t}^n + (1-\gamma) y_{f,t}^n$$

and

$$g_t = y_t - y_t^n$$

this implies that the second order approximation of the flow utility function is equal to

$$-\frac{YU_c}{2}\left\{(\omega+\sigma)g_t^2+\gamma(1-\gamma)(1+\omega)\left[y_{s,t}-y_{f,t}-c(y_{s,t}-y_{f,t})\right]^2+2S_m\eta_i i_t+\eta_i v^{-1} i_t^2+\gamma\vartheta(1+\omega\vartheta)Var_i(p(i)_{s,t})\right\}+t.i.p.+O(3)$$

The term $2S_m \eta_i i_t + (v\eta_i)^{-1} i_t^2$ can be further simplified by noting that

$$S_m v = \frac{K_M}{U_C} = \frac{I}{(1+I)} = \Delta$$

and, then,

$$2S_m \eta_i i_t + \eta_i v^{-1} i_t^2 = \eta_i v^{-1} (i_t + \Delta)^2 + t.i.p. = (v\eta_i)^{-1} (i_t - i^*)^2$$

where i^*=-Δ. Moreover, further tedious algebraic manipulations show that

$$[y_{s,t} - y_{f,t} - c(y_{s,t} - y_{f,t})]^2 = \frac{(1-\gamma)(1+\omega)}{\gamma} c g_t^2$$

and, then

$$L_t = E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_{st}^2 + \lambda_i (i_t - i^*)^2 + \lambda_g g_t^2 \right] \quad \text{(A.90)}$$

where

$$\lambda_i = \frac{v^{-1} \eta_i (1-\xi)(1-\xi\beta)}{\xi\gamma\vartheta(1+\omega\vartheta)} \quad \text{(A.91)}$$

and

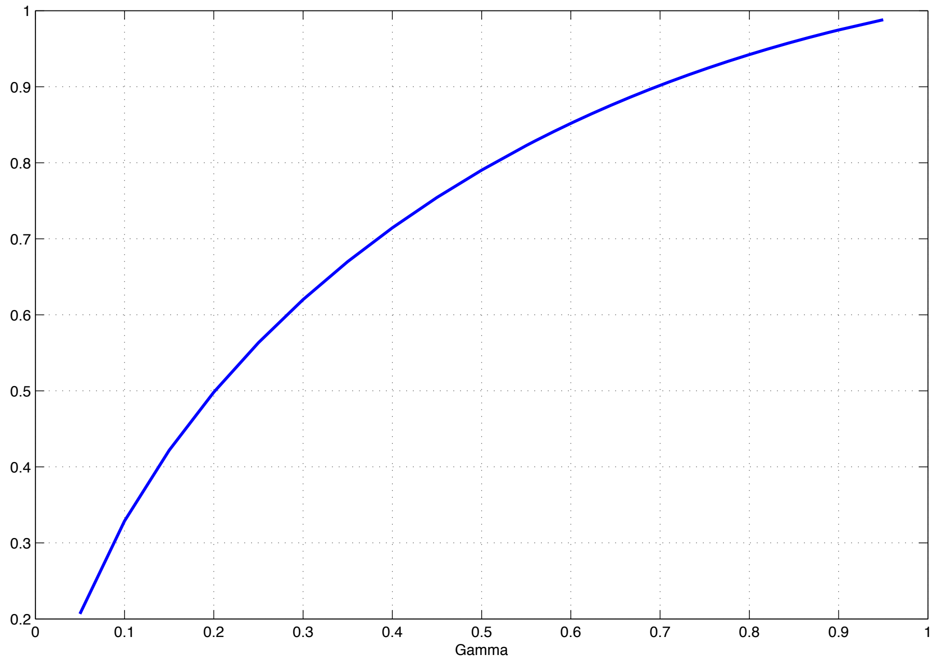
$$\lambda_g = \frac{(\omega+\sigma)(1-\xi)(1-\xi\beta)}{\xi\gamma^2\vartheta(1+\omega\vartheta)} \quad \text{(A.92)}$$

disregarding scaling parameters, terms independent of policy and $O(3)$ terms.

Appendix 2: Robustness checks

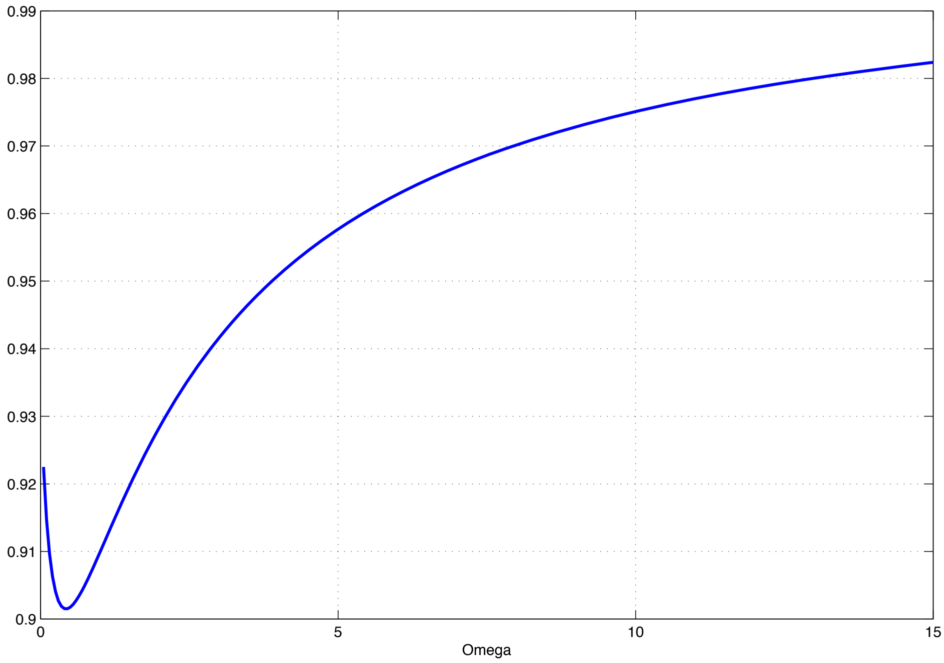
This appendix shows how the ratio of the impact reaction in the interest rates in the heterogenous economy relative to the baseline New Keynesian economy varies when the values of the parameters depart from those in the baseline calibration.

Figure 3: Ratio impact response interest rate in heterogeneous versus baseline NK model



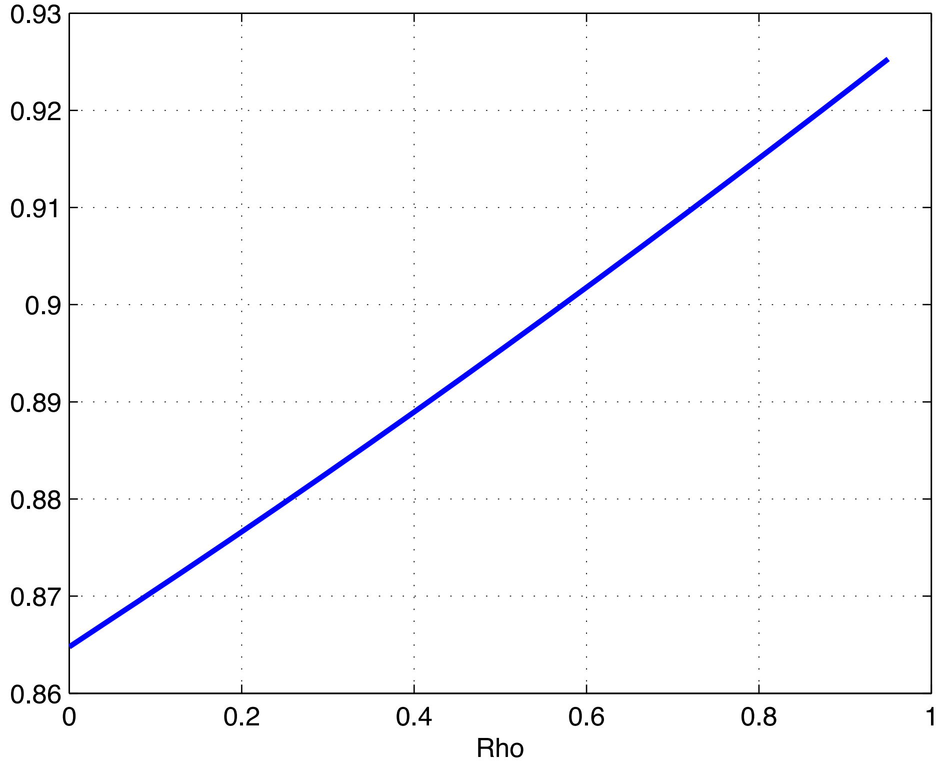
Obviously, when the percentage of firms with sticky price tends to 100%, the outcomes in the heterogeneous and baseline New Keynesian economy tend to become the same.

Figure 4: Ratio impact response interest rate in heterogeneous versus baseline NK model



High values of ω reduce the differences in the reaction of the interest rate to shocks in the heterogeneous economy compared to the baseline. However, the result does not disappear even for extremely high values of this parameter. The interpretation for the convergence of the results in the heterogeneous economy to those for the baseline economy is that, while the sensitivity of inflation to the output gap does not vary much with ω, the sensitivity of aggregate activity to the interest rate decreases a lot in the heterogeneous economy (conditional to the value assigned to σ) when ω increases. Then, the central bank engages more and more in macroeconomic stabilization relative to the baseline economy.

Figure 5: Ratio impact response interest rate in heterogeneous versus baseline NK model



The outcome of this simulation shows that the result of attenuation in the monetary policy reaction in a heterogeneous economy does not depend on the degree of autocorrelation ρ assumed for the stochastic shock in the economy.

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