EUROPEAN CENTRAL BANK

EUROSYSTEM

Working Paper Series

Heinrich Kick

Pricing of bonds and equity when the zero lower bound is relevant

No 1992 / January 2017

Abstract

This paper investigates the joint dynamics of nominal bond yields, real bond yields and dividend yields from the 80s up to the aftermath of the financial crisis by mapping them on a set of macro factors.

It builds on an existing discrete time affine Gaussian model of the term structure model of nominal bonds, real bonds and equity and extends it by three important innovations. Firstly, allowing for structural shifts in inflation expectations. Secondly, accounting for the relevance of the zero lower bound in the period after 2008 by modelling a so-called shadow rate and deriving asset prices by explicitly considering the zero lower bound. Finally, calculating the standard errors to correctly capture the multi-step nature of the estimation process, which results in substantially larger standard errors than previously reported for the model. We achieve statistically significant risk premia by imposing restrictions on the matrix of risk premia.

Taken together, these modifications allow to better model asset prices also during the financial crisis and the ensuing economic environment of sluggish growth, low inflation rates, interest rates close to zero and quantitative easing.

Keywords: Asset pricing, Zero lower bound, Financial crisis
JEL Classification: C13, E43, G12

Non-technical summary

This work adapts an empirical macro finance model for the joint dynamics of nominal bonds, real bonds and equity prices to an environment where the interest rate cannot fall below a lower bound.

Joint pricing of bonds and stocks has received increasing attention, recognising the need for a better understanding of the joint dynamics of asset prices. The stylised fact that bond yields and dividend yields are highly correlated, which is spelled out in the FED model, constitutes the empirical evidence that motivates investigating the joint dynamics. In the light of this apparent co-movement it is rather surprising that empirical models to price bonds and equity jointly are relatively scarce. The idea behind joint pricing models is that different asset prices, say stock prices and bond prices, are driven by the same underlying factors causing a co-movement of these prices that can only be captured when the different asset prices are modelled jointly.

In the joint pricing model as proposed in (Ang and Ulrich 2012) which serves as our starting point the dynamics of nominal bonds, real bonds and equity are explained via macro factors, namely inflation, the output gap, monetary policy shocks and dividend growth. The latter variable does not affect bond prices or the other factors but is also driven by the other factors and thereby creates the link between bond and equity prices. We extend the original sample to include post-crisis years so that our sample horizon spans 1982 - 2015. This extension motivates the adaption of the model to an environment when the lower bound for interest rates becomes relevant, as the fed funds rate is constraint by 0 during the years after the financial crisis. While we estimate the model only with US data this issue is also highly relevant and contemporary in Europe and Japan.

Taken the joint pricing model as proposed in (Ang and Ulrich 2012) as a starting point, we enhance it by introducing shadow rates as pioneered by (Krippner 2011) to account for the zero lower bound (ZLB). The reason for introducing the lower bound is that economic agents can hold cash instead of parking money at a negative rate, and therefore the storing and inconvenience costs of holding cash constitute a floor on the interest rate. The idea behind a shadow rate model is to declare the interest rate of the original model which can go below the lower bound as a shadow rate and derive the actual interest rate from that shadow rate by setting it to the lower bound whenever the shadow rate falls below this bound. Through the distance to the lower bound the level of the shadow rate thus contains information of how long the interest rate is expected to stay at the lower bound. There is a fast growing literature relying on this mechanism to adjust affine bond pricing models to situations where the lower bound is relevant. The lower bound directly impacts the shape of the entire yield curve when the short rate is close to its lower bound. As the short rate cannot become lower than the bound, but remain at this level for some time, the yield curve becomes compressed at the lower end, but the long end still can react to the inflow of macroeconomic news. Such news essentially affect agents' expectations of when the short rate might rise in the future rather than by

how much it moves now.

In addition we recognise that inflation displays structural shifts over the considered time horizon which we argue to be the consequence of paradigm shifts in monetary policy that have to be explicitly accounted for in the model. We thus perform a trend-cycle decomposition of inflation expectations to separate the persistent innovations from cyclical fluctuations.

We find that modifying the model results in a far better description of long term inflation expectation formation and a closer model fit. Adding the ZLB does not change model fit dramatically for past data, but it removes the possibility that simulated interest rate forecasts violate the ZLB, which is shown to be an important improvement. The modifications are needed to make the original model suitable to the current low interest rate environment. Given the current monetary policy stance and the shape of the forward curve, pricing models that take into account the ZLB will only increase in popularity.

1 Introduction

This paper investigates the joint dynamics of nominal bond yields, real bond yields and dividend yields from the 80s up to the aftermath of the financial crisis by mapping them on a set of macro factors.

For this purpose we extend the discrete time Gaussian affine term structure model (GATSM) of nominal bonds, real bonds and equity as proposed in (Ang and Ulrich 2012) by three important innovations. Firstly, we account for structural shifts in inflation expectations. Secondly, we include the relevance of the zero lower bound (ZLB) in the period after 2008 by modelling a so-called shadow rate and deriving asset prices by explicitly considering the ZLB[1]. Finally, we propose an alternative way of calculating the standard errors to correctly capture the multi-step nature of the estimation process, which results in substantially larger standard errors than previously reported for the model. We propose a simplification of the model that achieves statistical significant risk premia by restricting the matrix of risk premia based on economic reasoning.

Taken together, these modifications allow to better model asset prices also during the financial crisis and the ensuing economic environment of sluggish growth, low inflation rates, interest rates close to 0 and quantitative easing.

Allowing for structural shifts in inflation expectations addresses the unrealistic persistence in inflation under the risk neutral expectation by differentiating between cyclical variations with only moderate persistence and structural shifts that are permanent. We find empirical evidence that agents actually perform such a decomposition, by comparing the short term 1 year ahead forecasts with the 10 year ahead inflation forecasts, and the modification results in a roughly 15-20% better model fit.

Modelling the shadow rate explicitly and deriving the terms structures of nominal yields, real yields and equities transfers the advances in yield curve modelling pioneered by (Krippner 2011) to the pricing of bonds and equities in this particular macro-finance framework. It remedies the model of unrealistic agent's expectations of the probability of negative short rates when the actual short rate is approaching the ZLB.

Computing bootstrapped standard errors instead of relying on the maximum likelihood based standard errors used in the original paper recognises the fact that errors accumulate over the each step of the multi-step estimation process. This may appear a technicality, but in fact this changes the main conclusions about statistical significance of the model parameters in that it renders most of the risk premia insignificant. We address this problem by proposing a more parsimonious specification of the model with zero-restrictions on some of the risk premia based on economic reasoning. This restores statistically significant risk premia even when the standard errors are correctly calculated.

The applications of such an estimated model are manifold, as shown in the results section: it can be used to decompose the term structure of expected equity returns into

[1] Note that the term ZLB is used as synonym for the lower bound of interest rates due to the availability of cash - owing to technical reasons and practical inconvenience of holding cash, this bound is not necessarily equal to 0.

the real short rate, a real duration premium, expected inflation and the inflation risk premium and a real cash flow premium, as done in (Ang and Ulrich 2012). Furthermore it can be used to decompose the variations of each of these term structures into the contribution from the macro factors via variance decomposition. Linear expressions can be derived to show how yields depend on the macro factors. Impulse response analysis can show how shocks to any of the macro variables impact asset prices over time. A latent equity factor can be extracted, that contains all the information reflected in equity prices that are not included in the macro factors. All these results contribute to a better understanding on how macroeconomic factors affect asset prices, in particular in the medium and long term.

The paper proceeds as follows: in section 2, the model as introduced in (Ang and Ulrich 2012) is summarised, and all innovations are explained and motivated, together with some discussion of the literature. Section 3 treats the estimation methodology as used in (Ang and Ulrich 2012) and the modifications to calculation of the standard errors we propose here. Section 5 contains details on the data used in the empirical analysis. The following sections discussed model selection aspects and the results for the preferred specification, and section 6 concludes.

2 Model Setup

The stylised fact that motivates investigating bonds and equity pricing jointly, namely the high correlation of dividend yields and the 10 year nominal bond yields referred to as the FED-model (e.g. (Bekaert and Engstrom 2010)) has been long known, yet the literature on jointly pricing bonds and equities is surprisingly recent. Notable contributions include (Lemke and Werner 2009), (Baele, Bekaert, and Inghelbrecht 2010), (Bekaert and Engstrom 2010) and (Lettau and Wachter 2011).

A large body of literature has been devoted to the class of GATSMs (see (Dai and Singleton 2000), (Dai and Singleton 2003), (Duffee 2002), (Cochrane and Piazzesi 2005), (Ang, Bekaert, and Wei 2008)). The model setup we choose follows closely (Ang and Ulrich 2012), which extends the discrete time GATSM presented in (Ang and Piazzesi 2003) by consistently adding the pricing kernel for real claims and equities.

The model evolves around a vector autoregressive model (VAR) describing the evolution of the priced factors X. $X = [g, \pi_e, f, d, L]$ where g is the output gap, π^e a measure of inflation expectations, f the residuals of a Taylor rule regression of the short rate r on g and π_e, v signifies volatility of equity returns, d denotes dividend growth and L is a latent factor carrying information on future dividend growth. The coefficients of the VAR include a number of zero restrictions based on economic intuition. f and L are assumed to depend only on their lagged values. In addition, g and π^e do not depend on lagged values of d and L. A richer discussion of the background for these restrictions is found in (Ang and Ulrich 2012).

$$X_t = \mu + \Phi X_{t-1} + \Sigma\epsilon_t \tag{1}$$

The identification of Σ is achieved assuming orthogonality for the residuals of f, d and l to all other residuals, leaving the residuals of g and π^e potentially correlated among each other. Shock identification on this subsystem is achieved relying on short-term restrictions implying that the relevant submatrix of Σ is obtained by a Cholesky-decomposition of the relevant submatrix of the covariance matrix of the residuals. g, π^e and f are centred on 0.

We have $r_t^{\$} = \delta_0^{\$} + \delta_1^{\$} X_t$, where the coefficients are defined from the Taylor rule regression mentioned above.

Inflation depends on lagged inflation, X_{t-1} and ϵ_t and such that

$$\pi_t = \pi_c + \pi_{t-1}^e + \Sigma^{\pi'}\epsilon_t + \sigma_\pi \epsilon_t^\pi \tag{2}$$

The inflation dynamics is needed in order to link the nominal and real pricing kernels.

The nominal and real risk premia are assumed to be affine in X:

$$\lambda_t^{\$} = \lambda_0^{\$} + \lambda_1^{\$} X_t \tag{3}$$

and

$$\lambda_t^r = \lambda_0^r + \lambda_1^r X_t \tag{4}$$

where $\lambda_0^r = \lambda_0^{\$}$ and $\lambda_1^r = \lambda_1^{\$}$.

Both the nominal and real pricing kernel take standard exponential form

$$M_{t+1}^{\$} = \exp\left(-r_t^{\$} - \frac{1}{2}\lambda_t^{\$'}\lambda_t^{\$} - \lambda_t^{\$'}\epsilon_{t+1}\right) \tag{5}$$

and

$$M_{t+1}^r = \exp\left(-r_t - \frac{1}{2}\lambda_t^{r'}\lambda_t^r - \lambda_t^{r'}\epsilon_{t+1}\right) \tag{6}$$

where r_t denotes the real short rate which is related to X_t via

$$r_t = \delta_0^r + \delta_1^{r'} X_t \tag{7}$$

and

$$\delta_0^r = \delta_0^{\$} - \pi_c - \frac{1}{2}\Sigma^{\pi'}\Sigma^\pi + \Sigma^{\pi'}\lambda_0^{\$} - \frac{1}{2}\sigma_\pi^2 \tag{8}$$

and

$$\delta_1^r = \delta_1^{\$} - e_2 + \left(\lambda_1^{\$'}\Sigma^\pi\right) \tag{9}$$

with e_i being a column vector of zeros and a 1 in the ith place.

This implies a spread between nominal and real rate that consists of expected inflation, an inflation risk premium, and a part due to the Jensens's inequality:

$$r_t^\$ - r_t = \underbrace{\pi_c + e_2' X_t}_{\text{expected inflation}} \underbrace{-\Sigma^{\pi'}\lambda_0^\$ - (\Sigma^{\pi'}\lambda_1^\$)' X_t}_{\text{inflation risk premium}} + \underbrace{\frac{1}{2}\Sigma^{\pi'}\Sigma^\pi}_{\text{Jensen's inequality}} \tag{10}$$

Note that the real short rate is not observed empirically, but derived endogenously by the model.

As dividend growth is expressed in real terms, the pricing kernel for equities is the real pricing kernel.

2.1 Pricing nominal bonds, real bonds and equities within the model

The pricing of nominal bonds within such an affine structure has been first derived in (Ang and Piazzesi 2003), the following briefly summarises how the prices are obtained. Let $P_t^\$$ be a nominal zero-coupon bond of maturity n at time t. This price can be iteratively defined as

$$P_t^\$(n) = E_t\left[M_{t+1}^\$ P_{t+1}^\$(n-1)\right] \tag{11}$$

. Alternatively this can be expressed under the risk-neutral pricing measure Q:

$$P_t^\$(n) = E_t^Q\left[\exp\left(-\sum_{i=0}^{n-1} r_{t+i}^\$\right)\right] \tag{12}$$

In order to compute the Q-expectations of r, recall that $r_t = \delta_0 + \delta_1' X_t$, and note that the Q-dynamics of X_t follow

$$X_{t+1} = \mu^Q + \Phi^Q X_t + \Sigma\epsilon_{t+1}^Q \tag{13}$$

with $\mu^Q = \mu - \Sigma\lambda_0^\$$ and $\Phi^Q = \Phi - \Sigma\lambda_1^\$$. Therefore

$$P_t^\$(n) = \exp(A_n^\$ + B_n^{\$\prime} X_t) \tag{14}$$

where

$$A_{n+1}^\$ = A_n^\$ + B_n^{\$\prime}\mu^Q + \frac{1}{2}B_n^{\$\prime}\Sigma\Sigma' B_n^\$ + A_1^\$ \tag{15}$$

$$B_{n+1}^\$ = B_n^{\$\prime}\Phi^Q + B_1^{\$\prime} \tag{16}$$

with $A_1^\$ = -\delta_0^\$$ and $B_1^\$ = -\delta_1^\$$. Yields are recovered as

$$y_t^\$(n) = -\frac{A_n^\$}{n} - \frac{B_n^{\$\prime}}{n} X_t \tag{17}$$

The pricing of real bonds works in perfect analogy to the pricing of real bonds, working under the real risk neutral measure instead. The resulting bond prices and yields are

$$A^r_{n+1} = A^r_n + B^{r\prime}_n \mu^Q + \frac{1}{2} B^{r\prime}_n \Sigma\Sigma' B^r_n + A^r_1 \quad (18)$$

$$B^r_{n+1} = B^{r\prime}_n \Phi^Q + B^{r\prime}_1 \quad (19)$$

with $A^r_1 = -\delta^r_0$ and $B^r_1 = -\delta^r_1$ and

$$y^r_t(n) = -\frac{A^r_n}{n} - \frac{B^{r\prime}_n}{n} X_t \quad (20)$$

respectively.

Equity prices can be derived in a similar spirit, starting from

$$\frac{P^r_t}{D^r_t} = \frac{P^\$_t}{D^\$_t} = E^Q_t \left[\sum_{s=1}^{\infty} \exp\left(\sum_{k=1}^{s} d_{t+k} - r_{t+k-1} \right) \right]. \quad (21)$$

As before, we use the real risk neutral dynamics of X, noting that both d and r are functions of X. Then, the price-dividend ratio can be expressed as

$$\frac{P^r_t}{D^r_t} = \frac{P^\$_t}{D^\$_t} = \sum_{n=1}^{\infty} \exp(a_n + b'_n X_t) \quad (22)$$

where a_n and b_n follow the recursions

$$a_{n+1} = a_n - \delta^r_0 + (e_5 + b_n)' \mu^Q + \frac{1}{2}(e_5 + b_n)' \Sigma\Sigma' (e_5 + b_n) \quad (23)$$

$$b_{n+1} = -\delta^r_1 + \Phi^Q (e_5 + b_n) \quad (24)$$

where $a_1 = -\delta^r_0 + e'_5 \mu^Q + \frac{1}{2} e'_5 \Sigma\Sigma' e_5$, $b_1 = -\delta^r_1 + \Phi^{Q\prime} e_5$.

2.2 The term structures of risk premia

The expected k-period mean holding return on equity is defined as

$$E_t[R^{E,\$}_t(k)] = E_t \left[\sum_{s=1}^{k} ln \left(\frac{P^\$_{t+s} + D^\$_{t+1}}{P^\$_{t+s-1}} \right) \right] \quad (25)$$

Note that the expected k- period mean real return is defined in close analogy, and is linked to the previous expression via

$$E_t[R^{E,\$}_t(k)] = E_t[R^{E,r}_t(k)] + E_t[\sum_{s=1}^{k} \pi_{t+s}]. \quad (26)$$

The thus defined term structure of equity returns can be decomposed in the term structure of risk premia, the real short rate and expected inflation:

$$
\begin{array}{llll}
 & E_t[R_t^{E,\$}(k)] & \text{Total equity return} & \\
= & r_t & \text{Real short rate,} & r_t \\
+ & (y_t^r(k) - r_t) & \text{Real duration premium,} & DP_t(k) \\
+ & (y_t^{\$,r}(k) - y_t^r(k)) & \text{Inflation risk premium,} & IRP_t(k) \\
+ & (y_t^{\$}(k) - y_t^{\$,r}(k)) & \text{Expected inflation,} & E_t[\pi_t(k)] \\
+ & E_t[R_t^{E,\$}(k)] - y_t^{\$}(k) & \text{Real cash flow risk premium,} & CFP_t(k)
\end{array}
\tag{27}
$$

2.3 Accounting for the secular shift in inflation expectations

The model as laid out above implicitly assumes that all variables included in the VAR are stationary, as non-stationarity would imply explosive yields and equity price dividend ratios. However, both long short term and long term inflation expectations have shifted structurally over the observed horizon, due to changes in the way monetary policy is conducted. This is illustrated in Figure 1.

The early 80's are marked by a period of disinflationary monetary policy. This reflects the shift in monetary policy paradigm that occurred in October 1979 [2], abandoning the idea that monetary policy could be used to increase growth in the long term and putting inflation more in the focus of monetary policy (see (Meulendyke 1998) and (Judd and Rudebusch 1999)). The high inflation rates that prevailed in 1979 were brought down significantly during the early 80's, at the cost of a very sharp recession which marks the beginning of our observed sample. Inflation expectations then steadily decline from roughly 6.5% to 2.5% in 1998, and remain more or less stable during the later years. This is a reflection that long term inflation expectations are solidly anchored around the long term inflation target of the FED. It is striking that long term inflation expectations display a decline in parallel without any lead, which is in stark contrast to the inflation forecasts that one would obtain from a stationary VAR.

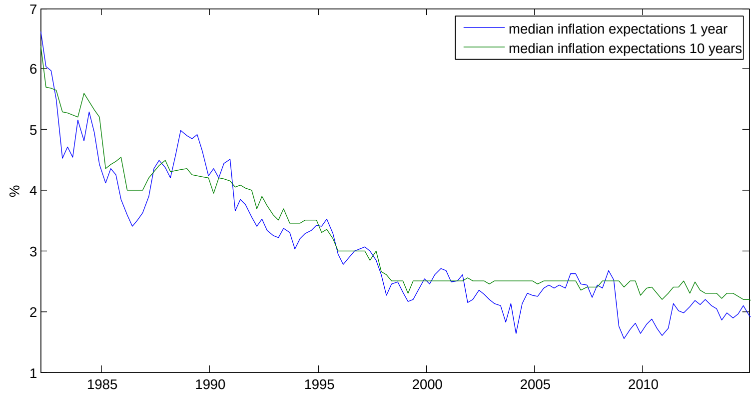
It is therefore doubtful that inflation dynamics can be adequately captured by the simple VAR specification as proposed in (Ang and Ulrich 2012). It rather appears natural to decompose short term inflation expectations into a structural trend that is reflected by 10 year inflation expectations, and a cyclical component the difference between 1 year and 10 year inflation expectations:

$$\pi_t^e = \bar{\pi}_t^e + \tilde{\pi}_t^e \tag{28}$$

where $\bar{\pi}_t^e$ denotes the 10 year inflation expectations and $\tilde{\pi}_t^e$ denotes the cyclical variation in inflation expectations. The long term trend absorbs the non-stationarity for the purpose of asset pricing we assume that agents treat the current level of the trend as the

[2] This shift follows the inclusion of price stability as a national policy goal in a 1977 amendment to the Federal Reserve Act, and the notion of natural rate of unemployment gaining traction in the academic debate.

Figure 1: 1 year and 10 year inflation expectations



The figure illustrates how the long term inflation forecast mirrors the "big picture variations" of the short term forecasts and only dampens out the short term variations - no mean reversion visible in 10 year inflation expectations. A stationary VAR(1) is not capable of reproducing such dynamics, motivating the need for a trend-cycle decomposition.

future long term average of inflation rates. The cyclical component can well be captured within the VAR, as it is stationary and reacts to cyclical fluctuations in the output gap in the intuitive way, i.e. $X = [g, \tilde{\pi}_e, f, v, d, L]$

The Taylor rule is left unchanged, so that the interest rate is set to respond to the sum of the structural and the cyclical component of inflation expectations. Also inflation is modelled to depend on the sum of the structural and cyclical component of inflation expectations. The yields remain an affine function of the state variables

$$y_t^{\$}(n) = -\frac{A_{n,t}^{\$}}{n} - \frac{B_n^{\$\prime}}{n} X_t \tag{29}$$

but the formerly constant element now includes the time variation of the trend inflation. It is obtained from the recursion relationship

$$A_{n+1,t}^{\$} = A_{n,t}^{\$} + B_n^{\$\prime} \mu^Q + \frac{1}{2} B_n^{\$\prime} \Sigma \Sigma' B_n^{\$} + A_{1,t}^{\$} \tag{30}$$

with $A_{1,t}^{\$} = -\delta_0^{\$} - \delta_1^{\$}[2] \bar{\pi}_t^e$.[3]

Similarly, for real yields, the trend cycle decompositions implies

$$y_t^r(n) = -\frac{A_{n,t}^r}{n} - \frac{B_n^{r\prime}}{n} X_t \tag{31}$$

[3]The notation $\delta_1^{\$}[2]$ refers to the second element in the vector $\delta_1^{\$}$ which is the coefficient of inflation expectations in the Taylor rule.

where

$$A^r_{n+1,t} = A^r_{n,t} + B^{r\prime}_n \mu^Q + \frac{1}{2} B^{r\prime}_n \Sigma\Sigma' B^r_n + A^r_{1,t} \tag{32}$$

with $A^r_{1,t} = -\delta^r_0 - \delta^r_1[2]\bar{\pi}^e_t$.
Finally, the price-dividend ratio can be expressed as

$$\frac{P^r_t}{D^r_t} = \frac{P^\$_t}{D^\$_t} = \sum_{n=1}^{\infty} \exp(a_{n,t} + b'_n X_t) \tag{33}$$

where $a_{n,t}$ follows the recursions

$$a_{n+1,t} = a_{n,t} - \delta^r_0 - \delta^r_1[2]\bar{\pi}^e_t + (e_5 + b_n)'\mu^Q + \frac{1}{2}(e_5 + b_n)'\Sigma\Sigma'(e_5 + b_n) \tag{34}$$

(35)

where $a_1 = -\delta^r_0 - \delta^r_1[2]\bar{\pi}^e_t + e'_5\mu^Q + \frac{1}{2}e'_5\Sigma\Sigma' e_5$.

2.4 Accommodating shadow rates within the model setup

The original model assumes that interest rates are an affine function of the macro factors, which is adequate for the pre-crisis times. However monetary policy has dramatically changed during recent years in response to the financial crisis. Notably, short rates reached the ZLB, at least from a practical perspective, and further monetary stimulus was achieved via large scale asset purchasing programs by the FED, targeting directly the medium and long maturities of yield curve. Traditional affine yield curve models that do not account for the ZLB result do not prevent expectations of rates below the ZLB in the future.[4] Furthermore, the shadow rate models enable to calculate the expected time to remain at the ZLB.

The approach we employ follows closely the dynamically growing literature on shadow rates, pioneered by authors such as Christensen and Rudebusch. (Krippner 2011), (Krippner 2013a), (Krippner 2013b) pioneer a tractable framework relating the adjustments in yields to the value of option prices that can be approximated within the GATSMs. (Lemke and Vladu 2014) rely on a brute-force Monte Carlo simulation to calculate the term structure when the zero bound is relevant. (Bauer and Rudebusch 2013), (Malik and Meldrum 2014) perform decompositions of the UK term structure, (Pericoli and Taboga 2015) propose a Bayesian estimation methodology for a shadow rate model.

The idea is to model a shadow short rate as an affine function of the factors, and to derive the actual short rate and the term structure from that shadow rate. The shadow rate can thus attain values below the ZLB. The short rate r is modelled as

$$r = \max\{zlb, r^s\} \tag{36}$$

[4] Note that the short rate within the original model environment is already prevented from going below the ZLB, since the monetary policy shock is extracted as a latent factor to ensure that the modelled short rate tracks the actual short rate without error.

where zlb is the bound below the actual short rate cannot go, and r^s is the shadow rate.

zlb is typically taken to be zero, but a recent study notes that policy targets have empirically not reached that bound, but rather complemented the low interest rates of levels around 10 bps by quantitative easing. It appears therefore that effectively, the ZLB can be set to values of 10 bps, and a similar value is found in (Lemke and Vladu 2014) when the bound is estimated.

The integration of the shadow rate requires some modifications to the model. Firstly, the Taylor rule is estimated via a Tobit regression instead of a linear regression, reflecting the truncation at the ZLB. This ensures that the estimation of monetary policy reaction as reflected by the shadow rate to the macro factors is not distorted by the implicit interest rate floor. Secondly, the monetary policy shock f is calculated as the difference between the truncated rate implied by the Taylor rule to the observed rate. This way, the shadow rate in the model can assume values below the bound, whenever the actual rate is at the ZLB. Effectively, a significant part of the forecast distribution of the shadow rate can be below the bound whenever the actual rate is close to the ZLB.

The lack of a closed form exact solution either requires a numerical procedure or a closed form approximation, as given in (Krippner 2013a) and (Krippner 2013b). It is rather trivial to implement a numerical solution for bond yields within the model setup, recalling that

$$P_t^{\$}(n) = E_t^Q\left[\exp\left(-\sum_{i=0}^{n-1} r_{t+i}^{\$}\right)\right] \tag{37}$$

which implies that

$$y_t^{\$*}(n) = \log\left(\frac{1}{E_t^Q\left[\exp\left(-\sum_{i=0}^{n-1} r_{t+i}^{\$}\right)\right]}\right)/n \tag{38}$$

The simulations based approach comes at the cost of high computational requirements, even after exploiting the antithetic procedure to improve accuracy of the Monte Carlo simulations. This calls for the use of the closed form solutions at least for the equity valuation, because the number of time horizons for which the Q-expectations of the interest rate process and dividend process are needed to calculate the price dividend ratio is not limited by the 20-year maturity of the longest bonds. In theory, these expectations are relevant up to infinity, but for practical purposes the infinite sum can be approximated by assuming that in the long term, the state variables revert to their mean[5]. Nevertheless, the purely simulations-based approach is not feasible to be extended to calculate the effect of the ZLB on the term structure of equity returns. This is achieved by realising that the ZLB affects the real rate in pretty much the same way as the nominal rate, so that ZLB adjustment leads to a nearly parallel shift in the real and the nominal

[5]Depending on the persistence of the macro-factors under the Q-measure, the approximation of the infinite sum in the equity valuation formula requires far more than the 80 terms corresponding to the expectations for the next 20 years.

yield curve. Hence, already for the real yield curve, the analytical solution can be taken and shifted by the difference of ZLB adjusted nominal yield curve and the nominal yield curve obtained from the analytical solution:

$$y_t^{r*}(n) = y_t^r + \left(y_t^{\$*}(n) - y_t^{\$}(n)\right) \tag{39}$$

Assuming that the ZLB does not significantly affect the covariance of the interest rate and dividend growth, the thus obtained real yields can be used for the discounting in the equity process rather than the analytical ones that ignore the ZLB. Again this is done by adding the adjustment of the

$$\frac{P_t^r}{D_t^r} = \frac{P_t^{\$}}{D_t^{\$}} = \sum_{n=1}^{\infty} \exp(a_n + b_n' X_t + + (y_t^{r*}(n) - y_t^r(n))) \tag{40}$$

In order to reflect the ZLB in term structure of expected equity returns, the adjustment is simply added, assuming that equities are priced off the term structure of nominal bonds:

$$E(r^e(n))^* = E(r^e(n)) + \left(y_t^{\$*}(n) - y_t^{\$}(n)\right) \tag{41}$$

Note that the shadow rates can be added to the model regardless of how inflation expectations are modelled. However, later we will argue that the trend-cycle decomposition in the modelling of inflation results in much more plausible Q-dynamics of the state variables and ultimately in much more plausible adjustments from the ZLB implementation.

3 Estimation Methodology

Following (Ang and Ulrich 2012) we perform a three-step estimation procedure. In the first step, the Taylor rule, the inflation process, the dividend dynamics and the p-dynamics of the state variables is estimated via ordinary least square (OLS) regression. The latent factor is set to 0 at this stage.

In a second stage the prices of risk are estimated by minimizing the sum of squared differences of model implied nominal yields, real yields and dividend yields and their empirical counterparts. Note that we have a longer history for nominal and real yields and also observe the nominal term structure for a larger range of maturities than real yields. Furthermore, there is only one dividend yield series. Since we sum over unweighted squared differences, this procedure can be expected to produce a relatively better fit to the nominal term structure than the real term structure or the dividend yield process.

In a third step, the latent equity factor is extracted, and the relevant dynamics under p are estimated. The procedure relies on having a linear approximation for the price dividend ratio of the form

$$\log(1 + DP_t) = (h_0 - d_0) + (h_1 - d_1) X_t \tag{42}$$

where h_0, d_0, h_1 and d_1 are all derived in (Ang and Ulrich 2012). For a given set of parameters, L can be extracted as

$$L_t = \frac{\log(1 + DP_t) - \log(DP^{\text{observed}})}{h_1[5] - d_1[5]}, \tag{43}$$

[6] which makes sure that the dividend yield is matched[7]. The vector of parameters to be estimated in this step is $[\Phi_{dL}, \sigma_d, \sigma_L, \lambda_d L, \Phi_{LL}]$ where Φ_{dL} is the coefficient corresponding to L in the equation governing the dividend dynamics, σ_d is the standard deviation of the residual of that equation, λ_{dL} governs the risk premium on L in the dividend process, σ_L is the standard deviation of the L-process which is specified to be purely autoregressive, and Φ_{LL} is the autoregressive parameter of L. These parameters are estimated jointly to maximise the log likelihood of the VAR $X_{t+1} = \mu + X_t\Phi$, which is given by

$$\begin{aligned} \mathcal{L}(\Phi, \Sigma) \quad = \quad & -(Tm/2)\log(2\pi) + (T/2)\log(|(\Sigma\Sigma')^{-1}|) \\ & -(1/2)\textstyle\sum_{t=1}^{T-1}[(X_{t+1} - X_t\Phi)'(\Sigma\Sigma')(X_{t+1} - X_t\Phi)] \end{aligned} \tag{44}$$

Instead of replicating the calculations of the standard errors via the score of the likelihood function as sketched in (Ang and Ulrich 2012) we calculate bootstrapped standard errors. This avoids the downward bias of the score based standard errors that is due to the presence of generated regressors (see (Pagan 1984)). The score of the likelihood function is inappropriate when the estimation is set up in various steps, with one step building on results of the previous. Furthermore, the estimation of the risk premia in step two is done with the objective of minimising the sum of squared differences to observed yields, which bears only indirectly a relationship with the likelihood function of the VAR by impacting the latent factor, so the scores of the likelihood function should not be used in any case to calculate standard errors. The extremely low standard errors of the risk premia reported in (Ang and Ulrich 2012), with t-statistics on the risk premium estimates ranging from 10 to over 100 despite t-statistics on the coefficients of the first step results in the range of 1 to 3 raise doubts about the calculations.

However, it appears to be common in the related literature to neglect the downward bias of the standard errors, for instance (Ang and Piazzesi 2003) also rely on the maximum likelihood scores that are not adjusted for the first stage uncertainty. (Lemke 2008) refers to the estimation of a similar model as calibration, probably for exactly this reason. (Hamilton and Wu 2012) point out that the estimation in several affine bond pricing models lacks robustness, citing lack of identification as the reason for this and proposing a new estimation procedure to solve this issue.

We propose to overcome the problems in calculating the standard errors related to the multi-step nature of the estimation via a bootstrap procedure. This procedure is essentially a parametric bootstrap, which is built up in several steps, mirroring the estimation

[6] $h_1[5]$ and $d_1[5]$ refer to the fifth element of h_1 and d_1 respectively.

[7] This procedure is in the spirit of (Chen and Scott 1993)

procedure.[8].

The first step is a residual bootstrap on the VAR to obtain a new vector X of the observable macro factors, including the monetary policy shock. The new interest rate series is obtained via the Taylor rule, using the newly drawn X:

$$r^{\$}_{t,new} = \delta^{\$}_0 + \delta^{\$}_1 X_{t,new} \tag{45}$$

. The new inflation series is built on the series of expected inflation π^e_{new} from the vector X, the new vector of errors of the VAR and a stationary bootstrap of the residuals of the inflation regression[9]. Finally, the new error terms of nominal yields, dividend yields and price-dividend ratio are constructed from a stationary bootstrap of the respective residuals. These error terms are added to the respective yield series constructed from the model with the original parameters for Φ and Λ, but the updated series for the macro factors X.

The newly calculated standard errors differ starkly from the standard errors in the paper, which can be replicated reasonably well by performing only the stationary bootstrap to construct new nominal yields, dividend yields and price-dividend ratios while leaving the VAR and the vector X unchanged. Virtually all risk premia become insignificant when calculated correctly.

The insignificance of risk premia motivates a more parsimonious specification of the model. There are two possibilities at hand to significantly reduce the number of parameters within this setup: a reduction of the model to its bare minimum of only two factors, π_e and f.[10] However, this implies that the central bank is modelled to set interest rates/shadow rates depending only on the one year ahead inflation expectations without paying attention to the output gap. This is hardly an appropriate characterisation of US monetary policy, both from an empirical viewpoint or considering the statutes of the FED. Note that the fit on the interest rate would still be perfect, since the information contained in the output gap would simply be absorbed into the monetary policy shock. For Europe, at least the statutes of the European Central Bank (ECB) would be consistent with such a modelling of monetary policy, even though it would be argued that the output gap or some other measure of economic activity contains additional information about future inflation expectations beyond the 1 year ahead forecast, and thus would not be neglected by the central bank. Taylor rule type regressions provide empirical evidence

[8]See also (Berkowitz and Kilian 1996), (Bühlmann 2002) and (Ruiz and Pascual 2002) for surveys on the difficulties to deal with when using bootstrapping procedure in the context of time series and financial data.

[9]A stationary bootstrap is a modification of a residual bootstrap that remains viable in the presence of auto-correlation in the error terms which could be material in the case of the residuals of the inflation regression. The bootstrapped residuals are built up of blocks of varying size of the original residuals, in order to maintain the persistence profile of the residuals. For more detail, see also (Politis and Romano 1994).

[10]π_e is needed for the connection between nominal and real interest rates, and f is needed so that the short rate can be modelled without error - otherwise the model could not be consistent, because the short rate r_t is known with certainty at t

for this line of argument (see e.g. (Blattner and Margaritow 2010)). Therefore reducing the number of factors implies a severe mis-specification of the model.

Alternatively, we can introduce zero restrictions on the matrix of risk premia based on economic considerations, which is more appealing. Without going deep into model selection issues, we restrict the off-diagonal elements of the matrix of risk premia λ_1 except in the row where they affect the dynamics of the real dividend growth[11].

4 Description of the variables

We use quarterly data from $1982 : Q1$ up to $2015 : Q1$, extending the sample used in (Ang and Ulrich 2012) by 7 years. While adding observations is per se useful, especially when standard errors are so large that they potentially influence the model selection, here the extended sample includes a deep recession and non-standard monetary policy. Hence, this sample extension actually is economically meaningful, with its particular challenges that are addressed by modifying the model appropriately. Wherever possible, we use the same data series or at least the same data sources.

The output gap g is calculated as the relative difference of real GDP and potential real GDP:

$$g = \frac{1}{4}\frac{Q_t - Q_t^*}{Q_t^*} \tag{46}$$

. The real GDP is obtained from the Bureau of Economic Analysis as provided by Haver (DGPH@USECON), the potential GDP comes from the Congressional Budget Office and is also obtained from Haver (GDPPOTHQ@USECON). Both measures are expressed in 2009 chained prices billion dollar and both are seasonally adjusted. The output gap is then demeaned.

The expected inflation π^e is the median 1 year ahead forecast obtained from the Survey of professional forecasters. The long term inflation forecast $\bar{\pi}$ is the median 10 year ahead forecast provided by the Survey of professional forecasters, backcasted using the combined Livingston and Blue Chip inflation forecasts. Whenever there is a gap in the series, it is interpolated linearly. All these series are provided by the Philadelphia FED. In order to obtain quarterly frequency the numbers are divided by 4. Both π^e and$\bar{\pi}$ are demeaned.

The realised inflation π_t is calculated from the CPI ex food and energy series provided by the Bureau of Labor Statistics as

$$\pi_t = \frac{1}{4}\log\left(\frac{CPI_t}{CPI_{t-4}}\right) \tag{47}$$

[11]Such a restrictions translates into restricting the change of measure affecting only the diagonal elements and the first element in the second row of the transition matrix of the VAR and the coefficients of the macro factors in the dividend growth regression. This coincides with the set of coefficients in the transition matrix that are statistically significant unequal 0.

We also perform the calculations with the CPI all items series, and although the parameter estimates obtained are significantly different, the main features of the model are stable.

The effective FED Funds rate r, the nominal yields $y(n)^{\$}$ and the real yields $y(n)^{r}$ are obtained from the Board of Governors. The nominal yields are the yields on treasuries at constant maturities at 3 month, 6 month, 1 year, 2 year, 3 years, 5 year, 7 year, 10 year and 20 year maturities. The real yields are the yields on inflation indexed treasuries at 5 year, 7 year, 10 year and 20 year maturities. The real yields are available starting from $2003 : Q1$ except for the 20 year maturity which start becoming available in $2004 : Q3$.

The calculations of the dividend growth and the dividend yield are based on the CRSP Value Weighted Index obtained from WRDS. The nominal quarterly dividends D_t are calculated as

$$D_t = P_t(vwretd_t - vwretx_t) \tag{48}$$

where $vwretd_t$ and $vwretx_t$ are the relative changes in the index of the market with and without dividend reinvestment respectively. Since D_t display a seasonal behaviour, the yearly dividend yield dy_t is calculated as the sum of the yearly dividends divided by the capitalisation at the end of the year. The dividend growth process[12] is then obtained as

$$d_t^{\$} = \frac{1}{4}\log\left(\frac{(dy_t)}{dy_{t-4}}\frac{P_t}{P_{t-4}}\right) \tag{49}$$

Finally, the real dividend growth is obtained as

$$d_t = d_t^{\$} - \pi_t \tag{50}$$

5 Application

5.1 Model Selection

As described in the previous section, we propose several key modifications to the model described in (Ang and Ulrich 2012), which can be implemented independently from each other, we therefore can choose from a number of specifications. These specifications differ (I) in the way that inflation expectations are modelled, (II) the modelling of the shadow rate and (III) the way of reducing the number of free parameters in order to obtain significant estimates on the risk premia.

As argued in the previous section, based on economic and econometric reasoning we prefer the specification where inflation expectations are decomposed into a persistent trend inflation and a stationary component that evolves jointly with the other macro factors, where the ZLB is recognised explicitly by modelling in a shadow rate model,

[12] Note that the expression given here differs from the one used in (Ang and Ulrich 2012), as we divide the annual dividend growth divided by four to bring it to the same frequency as the other series, which is needed so that for instance inflation can be subtracted to obtain real dividend growth.

and the significance of the standard errors is achieved by reducing the number of free parameters via restrictions of the matrix of prices of risk.

Before giving more detailed results for this preferred set up, in the following we outline the effects of the modifications separately to convince the reader that the rationale we have provided is indeed also confirmed by the estimation results. Table 1 displays the root mean square error (RMSE) to give an impression of how the introduction of the ZLB and of the trend-cycle decomposition as well as the restriction on the prices of risk affect the model fit. The main conclusion from this table is that the trend-cycle decomposition improves the bond pricing immensely, but fits equity prices worse than the reference model. The restricted setups perform slightly worse in terms of fit as expected, but this increase in RMSE is marginal and more than outweighed by the reduction in the standard errors. The preferred setup performs reasonably well in terms of RMSE - the fit is closer than the reference specification but second to the specification without the ZLB.

Table 1: Comparison of model fits (RMSE)

Setup	Nominal bonds	Real bonds	PD-Ratio	Total
Basic	1.68E-03	7.39E-04	3.91E-04	2.80E-03
trend-cycle	1.46E-03	5.70E-04	4.54E-04	2.49E-03
ZLB	1.73E-03	7.29E-04	3.90E-04	2.85E-03
ZLB and trend-cycle	1.55E-03	6.18E-04	4.34E-04	2.60E-03
basic restricted	1.69E-03	7.00E-04	3.94E-04	2.78E-03
trend-cycle restricted	1.51E-03	5.57E-04	4.86E-04	2.56E-03
ZLB restricted	1.76E-03	7.30E-04	4.00E-04	2.89E-03
ZLB and trend-cycle restricted	1.56E-03	6.25E-04	4.74E-04	2.66E-03

Modelling inflation expectations via a trend-cycle decomposition has two major advantages: firstly, it results in significantly better fit of the model and secondly, it obtains a more realistic impact of the shadow rate modelling. The fit is evaluated as the RMSE calculated across all yield series considered in the estimation process and thus coincides with the loss function in the second stage estimation. It is in the range of 2.78E-03 to 2.89E-03 for models without the trend-cycle decomposition and improves to 2.49E-03 - 2.66E-03 in specifications with trend-cycle decomposition.

When using the trend-cycle decomposition in combination with the shadow rate model, the adjustments of yields due the ZLB are in line with the qualitative and quantitative aspects of the adjustment found in the literature mentioned above. Notably, the adjustment found in the literature is typically hump-shaped and declines asymptotically to 0 with increasing maturity, so that it is already very close to 0 after 20 years. This is exactly the adjustment found when modelling shadow rates in a model with the trend-cycle decomposition of inflation. The resulting adjustments to the yields curve due to the ZLB for the simple model and the model with the trend-cycle decomposition are displayed in Figure 2. The interpretation of this graph is that the value of the option of holding the numeraire directly first increases more than linearly with maturity for short maturities, and later much less than linearly. Initially, the increase of uncertainty associated with the longer maturity dominates, but after a certain time the mean reversion element of the factors and thus the shadow rate takes over.[13]

However, when the inflation expectations are not decomposed into trend and cycle, the persistence under the q-measure of the macro-factors is required to be very high in order to match yields well, the eigenvalues of transition matrix under the q-dynamics are extremely close to unity, therefore the increase in uncertainty is not dominated by the mean reversion component over the calculated horizon of 80 quarters, so the value of the option of earning at least the lower bound yield is an increasing function over that period.

When the trend is isolated with an assumed unit root, this trend takes care of matching the persistent changes in the yields, and the cyclical component is not required to be excessively persistent to match the remainder of the yields, so in that case our model results in the familiar hump shape form of the adjustment.

Table 4 shows the coefficients of the macro factors in the linear expressions for the nominal and real bond yields. The coefficient of π_e in the linear expressions of nominal and real yields differ substantially between setups with and without the trend-cycle decomposition of inflation expectations - while they are identical in the expression for the short rate.[14] However at longer maturities, when the inflation expectations are modelled as trend and cycle, the coefficient on the cyclical component decays rapidly with maturity, and approach 0 at the 10 year horizon, while in the original specification the decay is much slower, the coefficient is still around 2 after 20 years. The coefficients on the other factors are not affected much by the changing specification. The interpretation does not change much - the structural decline in inflation has driven the decline in yields, in particular the parallel part of it. The cyclical variations only have a short term impact on yields. When the decomposition into trend and cycle is not performed then influence of inflation expectations on asset prices is large across all horizons. Note also, that the effect of the ZLB are generally larger in the specification without the trend-cycle decomposition.

Standard errors of the risk premia actually increase slightly when the inflation expectation process is decomposed into a trend component and a cyclical component.

Including the ZLB does not improve the model fit per se. The appeal of a shadow rate model stems from the fact that it indeed excludes the possibility of yields falling below the lower bound, while the simple specification always attributes some density to significantly negative yields. In the model without shadow rates, a forward simulation of the model includes paths where yields are significantly negative for an extended period of time. This is illustrated in Figure 3, which illustrates that whenever the yields are

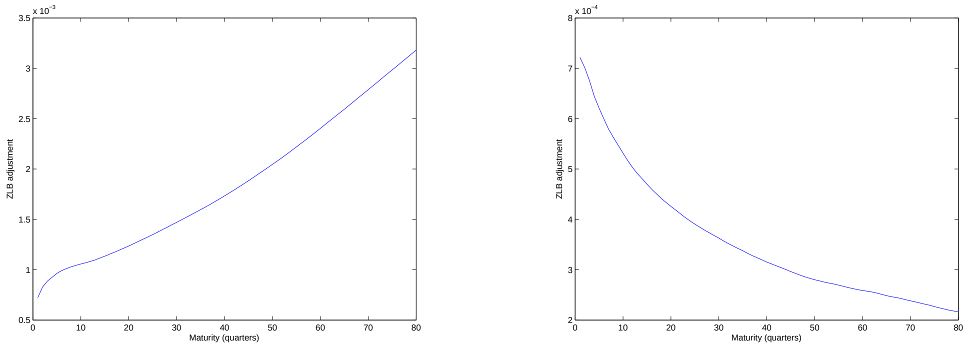
[13] When the λ_1 is restricted as in specification then the adjustment stemming from the ZLB displays a hump shape in both specifications - the decay is still much faster when inflation is modelled with a trend-cycle decomposition.

[14] The Results for the setup without the trend-cycle decomposition is available upon request.

Figure 2: Average adjustment to nominal yield curve over the sample as function of maturity

Simple specification

Specification with trend-cycle decomposition



The figure illustrates the adjustment of the yield curve due to the ZLB for (i) the VAR(1) specification of inflation expectations on the left hand side and (ii) the trend-cycle modelling of inflation expectations as proposed in this paper on the right hand side. The hump-shaped curve on the right hand side, where the adjustment becomes less important with increasing maturity is consistent with finding in the literature.

approaching the lower bound, there is a significant probability, up to nearly 50% when the short rates attain the ZLB, for the rates in the next period to be below the ZLB. The Q-Probabilities of negative rates are higher, as the persistence of the macro factors is larger under the Q measure, implying that the mean reversion of the macro factors that effectively pulls the short rates into positive territory is not as strong as under the P-measure, and a relatively smaller shock is sufficient to attain yields below the lower bound. The shadow rate model by construction sets these probabilities to 0.

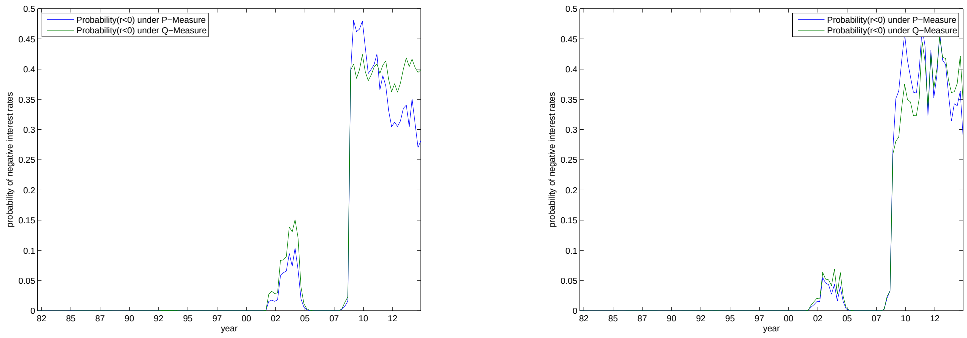
Standard errors cannot be calculated via the proposed bootstrap method due to excessive computational burden in the case of a shadow rate specification. We postulate that they should be of similar magnitude as without modelling shadow rates as there is no reason why this modification should significantly change the variability of the parameters. This is supported by the fact that the estimated values of the risk premia do not change materially, which would be highly unlikely if the standard deviation of the parameters increased significantly by the modification.

Restricting the number of parameters via reduction of macro factors or restrictions on λ_1 achieves statistical significance of the estimated risk premia. It comes at little cost in terms of model fit, with the sum of squared errors only increasing marginally.

Restrictions on the matrix of coefficients, on the other side, despite lacking support from a theoretical model, appear less of a misspecification. They stem from the intuition that persistence of the Q-dynamics is the crucial element that affects the pricing, therefore the risk premia affecting this persistence of the factors directly, i.e. the diagonal elements

Figure 3: Probabilities under the objective and risk neutral measure of nominal interest rates in t+1 below 0 given the information available in t.

Simple specification | Specification with trend-cycle decomposition



The figure displays the probabilities under the objective measure and the risk neutral measure the one-period ahead short rate forecasts is below the ZLB, both for the VAR(1) specification of inflation expectation on the left hand side and the trend-cycle decomposition of inflation expectation on the right hand side. The chart illustrates the importance of explicitly accounting for the ZLB, given that these probabilities have risen to above 40% in recent years as the short rates remain close to 0.

of the matrix λ_1, should therefore be easily identifiable from the observed yields. Ideally, this sort of argument should be backed up by a theoretical model that shows that the off-diagonal elements of λ_1 are 0 within the model framework. This could be a challenge for future work.

Given that the restrictions are quite beneficial from an econometric standpoint and do not imply obvious misspecification, the setup with zero restrictions, with shadow rates and a trend cycle specification of inflation expectations is the preferred setup and the results are discussed further in what follows. Note that the standard errors in the shadow rate specification are not available and thus drawn from the specification that ignores the ZLB.

5.2 Results

In Table 2 in AppendixA we display the estimation results of the physical dynamics of the model. The standard deviations do not necessarily coincide with the OLS errors even where these results are not affected by generated variable bias, because the bootstrap is also designed to deal with autocorrelation of the error term. The coefficients here deviate significantly from what is shown in (Ang and Ulrich 2012), because of (i) a larger sample used here, (ii) the trend-cycle decomposition in the modelling of inflation expectations and (iii) the correct calculation of real dividend growth here. The Taylor rule coefficients are in line with consensus estimates. The cyclical component of inflation expectation is much less persistent than the original inflation expectation series, as expected. Note that all off-diagonal elements in the transition matrix are statistically insignificant.

In Table 3 in Appendix A we show the estimation results of the risk premia and the transition matrix under the q measure. Note that despite the restriction on the risk premia, only a small number is significant. Essentially, the persistence of the monetary policy shock is much higher under the Q-measure than the P-measure. Also note that the standard deviation of the autoregressive parameter of the cyclical component of inflation expectations is close to 0 and the standard deviation of the risk premium affecting it is driven by the first stage estimation error only. As expected, the persistence of the macro factors increases under the Q-measure, which essentially increases the long term risk associated with them. This long term risk needed to generate sufficient impact of variations in the macro factors on asset prices. The Table 5 shows that without restriction the risk premia, the estimates of the risk premia are all insignificant. This highlights the importance of taking into account the accumulation of estimation error over the three steps of the approach. Negligence of this issue leads to entirely invalid conclusions about the significance of parameters. Estimating the standard errors based on other parameters being fixed gives invalid standard errors leading to invalid inference. It would be interesting to review the results reported in e.g.(Ang and Piazzesi 2003) with bootstrapped standard errors. Calculations not shown here confirm that this result is not driven by the inclusion of equity pricing in this particular model, but also extends to a pure bond pricing version of this model.

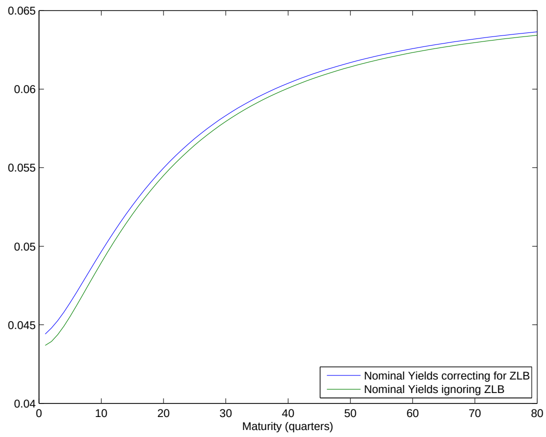
In Table 4 in Appendix A we present the coefficients of the macro factors in linear expressions for the yields. The price dividend yields does not significantly depend on macro series[15]. On the other hand, macro series are the main drivers of nominal and real yields. The importance of cyclical variations of inflation expectations washes out after 5 years, the output gap and the monetary policy shock are significant drivers of nominal yields over all observed horizons and real yields over most horizons. This highlights the usefulness of a macro-finance model to price bonds and shows the need to better understand the impact of macro factors on dividend dynamics. The VAR(1) setup of the model appears to be insufficient to capture the links between dividend growth and the macro factors. Again the comparison with the results of the unrestricted model shown in Table 6 shows that the restriction manifests itself in smaller standard errors making the model more useful.

As shown in Figure 2, the explicit modelling of the ZLB makes the yield curve flatter at the lower end, where the impact of the ZLB on yields is the strongest. At longer maturities, the effect becomes negligible.

The latent equity factor is not robust to the specification used and we therefore do not attempt an interpretation of it. Figure 5 displays the model predictions for nominal

[15] Here our results deviate strongly from (Ang and Ulrich 2012) - this might be due to the fact that the dividend yield series is calculated differently here, putting it on the same frequency as the other series.

Figure 4: Effect of ZLB on term structure of nominal bond yields



The figure displays the model implied average yield curve across the sample both for the exponentially affine model of interest rates and the non-linear shadow rate model.

and real bond yields over the sample period. It shows that the major part of yield curve variation is done by a shift in level, the gradient is driven by a decline of fluctuations with increasing maturity.

From Figure 6 we see that the model (blue line) achieves a tight fit to the actual 10 year bond yields (green line), but towards the end of the sample where the ZLB becomes relevant, the yields implied by the model are less volatile and do not approach the bound as much as the actual series do. This explains why modelling the ZLB explicitly does not improve the fit of the model, because if anything neglecting it brings 10 year yields further down in line with the actual series.

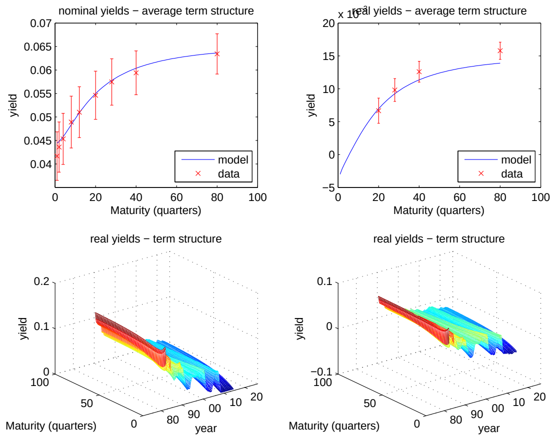
From Figure 7 we obtain that the volatility of the short end of the term structure of equity is much larger than at the long end. The terms structure of expected equity returns is downward sloping, as also reported by (Ang and Ulrich 2012) and in line with theoretical models on the term structure of equity as well as with evidence from dividend strips.

Figure 8 shows the decomposition of the average term structure of equity returns as laid out in section 2, and Figure 13 in the Annex depicts the same decomposition for the 10 year horizon for the sample period.

Despite the modifications of the model to adjust it to the low yield environment, qualitative evaluation of the model fit of bond yields is poorer for the low yield phase of the past year than the previous years[16], and oddly the model overestimates yields during those five years consistently. This suggests that the decompression of the term

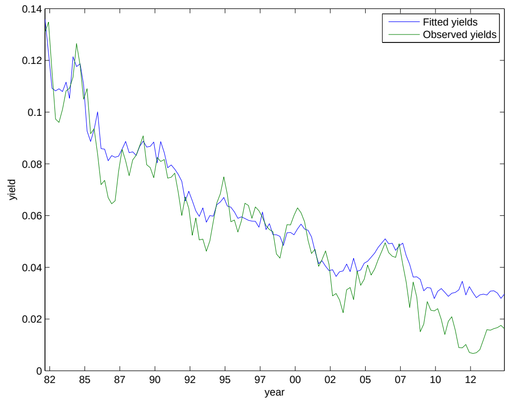
[16]See also Figure 11 in the Appendix.

Figure 5: Nominal and real bond pricing



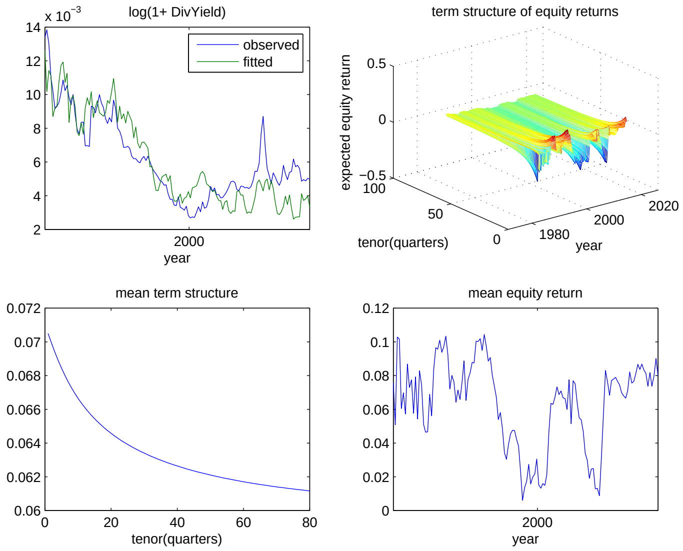
The figure shows the model implied average nominal and real yield curve for the shadow rate model specified with trend-cycle decomposition for inflation expectations in the top row. In the bottom row, the evolution of the model implied yield curve over the sample is displayed, showing how it changes its shape as a function of the macro factors.

Figure 6: 10 year bond yields



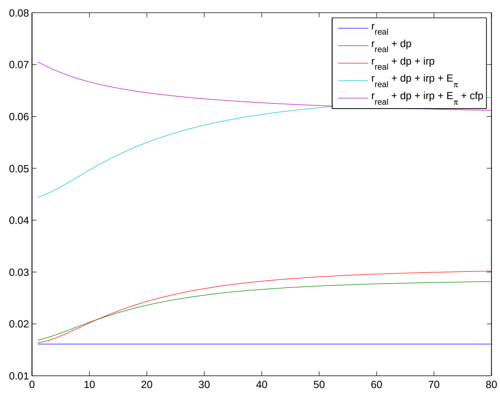
The figure illustrates the model fit of 10 year bond rates, showing that the fit is slightly less volatile and appears to be worst at the end of the sample.

Figure 7: Equity pricing



The figure displays an evaluation of the equity pricing capabilities of the model. The upper left hand chart shows the model fit of the price dividend ratio against the observed time series. The top right hand chart displays the model implied dynamics of the term structure of equity returns over the tenors from 1 - 80 months. Below, the left hand chart illustrates the average term structure of equity returns, which is sloping downward as suggested by recent theoretical and empirical research. The right hand chart displays the evolution of the expected equity return over time and illustrates the immense time variation in expected returns.

Figure 8: Decomposition of the term structure



The figure shows the decomposition of the model implied average term structure of expected equity returns into its average component term structures.

premium during that period is due to factors outside the scope of the model. Frictions in the financial sector which are targeted by quantitative easing might play a role.

The modification to account for the ZLB reduces the steepness of the yield curve at low maturities, which is mirrored by the observed terms structure of bond yields. The term structure of expected equity returns is confirmed to be downward sloping, in line with the results of (Ang and Ulrich 2012) and empirical works using evidence from dividend strips (see (van Binsbergen, Brandt, and Koijen 2012)). The decomposition of the term structure of equity returns (8) is qualitatively similar to the results in the original paper, although the level of expected equity returns is found to be lower here, which might be partially driven by the extended sample, of which the equity premium has most likely fallen significantly in line with term premia and credit spreads as part of the global search for yield.

6 Conclusion

In this paper we propose three important modifications to the macro finance model for joint pricing of bonds and equities as introduced in (Ang and Ulrich 2012), (i) to account for structural shifts in inflation expectations by decomposing the inflation expectation series into a trend component assumed to persist forever, and a stationary component, (ii) to explicitly considering the ZLB by introducing shadow rate and (iii) calculate the standard errors via a bootstrapping procedure to correctly capture the multi-step nature of the estimation process.

These extensions of the reference model have a significant impact on the results of the model and address its important shortcomings. Firstly, we tackle the fact that there is a secular trend in bond prices which is driven by a secular trend in inflation. This cannot be approached within a pure VAR(1) setup as it basically is the consequence of a regime shift in monetary policy, and requires unrealistically high persistence of all the macro variables to reproduce the observed asset prices. We show how by a simple trend-cycle decomposition, this problem of the original approach can be solved. Coincidentally, this modification also significantly improves the in sample fit without requiring additional free parameters.

The extension to a shadow rate model rather improves the economic properties of the model, notably it prevents the future interest rate under P and Q dynamics to violate the ZLB, which translates into a more realistic modelling of bond prices which are derived from the Q-expectations of the short rate. Furthermore, such a model allows to estimate the duration of the short rate staying at the ZLB. It also allows to replicate significant movements in bond yields when the short rate is at the ZLB. All these properties make it much more adequate for pricing bonds and equities during the low yield environment that currently persists. It is the first model of that sort that features joint pricing of nominal and real bonds and equities.

The correct calculation of the standard errors could be belittled as a technical point, but it proves to be of central importance as it has direct consequences on the statistical significance of all the key model parameters and therefore also on the selection of the best model. The proposed bootstrap procedure fully takes into account the additional uncertainty introduced by estimated or generated variables during several estimation steps. Ignoring it leads to substantially incorrect inference: the estimates of all risk premia in the reference model are statistically insignificant when calculated correctly which stands in stark contrast in the reported statistical significance in (Ang and Ulrich 2012). We address this by imposing zero restrictions on the off-diagonal elements of the matrix of prices of risk. We show that the restricted model can achieve similar fit as the unrestricted model, and a good share of the estimated parameters is again statistically significant.

All these modifications to the reference model allow to gain insights into the impact of macro factors on the term structure of bond yields and the price dividend ratio. However, the conclusions on the impact of macro factors on price dividend ratios cannot be confirmed, owing to a weak connection of the macro factors and real dividend growth as estimated within the VAR(1) specification. While this is hardly satisfactory, the sample length does not permit a more detailed model where an extended lag structure might help detecting the true relationship between real dividend growth and the macro factors. Further investigations are certainly warranted into the joint pricing of bonds and equity, and the work here provides first inroads.

Having performed an extensive model selection, while not exploring systematically all possible restriction on the risk premia, this work highlights the need of theoretical support for restrictions on the prices of risk - or patience, as with longer time series, the unrestricted model might also result in significant estimates.

References

Alvarez, F. and U. J. Jermann (2005). Using asset prices to measure the persistence of the marginal utility of wealth. *Econometrica 73*(6), 1977–2016.

Ang, A. and G. Bekaert (2007). Stock return predictability: Is it there? *Review of Financial Studies 20*(3), 651–707.

Ang, A., G. Bekaert, and M. Wei (2008). The term structure of real rates and expected inflation. *The Journal of Finance 63*(2), 797–849.

Ang, A., J. Boivin, S. Dong, and R. Loo-Kung (2011). Monetary Policy Shifts and the Term Structure. *Review of Economic Studies 78*(2), 429–457.

Ang, A. and M. Piazzesi (2003, May). A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. *Journal of Monetary Economics 50*(4), 745–787.

Ang, A. and M. Ulrich (2012). Nominal bonds, real bonds, and equity. *Netspar Discussion Paper* (12).

Baele, L., G. Bekaert, and K. Inghelbrecht (2010). The determinants of stock and bond return comovements. *Review of Financial Studies 23*(6), 2374–2428.

Bauer, M. D. (2015). Restrictions on Risk Prices in Dynamic Term Structure Models. CESifo Working Paper Series 5241, CESifo Group Munich.

Bauer, M. D. and G. D. Rudebusch (2013). Monetary policy expectations at the zero lower bound. Working Paper Series 2013-18, Federal Reserve Bank of San Francisco.

Bekaert, G. and E. Engstrom (2010, April). Inflation and the stock market: Understanding the "Fed Model". *Journal of Monetary Economics 57*(3), 278–294.

Belo, F., P. Collin-Dufresne, and R. S. Goldstein (2015). Dividend dynamics and the term structure of dividend strips. *The Journal of Finance 70*(3), 1115–1160.

Berkowitz, J. and L. Kilian (1996). Recent developments in bootstrapping time series. Finance and Economics Discussion Series 96-45, Board of Governors of the Federal Reserve System (U.S.).

Bühlmann, P. (2002, 05). Bootstraps for time series. *Statist. Sci. 17*(1), 52–72.

Black, F. (1995). Interest rates as options. *The Journal of Finance 50*(5), pp. 1371–1376.

Blattner, T. S. and E. Margaritow (2010, June). Towards a Robust Monetary Policy Rule for the Euro Area. *European Central Bank Working Paper Series* (1210).

Buraschi, A. and A. Jiltsov (2005). Inflation risk premia and the expectations hypothesis. *Journal of Financial Economics 75*(2), 429 – 490.

Campbell, J. Y. and R. J. Shiller (1988). The dividend-price ratio and expectations of future dividends and discount factors. *Review of Financial Studies 1*(3), 195–228.

Chen, R.-R. and L. Scott (1993). Maximum likelihood estimation for a multifactor equilibrium model of the term structure of interest rates. *Journal of Fixed Income 3*(3), 14–31.

Cochrane, J. H. and M. Piazzesi (2005). Bond risk premia. *American Economic Review 95*(1), 138–160.

Dai, Q. and K. Singleton (2003). Term structure dynamics in theory and reality. *Review of Financial Studies 16*, 631–678.

Dai, Q. and K. J. Singleton (2000). Specification analysis of affine term structure models. *The Journal of Finance 55*(5), 1943–1978.

Duffee, G. R. (2002). Term premia and interest rate forecasts in affine models. *The Journal of Finance 57*(1), 405–443.

Fama, E. F. (1981, September). Stock Returns, Real Activity, Inflation, and Money. *American Economic Review 71*(4), 545–65.

Fama, E. F. and K. R. French (1988). Dividend yields and expected stock returns. *Journal of Financial Economics 22*(1), 3 – 25.

Gilchrist, S. and E. Zakrajsek (2012). Credit spreads and business cycle fluctuations. *American Economic Review 102*(4), 1692–1720.

Gurkaynak, R. S., B. Sack, and J. H. Wright (2010, January). The TIPS Yield Curve and Inflation Compensation. *American Economic Journal: Macroeconomics 2*(1), 70–92.

Hamilton, J. D. and J. C. Wu (2012). Identification and estimation of Gaussian affine term structure models. *Journal of Econometrics 168*(2), 315–331.

Ibbotson, R. G. and P. Chen (2003). Long-run stock returns: Participating in the real economy. *Financial Analysts Journal 59*(1), 97 – 109.

Judd, J. and G. Rudebusch (1999). The goals of u.s. monetary policy.

Krippner, L. (2011, October). Modifying Gaussian term structure models when interest rates are near the zero lower bound. CAMA Working Papers 2011-36, Centre for Applied Macroeconomic Analysis, Crawford School of Public Policy, The Australian National University.

Krippner, L. (2013a, January). A tractable framework for zero lower bound Gaussian term structure models. Reserve Bank of New Zealand Discussion Paper Series DP2013/02, Reserve Bank of New Zealand.

Krippner, L. (2013b). Measuring the stance of monetary policy in zero lower bound environments. *Economics Letters 118*(1), 135–138.

Lemke, W. (2008). An affine macro-finance term structure model for the euro area. *The North American Journal of Economics and Finance 19*(1), 41 – 69. Relation between Monetary Policy and Financial Markets.

Lemke, W. and A. L. Vladu (2014, 9). A Shadow-Rate Term Structure Model for the Euro Area. Working paper series, European Central Bank.

Lemke, W. and T. Werner (2009, April). The term structure of equity premia in an affine arbitrage-free model of bond and stock market dynamics. Working Paper Series 1045, European Central Bank.

Lettau, M. and S. C. Ludvigson (2005, June). Expected returns and expected dividend growth. *Journal of Financial Economics 76*(3), 583–626.

Lettau, M., S. C. Ludvigson, and J. A. Wachter (2008). The declining equity premium: What role does macroeconomic risk play? *Review of Financial Studies 21*(4), 1653–1687.

Lettau, M. and J. A. Wachter (2011, July). The term structures of equity and interest rates. *Journal of Financial Economics 101*(1), 90–113.

Lopez, P., J. D. Lopez-Salido, and F. Vazquez-Grande (2015, June). Nominal Rigidities and the Term Structures of Equity and Bond Returns. Finance and Economics Discussion Series 2015-64, Board of Governors of the Federal Reserve System (U.S.).

Malik, S. and A. Meldrum (2014). Evaluating the robustness of uk term structure decompositions using linear regression methods. Bank of England working papers 518, Bank of England.

Mehra, R. and E. C. Prescott (1985). The equity premium: A puzzle. *Journal of Monetary Economics 15*(2), 145 – 161.

Meulendyke, A.-M. (1998). *U.S. Monetary Policy and Financial Markets.*

Pagan, A. (1984). Econometric issues in the analysis of regressions with generated regressors. *International Economic Review 25*(1), 221–247.

Pericoli, M. and M. Taboga (2015). Understanding policy rates at the zero lower bound: insights from a bayesian shadow rate model. Temi di discussione (Economic working papers) 1023, Bank of Italy, Economic Research and International Relations Area.

Pflueger, C. and L. Viceira (2011). An empirical decomposition of risk and liquidity in nominal and inflation-indexed government bonds.

Piazzesi, M. and M. Schneider (2009). Trend and cycle in bond premia.

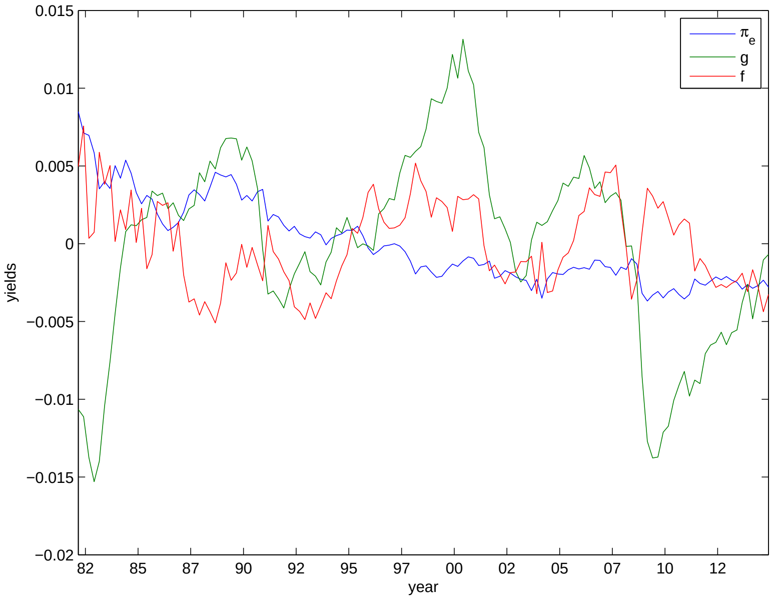
Politis, D. N. and J. P. Romano (1994). The stationary bootstrap. *Journal of the American Statistical Association 89*(428), pp. 1303–1313.

Ruiz, E. and L. Pascual (2002). Bootstrapping financial time series. *Journal of Economic Surveys 16*(3), 271–300.

van Binsbergen, J., M. Brandt, and R. Koijen (2012, June). On the Timing and Pricing of Dividends. *American Economic Review 102*(4), 1596–1618.

A Summary statistics

Figure 9: Macro Variables



The figure displays the demeaned time series of the macro factors that drive the model.

Table 2: Dynamics under physical measure - trend-cycle decomposition

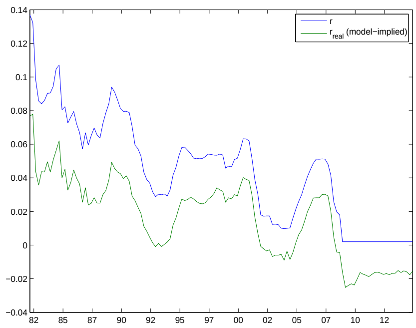
	constant	π_e	g	f	d	L	σ_π
			Taylor Rule parameters				
	$\delta_0^{\$}$		$\delta_1^{\$}$				
$r^{\$}$	0.011	2.599	0.364	1.000			
	(0.000)	(0.091)	(0.040)	(0.000)			
			Taylor Rule (real interest rate)				
	δ_0^r		δ_1^r				
r	0.004	1.623	0.363	1.026			
	(0.000)	(0.091)	(0.183)	(0.014)			
	π_c			$\Sigma_p i$			
π	0.0069	-0.0002	-0.0001	-0.0003			0.0011
	(0.0001)	(0.0001)	(0.0001)	(0.0001)			(0.0000)
					VAR Parameters		
	μ			Φ			
π_e	0.000	0.569	0.032	0.000	0.000	0.000	
	(0.000)	(0.066)	(0.011)	(0.000)	(0.000)	(0.000)	
g	0.000	-0.537	0.991	0.000	0.000	0.000	
	(0.000)	(0.197)	(0.032)	(0.000)	(0.000)	(0.000)	
f	0.000	0.000	0.000	0.795	0.000	0.000	
	(0.000)	(0.000)	(0.000)	(0.056)	(0.000)	(0.000)	
d	0.002	-1.130	-0.092	-0.455	0.864	0.000	
	(0.002)	(1.826)	(0.300)	(0.439)	(0.055)	(0.000)	
L	0.000	0.000	0.000	0.000	0.000	0.000	
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	
	Volatility						
			Σ				
π_e	0.001	0.000	0.000	0.000	0.000		
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)		
g	0.000	0.001	0.000	0.000	0.000		
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)		
f	0.000	0.000	0.002	0.000	0.000		
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)		
d	0.000	0.000	0.000	0.010	0.000		
	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)		
L	0.000	0.000	0.000	0.000	0.000		
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)		

Table 3: Risk premia and dynamics under risk neutral measure - restricted specification with trend-cycle decomposition

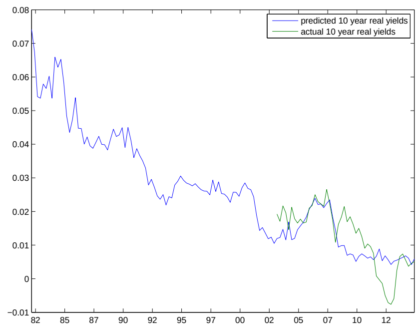
						Risk Premia Parameters
	λ_0			λ_1		
π_e	-0.050	-120.815	0.000	0.000	0.000	0.000
	(0.000)	(533.021)	(0.000)	(0.000)	(0.000)	(0.000)
g	-1.118	0.000	8.527	0.000	0.000	0.000
	(4.294)	(0.000)	(40.070)	(00.000)	(0.000)	(0.000)
f	0.326	0.000	0.000	-88.248	0.000	0.000
	(1.276)	(0.000)	(0.000)	(35.552)	(0.000)	(0.000)
d	-0.200	-68.261	-9.063	-81.884	0.000	0.000
	(2.129)	(575.711)	(65.527)	(59.631)	(0.000)	(0.000)
L	0.000	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	μ_q			Φ_q		
π_e	0.000	0.630	0.032	0.000	0.000	0.000
	(0.000)	(0.000)	(0.011)	(0.000)	(0.000)	(0.000)
g	0.002	-0.505	0.979	0.000	0.000	0.000
	(0.006)	(0.208)	(0.055)	(0.000)	(0.000)	(0.000)
f	-0.001	0.000	0.000	0.941	0.000	0.000
	(0.002)	(0.000)	(0.000)	(0.037)	(0.000)	(0.000)
d	0.004	-0.414	0.003	0.405	0.864	0.000
	(0.023)	(6.605)	(0.640)	(0.620)	(0.055)	(0.000)
L	0.000	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Figure 10: Short rates and real yields

Nominal and real short rate



10 year real bond yields
observed vs. model



The chart on the right hand side shows the model implied nominal and real short rates. Note that the nominal rate coincides with the observed time series as the model is constructed to achieve perfect fit, while the real short rate is unobserved. The chart on the left hand side displays the model implied 10 year real yields vs. the observed 10 year real yields. Note that the observed series is only available after 2002.

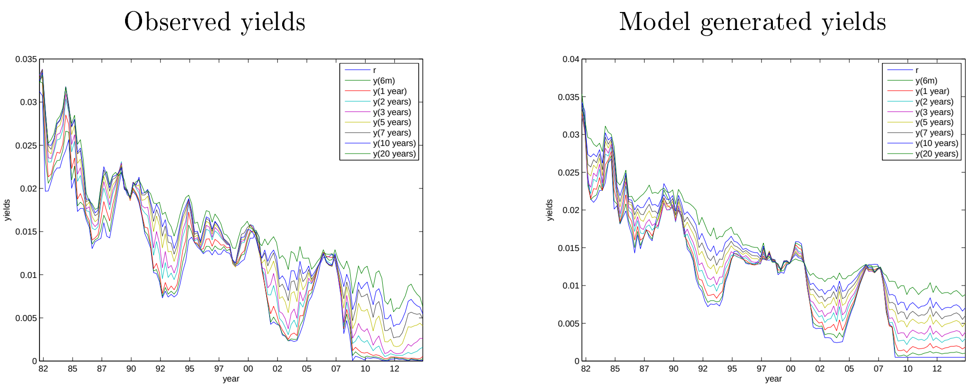
Table 4: Linear expressions for nominal yields, real yields and dividend yields - restricted specification with trend-cycle decomposition

	Linear approximation of Log(1+D/P)				
	0.004	-0.008	0.038	-0.104	-0.022
	(0.000)	(0.240)	(0.073)	(0.089)	(0.008)
	Nominal Yields				
$r^{\$}$	0.011	2.599	0.364	1.000	
	(0.000)	(0.091)	(0.040)	(0.000)	
3 months	0.011	2.026	0.401	0.970	
	(0.001)	(0.419)	(0.145)	(0.018)	
6 months	0.011	1.236	0.441	0.914	
	(0.001)	(0.390)	(0.157)	(0.045)	
1 year	0.012	0.445	0.449	0.815	
	(0.001)	(0.388)	(0.167)	(0.074)	
2 years	0.013	0.120	0.421	0.730	
	(0.001)	(0.402)	(0.163)	(0.084)	
5years	0.014	-0.085	0.347	0.595	
	(0.001)	(0.379)	(0.140)	(0.083)	
7 years	0.014	-0.123	0.285	0.493	
	(0.000)	(0.327)	(0.117)	(0.075)	
10 years	0.015	-0.117	0.217	0.385	
	(0.001)	(0.256)	(0.090)	(0.061)	
20 years	0.016	-0.068	0.114	0.209	
	(0.001)	(0.136)	(0.048)	(0.034)	
	Real Yields				
5 years	0.006	-0.158	0.300	0.610	
	(0.000)	(0.299)	(0.127)	(0.086)	
7years	0.006	-0.165	0.244	0.506	
	(0.001)	(0.260)	(0.106)	(0.077)	
10 years	0.007	-0.141	0.186	0.395	
	(0.001)	(0.205)	(0.081)	(0.063)	
20 years	0.007	-0.078	0.098	0.214	
	(0.000)	(0.109)	(0.043)	(0.035)	

Table 5: Risk premia and risk neutral dynamics - unrestricted specification with trend-cycle decomposition

						Risk Premia Parameters
	λ_0			λ_1		
g	-0.261	-26.175	-549.711	-70.216	0.000	0.000
	(36.780)	(421.654)	(2,464.882)	(1,157.266)	(0.000)	(0.000)
π_e	-0.069	67.266	-709.828	15.629	0.000	0.000
	(10.995)	(102.890)	(428.448)	(200.596)	(0.000)	(0.000)
f	-0.059	15.763	184.935	-66.438	0.000	0.000
	(11.029)	(162.000)	(802.170)	(420.095)	(0.000)	(0.000)
d	0.216	-22.968	-245.953	-98.730	0.000	0.000
	(6.250)	(231.724)	(768.669)	(766.668)	(0.000)	(0.000)
L	0.000	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	μ_q			Φ_q		
g	0.000	1.029	0.251	0.101	0.000	0.000
	(0.053)	(0.598)	(3.076)	(1.587)	(0.000)	(0.000)
π_e	0.000	0.000	0.975	-0.001	0.000	0.000
	(0.002)	(0.039)	(0.164)	(0.077)	(0.000)	(0.000)
f	0.000	-0.026	-0.306	0.905	0.000	0.000
	(0.017)	(0.268)	(1.259)	(0.665)	(0.000)	(0.000)
d	-0.001	0.149	1.451	0.581	0.864	0.000
	(0.060)	(2.342)	(6.705)	(7.198)	(0.051)	(0.000)
L	0.000	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Figure 11: Observed nominal yields vs. model generated nominal yields



The figure shows the time series of the yield curve at all observed maturities on the left hand side and the model implied series on the right hand side.

Table 6: Yields in the unrestricted case, trend-cycle decomposition of inflation expectations

	Linear approximation of Log(1+D/P)				
	0.004	0.038	0.101	-0.090	-0.024
	(0.001)	(0.071)	(0.282)	(0.081)	(0.007)
	Nominal Yields				
$r^{\$}$	0.011	0.364	2.599	1.000	
	(0.000)	(0.040)	(0.091)	(0.000)	
3 months	0.011	0.357	2.459	0.969	
	(0.000)	(0.042)	(0.120)	(0.027)	
6 months	0.012	0.343	2.201	0.912	
	(0.001)	(0.043)	(0.191)	(0.048)	
1 year	0.012	0.320	1.762	0.813	
	(0.001)	(0.047)	(0.293)	(0.067)	
2 years	0.013	0.301	1.406	0.733	
	(0.001)	(0.051)	(0.347)	(0.076)	
5years	0.014	0.272	0.879	0.613	
	(0.001)	(0.057)	(0.383)	(0.084)	
7 years	0.014	0.251	0.520	0.529	
	(0.000)	(0.061)	(0.383)	(0.084)	
10 years	0.015	0.229	0.175	0.444	
	(0.000)	(0.064)	(0.371)	(0.080)	
20 years	0.016	0.189	-0.282	0.313	
	(0.000)	(0.064)	(0.332)	(0.071)	
	Real Yields				
5 years	0.006	0.248	0.138	0.619	
	(0.001)	(0.051)	(0.304)	(0.080)	
7years	0.007	0.222	-0.177	0.527	
	(0.001)	(0.054)	(0.304)	(0.081)	
10 years	0.007	0.194	-0.459	0.433	
	(0.001)	(0.058)	(0.304)	(0.079)	
20 years	0.007	0.140	-0.730	0.280	
	(0.001)	(0.064)	(0.329)	(0.077)	

Figure 12: Difference between shadow rate implied yields and model yields after applying the ZLB restriction in the preferred model setup

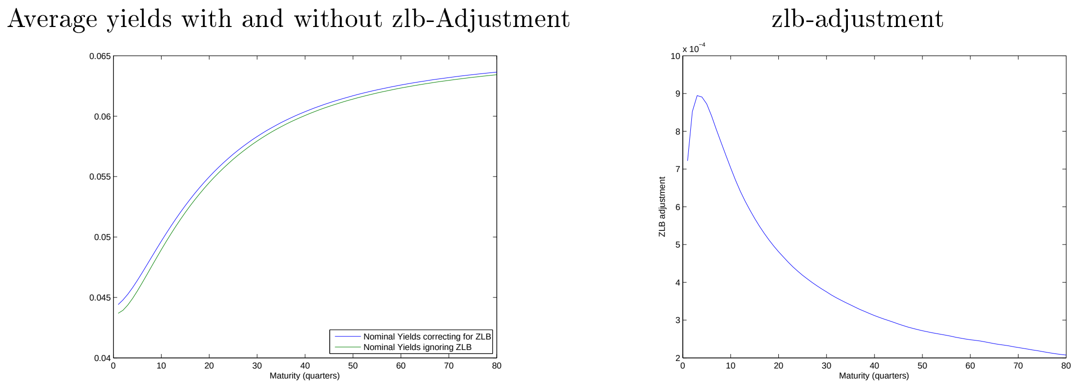
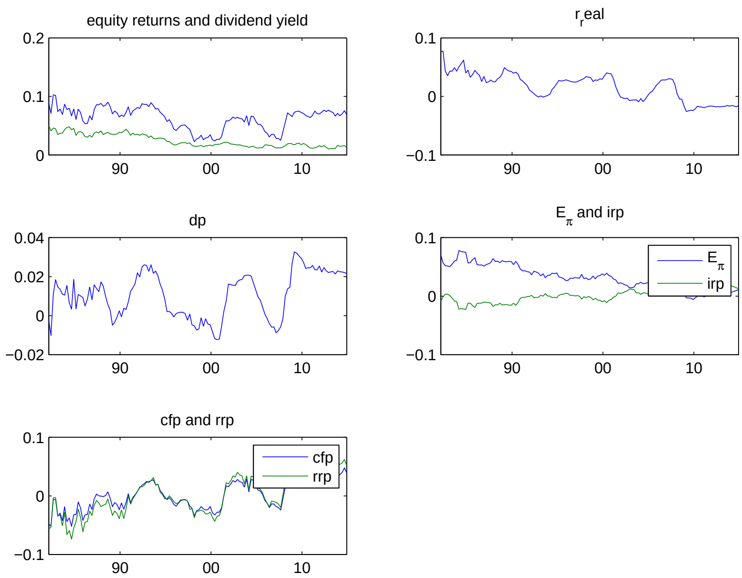


Figure 13: Decomposition of 10 year premia



Heinrich Kick
European Central Bank, Frankfurt am Main, Germany; email: heinrich.kick@ecb.int

Postal address 60640 Frankfurt am Main, Germany
Telephone +49 69 1344 0
Website www.ecb.europa.eu

ISSN	1725-2806 (pdf)	DOI	10.2866/003067 (pdf)
ISBN	978-92-899-2714-7 (pdf)	EU catalogue No	QB-AR-17-004-EN-N (pdf)