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Multiplex interbank networks and systemic importance: an application to European data

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Abstract

Research on interbank networks and systemic importance is starting to recognise that the web of exposures linking banks balance sheets is more complex than the single-layer-of-exposure approach. We use data on exposures between large European banks broken down by both maturity and instrument type to characterise the main features of the multiplex structure of the network of large European banks. This multiplex network presents positive correlated multiplexity and a high similarity between layers, stemming both from standard similarity analyses as well as a core-periphery analyses of the different layers. We propose measures of systemic importance that fit the case in which banks are connected through an arbitrary number of layers (be it by instrument, maturity or a combination of both). Such measures allow for a decomposition of the global systemic importance index for any bank into the contributions of each of the sub-networks, providing a useful tool for banking regulators and supervisors in identifying tailored policy instruments. We use the dataset of exposures between large European banks to illustrate that both the methodology and the specific level of network aggregation matter in the determination of *interconnectedness* and thus in the policy making process.

Keywords: *interbank networks, systemic importance, multiplex networks*

JEL Classification: *G21, D85, C67.*

Non-technical summary

The recent financial crisis and the stress suffered in interbank markets brought to the fore the relevance of interconnectedness between banks and the importance of higher-order feedback loops embedded in the reciprocal web of exposures linking financial institutions. Network theory and network-based analytics have proven useful in extracting relevant information from systems that due to their inherent complexity do not lend themselves to simple apprehension.

Of critical importance in macro prudential policy is the identification of key players in the financial network. In the context of interconnectivity analysis, the identification of critical nodes within a network has been a problem long studied in disciplines like sociology, under the heading of "centrality" analysis.

While early contributions on interbank contagion and networks have focused on aggregated exposures, it is now increasingly recognised that the web of credit relationships linking banks' balance sheets is in general more intricate and complex.

Macro prudential policy addressing banks' systemic importance could indeed benefit from the consideration of subnetworks and aggregated network separately: systemic importance may depend on which activity is at the time more critical or the target of a specific policy.

The paper presents a unique dataset of exposures between large European banks that features a high level of disaggregation in terms of instruments and maturity. We analyse its multiplex structure by means of correlated multiplexity, core-periphery and similarity analyses.

Additionally, we introduce two new accounting-based measures designed for multiplex (or multilayer) networks, which build a systemic importance score for each bank in the aggregated network that can be decomposed into the contributions by each sub-network. This provides a holistic analysis that truly incorporates the multiplex structure of the network, instead of doing separate analyses for the different layers and the aggregate network. We use the dataset of exposures between large European to illustrate the measures. Our approach builds on the logic that drives the policy process of assessing banks' importance at the Financial Stability Board (FSB) in its recognition of the importance of interconnectedness, but delves deeper into this aspect by considering the different layers in an integrated accounting framework. The insights from the measures proposed can be of policy relevance for supervisors and regulators.

1 Introduction

Growing interest in the analysis of financial interconnectedness and the assessment of systemic risk reflect policy interest extending well beyond traditional micro-prudential supervision. Risk externalities of bank behaviour, which are not taken into account by micro-prudential policies, call for a macroprudential approach. The recent financial crisis and the stress suffered in interbank markets brought to the fore the relevance of bank interconnectedness and the importance of higher-order feedback loops embedded in the reciprocal web of exposures linking financial institutions.

Of critical importance in macroprudential policy is the identification of key players in the financial network, as exemplified by recently introduced *Basel Committee on Banking Supervision* (BCBS) requirements on globally systemic important banks (G-SIBs). In the context of network interconnectivity analysis, the use of "centrality" indicators attempt to assess, based on different criteria, how important a node is for the functioning of the network under study.

While early contributions on interbank contagion and networks have focused on aggregated exposures, it is now increasingly recognised that the web of credit relationships linking banks' balance sheets is in general more intricate and complex. The empirical literature thus far has either disregarded heterogeneity in credit relationships or worked with only one layer (typically the overnight unsecured market) resting on the tenet that it is representative of the whole web of exposures.[1] Of course, there is a very good reason why most of the extant literature on interbank networks has worked with the simplification of a single layer of exposures, namely data availability.

Macro prudential policy addressing banks' systemic importance could indeed benefit from the consideration of subnetworks and aggregated network separately. To the extent that transmission channels' magnitude and speed differ across layers, institutions' systemic importance may differ at the aggregate and more granular levels. Furthermore, systemic importance may depend on which activity is at the time more critical or more directly addressed by the specific policy being considered. This is not hard to understand given the derivation of systemic importance: a given bank's aggregated systemic importance could be seen as being composed of importance in the different subnetworks, thus possibly accounting for the multiple feedback loops that can exist when banks are connected at different levels or layers of exposure. This is indeed the logic that drives the policy process of assessing banks' importance at the *Financial Stability Board (FSB)* and the *Basel Committee on Banking Supervision*, whereby global importance is constructed by an aggregation of the size of the banks in relevant activities (see Basel Committee on Banking Supervision (2013)). Distinguishing aggregate from granular components of systemic importance is an improvement over, for example, past practice of considering only banks' size. Explicitly addressing banks' importance and complexities associated with interconnections within a financial system through activities not recognised by the size of their balance sheets is key for financial stability assessment. Thus, the

[1] Exceptions to this are the recent contributions by Bargigli et al. (2015) and Langfield et al. (2014) among others. See the literature review section below for more details.

regulatory definition of systemically important banks adopted by the BCBS and the FSB rests on distinctly recognising all the bank activities central to the financial system and (exogenously) weighing them to derive a unique ranking of systemic importance.[2] This recognition dates back to the ECB's surveillance of large and complex banking groups, which acknowledged the usefulness of refining the degree of centrality of banks in such a way that banks' importance relative to a given fragility can be identified. In an environment of scant or rapidly changing liquidity, for instance, it is central to policy makers' interest to identify banks important in the provision of short term loans. Likewise, the more structural provision of long term financing among banks is best understood in light of banks' importance vis-à-vis longer term funding. The policy interest may centre on a bank's importance relative to the activity of a given market, and the picture provided by an aggregate interconnectivity analysis that does not allow for a decomposition may be misleading. Without granular information allowing to observe a bank's centrality in the interconnectivity of these activities, the scope of policy action remains limited.

The papers addressing different layers of exposures between banks typically perform separate analyses for each layer and the aggregated network.[3] Our paper contributes to this literature by building on a holistic representation of the balance sheet of the banking system that allows for a consistent decomposition of systemic importance.

To this end, in the present paper we build on the framework introduced by Aldasoro and Angeloni (2015) and expand two of their systemic importance measures to the case in which banks are connected through different layers, with the goal of attributing to each subnetwork its contribution to the systemic importance index for any given bank. We then present a unique dataset of exposures between large European banks that features a high level of disaggregation in terms of instruments and maturity, originally introduced into the literature in Alves et al. (2013). We analyse its multiplex structure and use it to illustrate the proposed measures.

The remainder of the paper is structured as follows. Section 2 briefly outlines the relation to the literature, whereas in Section 3 we develop the logic behind multilayer networks, present the approach to systemic importance in single layer networks and its extension to the multiplex case. Section 4 presents the analysis of the multiplex structure of the network of large European banks and uses the data to illustrate the measures of systemic importance. Finally, Section 5 concludes.

[2]Likewise and as an example, an approach based on banks' static characteristics observable through different activity shares could benefit from explicitly recognising the importance of the banks' degree of interconnectedness across the different activities, thus completing the decomposition of systemic importance on the basis of both activities and interconnectivity degrees.

[3]See for instance León et al. (2014) and Molina-Borboa et al. (2015). An exception is the recent contribution by Poledna et al. (2015), which uses the algorithm developed in Battiston et al. (2012) to study the multiplex Mexican interbank system.

2 Related Literature

The analysis of the systemic importance of financial institutions attracted attention recently with the collapse of some large institutions and near collapse of many others. Developments made all too clear the dangers of the too-big-to-fail and too-big-to-bail problems. Our paper is related to strands of literature which can be traced back to the seminal contributions by Allen and Gale (2000) and Freixas et al. (2000). These theoretical papers were pivotal in their recognition of the importance of the structure of interconnections between financial institutions. In the wake of these contributions distinct approaches emerged in the literature, ranging from a static understanding of financial relationships to dynamic approaches built on assumed interactive frameworks.[4] Whereas interactive models include those dealing with contagion simulations (with specific assumptions on the reaction function of banks), the static types look at empirical data of some form of link between banks and remain mute on the mechanisms characterising contagion. Many contributions included proposals of new metrics of systemic importance.[5] Our paper relates to the empirical approach to systemic importance on the basis of static networks, and highlights the important policy content of the choice of granularity of information in the analysis of systemic importance.

Empirical approaches in the financial networks literature had to cope with the limited amount of information available. In this context, two methods for deriving financial interlinkages have been broadly used - depending on the perspective taken on the type of interconnectedness sought. The first and most easily available set of measures relies, directly or indirectly, on a given multinomial asset price distribution (which associates in some non trivial manner financial institutions). It typically exploits the asset price correlations observed among institutions through time (see for instance Billio et al. (2012)).

The second method, used in this paper, approaches instead interconnectedness as observable in "traditional" network measures of financial institutions' balance sheets. In recent years a number of papers on real-world interbank networks and payment systems have built on specific datasets by constructing indicators with a financial focus.[6] This approach has mostly remained in the supervisory community, as information of this type has proven highly confidential. Therefore this approach has almost entirely remained at the national level. Starting with the contribution by Boss et al. (2004) based on the Austrian interbank market, other studies made use of alternative country-specific datasets: Craig and von Peter (2014) for Germany, Soramäki et al. (2007) and Bech and Atalay (2008) for the U.S.A., Degryse and Nguyen (2007) for Belgium, van Lelyveld and In't Veld (2012) for the Netherlands, Fricke and Lux (2012) for Italy, Langfield et al. (2014) for the

[4] For a good overview of simulation studies and methods applied to interbank contagion see Upper (2011).

[5] For the most comprehensive summary of measures to date see Bisias et al. (2012). For recent overviews of the literature on interbank exposure networks and interbank networks at large see Langfield and Soramäki (2014) and Hüser (2015) respectively.

[6] Examples of these are *DebtRank* by Battiston et al. (2012), *SinkRank* by Soramäki and Cook (2013) or the contributions by Denbee et al. (2014) and Greenwood et al. (2015) among others.

U.K., and Alves et al. (2013) for large European banks, among others. Taken together, these and similar studies provide a series of "stylised facts" of real-world interbank networks: a tiered banking network structure (with a core of highly connected institutions to which other periphery banks are linked), low density, low average path length, a scale-free degree distribution, high clustering relative to random networks and disassortative behaviour.

Our paper builds on the dataset of Alves et al. (2013) (more on the dataset in Appendix A) and the recent contribution by Aldasoro and Angeloni (2015) that uses measures of the traditional input-output literature translated to a banking context. As noted in Alves et al. (2013), financial networks are in fact a multiplicity of networks through which large financial institutions interact, thus altering the notion of systemic importance.

At the same time our contribution elaborates on multiplex or multilayer interbank networks. The literature on multi-layer networks in general is only recently starting to take off in full force (for more details see section 3). While it is usually recognised that the web of exposures linking financial institutions is much more complex than a single matrix of exposures would suggest, data limitations have typically hindered more holistic analyses of interbank networks in which financial institutions' balance sheets are intertwined through a variety of layers. Recent contributions are starting to fill this gap. Bargigli et al. (2015) study the multiplex structure of interbank networks using Italian data broken down by maturity and by the nature of the contract involved (secured versus unsecured). They find that different layers present several topological and metric properties which are layer-specific, whereas other properties are of a more universal nature. Furthermore, they find that the overnight layer is representative of the total interbank network, whereas other layers do not bear much resemblance to the overall aggregated network. Financial layers, they established, are substantially different in terms of the characteristics that are known to determine network fragility and the relative importance of banks. Langfield et al. (2014) present an analysis of different layers of the U.K. interbank system by constructing an exposure and a funding network from very granular data. Their dataset allows them to analyse the interbank market structure by banks' sectors and their role in the interbank system, as well as by instrument class by means of cluster analysis. Their findings again point to the importance of considering different layers, as structure typically differs among them. For instance, how close the network resembles a core-periphery structure depends on the asset class considered. Further related contributions are the papers by León et al. (2014), Molina-Borboa et al. (2015) and Montagna and Kok (2013). León et al. (2014) study the network of Colombian sovereign securities settlements by combining data on transactions from three different trading and registering individual networks. Interestingly, they find strong non-linear effects in the aggregation of networks, as the most important layer in terms of market value transacted does not transfer its properties to the multiplex (or aggregated) network. Molina-Borboa et al. (2015) present an analysis of the persistence and overlap of relationships between banks in a decomposition of the Mexican banking system's exposures network. Finally,

Montagna and Kok (2013) present an agent-based model of a multi-layered network consisting of three different subnetworks: short term interbank loans, longer term bilateral exposures and common exposures in banks' securities portfolios. Their model is calibrated to European data and further balance sheet items are considered for a more comprehensive assessment of systemic risk. These studies have in common addressing in different systems or under different conceptual frameworks issues that in Europe have proven relevant for the surveillance of financial stability and macroprudential policy. Finally, a recent contribution by Poledna et al. (2015) uses a rich dataset for the Mexican interbank system, studies its multiplex structure and makes the point that the consideration of different layers in isolation delivers an underestimation of systemic risk.

3 Analysing complexity in multilayered banking networks

Following a brief review of the convenient input-output technique adopted by Aldasoro and Angeloni (2015) in approaching financial networks of a single layer (or aggregate) network (i.e. a *monoplex* interbank network) and an outline of the notion of systemic importance in section 3.1, section 3.2 develops the analytical foundation for connections through different "layers" (i.e. a *multiplex* interbank network).

3.1 The Input-Output approach to banking and the notion of systemic importance

We consider a banking system composed of n banks, each of which collects deposits and equity, lends to non bank customers and lends to and borrows from other banks. The aggregated balance sheet in matrix notation can be expressed as follows:[7]

$$\mathbf{e} + \mathbf{d} + \mathbf{X}'\mathbf{i} = \mathbf{X}\mathbf{i} + \mathbf{l} \tag{1}$$

where $\mathbf{e}$, $\mathbf{d}$, $\mathbf{l}$ are column vectors denoting respectively, equity, deposits and total non-interbank lending (composed of loans, net securities holdings and lending to (reserves at) the central bank), $\mathbf{i}$ is a unit (i.e. summation) vector of appropriate size, and $\mathbf{X}$ is the matrix of interbank gross bilateral positions,[8] where an element x_{ij} represents lending from bank i to bank j and where by construction $x_{jj} = 0$, $\forall j = 1, ..., n$. All magnitudes are expressed in monetary terms, say euros.

[7] Unless otherwise specified, we use standard notation from matrix algebra. By capital bold fonts (e.g. $\mathbf{X}$) we denote an $n \times n$ matrix with generic elements x_{ij}, whereas lower case bold fonts (e.g. $\mathbf{x}$) represent $n \times 1$ column vectors with generic elements x_i. The transpose of a matrix or vector is indicated with a prime (as in $\mathbf{X}'$ or $\mathbf{x}'$). The vector $\mathbf{x}_j$ denotes de j^{th} column of matrix $\mathbf{X}$, whereas $\mathbf{x}_i'$ stands for the ith row of matrix $\mathbf{X}$. The identity matrix is indicated by $\mathbf{I}$, the unit (column) vector is indicated by $\mathbf{i}$, and $\mathbf{i}_j$ stands for the jth column of $\mathbf{I}$. Finally, a lower case bold letter with a "hat" on it (e.g. $\hat{\mathbf{x}}$) denotes an $n \times n$ diagonal matrix with the vector $\mathbf{x}$ on its main diagonal.

[8] We can think of $\mathbf{X}$ as aggregating different types of exposures between banks. Such a matrix has been the focus of analysis of much of the work on financial networks and we refer to it as the *monoplex* interbank network.

Let $\mathbf{q}$ be a vector with total bank assets/liabilities and $\hat{\mathbf{q}}$ be a corresponding diagonal matrix, such that $\hat{\mathbf{q}}\mathbf{i} = \mathbf{q}$. Then the right hand side of Equation 1 can be written in the following form:

$$\mathbf{q} = \mathbf{X}\hat{\mathbf{q}}^{-1}\hat{\mathbf{q}}\mathbf{i} + \mathbf{l} = \mathbf{A}\mathbf{q} + \mathbf{l} \tag{2}$$

where $\mathbf{A} \equiv \mathbf{X}\hat{\mathbf{q}}^{-1}$ is the matrix of interbank positions in which each *column* is divided by the total assets of the *borrowing* bank. Hence, the columns of $\mathbf{A}$ are fractions of unity and express, for each bank, the share of funding from other banks as a ratio to total funding. Equation 2 is similar in form and interpretation to the familiar input-output system.[9] For a given matrix $\mathbf{A}$, the relation between loans and total assets is given by the well-known Leontief inverse $\mathbf{B} \equiv (\mathbf{I} - \mathbf{A})^{-1}$:

$$\mathbf{q} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{l} \equiv \mathbf{B}\mathbf{l} \tag{3}$$

The Leontief inverse $\mathbf{B}$ captures all direct and indirect connections between banks,[10] a feature that relates it to a standard result in graph theory: matrix $\mathbf{B}$ can be expressed as an infinite series as $\mathbf{B} = \mathbf{I} + \sum_{k=1}^{\infty} \mathbf{A}^k$ (we use this property later in the paper). Self impact is captured by the identity matrix, direct impact by $\mathbf{A}$ and second and higher rounds of indirect interconnections are captured by $\mathbf{A}^k$, with $k \geq 2$. The (i, j)-th element of matrix $\mathbf{A}^k$ will be positive if there exists a path of length k between banks i and j.[11] In our context this implies that banks i and j are linked indirectly in terms of borrowing/lending relationships via $k-1$ intermediaries. The Leontief inverse captures in a unique way the magnifying and distributive role shocks on lending or investment have on the interbank system.

According to this setting, we can study how a shock to non-interbank loans to one bank (l_i) affects all banks in the system, in a way that depends on the matrix of bank interconnections $\mathbf{B}$. Additionally, we may assess the effect that shocks to the primary sources of funding (equity and deposits) have on the total size of banks' balance sheets. To this end we need a slight transformation of the framework just presented, by focusing on the liability side instead of the asset side:

[9] For a complete treatment of input-output analysis including several different applications see the manual by Miller and Blair (2009) or the classic work by Pasinetti (1977).

[10] See Aldasoro and Angeloni (2015) for the conditions needed to express total assets in terms of non-interbank lending and the Leontief inverse, and for a discussion on the stability of matrix $\mathbf{X}$.

[11] Battiston et al. (2012) propose the DebtRank algorithm to identify systemic banks, within a framework that is formally related to the one presented here. Two important differences in approach are worth noting. First, in the computation of their systemic importance measure they focus on the first two elements in the infinite series outlined above, under the premise that otherwise cycles are considered. While we do not question the wisdom of their approach, our framework instead considers all indirect connections as they provide for a benchmark of potential impact (akin to an impulse-response function). Second, our framework is constructed directly from an accounting representation of the balance sheet of the banking system, so centrality is embedded within a consistent system larger than the interbank network itself.

$$\mathbf{q} = \underbrace{\mathbf{e} + \mathbf{d}}_{\equiv \mathbf{v}} + \mathbf{X}'\mathbf{i} = \mathbf{v} + \mathbf{X}'\mathbf{i} \tag{4}$$

$$\iff \mathbf{q}' = \mathbf{v}' + \mathbf{i}'\hat{\mathbf{q}}\hat{\mathbf{q}}^{-\mathbf{1}}\mathbf{X} = \mathbf{v}' + \mathbf{q}'\mathbf{O} \tag{5}$$

where $\hat{\mathbf{q}}^{-\mathbf{1}}\mathbf{X} \equiv \mathbf{O}$ is the matrix of output coefficients. This is the matrix of interbank positions in which each *row* is divided by the total assets of the *lending* bank. Hence the *rows* of $\mathbf{O}$ are fractions of unity that express, for each bank, the share of funding provided to other banks as a share of total funding provided.

It is then straightforward to see that Equation 5 yields the following:

$$\mathbf{q}' = \mathbf{v}'\mathbf{G}, \tag{6}$$

where $\mathbf{G} \equiv (\mathbf{I} - \mathbf{O})^{-1}$. Equation 6 represents the supply-side version of the input-output scheme (known as the "Ghosh inverse").

There are different ways in which the importance of nodes in a network can be characterised, ranging from simple degree metrics (measuring the number of a node's linkages), to more elaborate metrics trying to ascertain the specific role a node may play within the distribution function served by the network. Although this issue is also central in the context of multiplex networks, we seek here the simplest tractable decomposition of standard measures of importance that illustrate the lessons to be drawn from the decomposition itself.

Aldasoro and Angeloni (2015) present several measures to assess the systemic importance of banks in the interbank system, adapted from the input-output literature. Of particular interest here are two measures: namely "backward" and "forward" linkages related to Equation 3 and Equation 6 respectively. These measures are related to interconnectedness risks stemming from credit issuance and funding requirements.

Backward linkages are useful in illustrating the transmission mechanism associated to distress in non-interbank assets. For example, when bank j suffers a shock to non-interbank lending, the first impact is a reduction in bank j's balance sheet. Subsequently, however, other banks may be affected: bank j may curtail credit to other banks, spreading the effect through the system. In the end, a unitary drawdown in non-interbank lending for bank j would affect the system to an extent given by the jth column of matrix $\mathbf{B}$. The Rasmussen-Hirschman (RH) backward linkage index for bank j, which we denote as h_{b_j}, can therefore be computed as:

$$h_{b_j} \equiv \mathbf{i}'\mathbf{B}\mathbf{i}_j \tag{7}$$

The notion of a *backward* linkage stems from the fact that the hypothetical shock originates in

non-interbank lending and the index traces *back* its effect through the entire system, with interbank linkages playing a crucial role in the process. The backward index can be normalised by, for instance, relating it to the mean of the system (which we set equal to one):

$$\bar{h}_{b_j} \equiv n \frac{\mathbf{i}'\mathbf{B}\mathbf{i}_j}{\mathbf{i}'\mathbf{B}\mathbf{i}} \tag{8}$$

Similarly, the forward index captures the transmission of shocks on equity and deposits. It builds on Equation 6 by summing along the row dimension of the so-called "Ghosh" inverse $\mathbf{G}$ (based on the output matrix $\mathbf{O}$):

$$h_{f_j} \equiv \mathbf{i}_j'\mathbf{G}\mathbf{i} \tag{9}$$

The hypothetical shock in this setting comes from the non-interbank funding side of the balance sheet of the relevant bank. As opposed to the backward linkage, the original stimulus here comes from the *sources* of funds, and the index evaluates how this affects the *uses* of funds, again with the matrix of interconnections playing a key role. That is, matrix $\mathbf{A}$ starts at the end of the "banking process" with a change in non-interbank lending and traces its effect *backward* through the system,[12] whereas matrix $\mathbf{O}$ starts at the beginning with a change in primary sources of funding and traces the effects *forward* through the system.

As with the backward index, the forward index can also be normalised relative to the mean of system:

$$\bar{h}_{f_j} \equiv n \frac{\mathbf{i}_j'\mathbf{G}\mathbf{i}}{\mathbf{i}'\mathbf{G}\mathbf{i}} \tag{10}$$

Based on the normalised version of the indices one can construct a taxonomy of systemic importance as seen from either the credit or funding activities of the institutions. By normalising the indicators with the mean of the system (where such mean is set to 1), a bank showing an indicator above 1 will present an above average score of systemic importance, i.e. a shock to this bank will affect the system more than the same shock to the average of all banks. This taxonomy is summarized in Table 1 and we use it in Section 4 in order to identify the set of systemically important banks as those presenting both normalised indicators above one, i.e. banks having above average institutional levels of interconnectedness.[13] On the other hand, for the purposes of illustrating the multiplex extension of the measures, we stick to the unnormalised version of the indices as they allow for better comparability.

[12] It is implicitly assumed that the final goal of banking is the provision of credit to the non-financial sector.

[13] Battiston et al. (2015) present a taxonomy in a similar spirit by classifying banks as *impactful* or *vulnerable*, i.e. those that cost most equity depletion to others and those that suffer the most in terms of equity depletion because of others, respectively. Parallels can be drawn between the categorizations, as for instance banks that are more *dependent on funds* by definition are high borrowers and through their distress can therefore severely impair other actors in the network (i.e. they are more *impactful*).

Table 1: Classification based on backward and forward linkages

		$\bar{h}_{f_j}$	
		< 1	> 1
$\bar{h}_{b_j}$	< 1	Generally independent	Important provider of funds
	> 1	Dependent on funds from others	Key bank

3.2 Systemic importance in a multiplex context

Interconnectedness analysis generally focuses on a single network. However, in many, if not most, complex systems, the web of links connecting the actors of interest is more intricate and involved. In social networks, this is so because the links connecting different actors can take place at distinct levels: for instance people might be connected in the "workplace" network and not in the "gym" network. If one were to aggregate all connections into a single matrix of interconnections, the implications for, say, the spread of rumours, could be non-trivially altered. In transportation networks the multi-level nature of connections is even more evident: think for example of the bus, tram, subway and suburban train networks in any modern city. These networks share many nodes (in many stations one can commute from one network to the other in order to reach the final destination) and serve the overall purpose of taking people from point A to point B in a cost-effective manner. Inoperability of one node might be quite consequential if for example the node is an important nexus between different layers of the transportation network.

The fact that in many networks the edges or links connecting nodes can be of multiple types has been termed *multiplexity*,[14] as opposed to a single-layer type network which is referred to as a *monoplex* network.[15]

A single-layer graph is typically characterised as a tuple $G = (E, V)$ where V is the set of vertices or nodes and $E \subseteq V \times V$ is the set of edges or links. The characterisation of multi-layer networks requires the specification of levels or layers of connectivity between the nodes.[16]

To keep a high level of generality, one can note that a multi-layer network can have different *aspects*. A clear example of what an *aspect* represents is given by the case that concerns us in this paper, namely interbank networks. Different aspects could in this context reflect different criteria for grouping types of exposure. Thus: one *aspect* of banks' exposures depends on the "instrument

[14] See De Domenico et al. (2013), D'Agostino and Scala (2014), Kivelä et al. (2014) and references therein. As noted by these authors, the engineering and in particular the sociology literature have pioneered the study of multilayer networks.

[15] In many applications such single-type or single-layer network results from the aggregation of different subnetworks.

[16] Regarding the terminology and technical notation that we use to discuss multi-layer networks, we follow the review article by Kivelä et al. (2014).

type", whereas another *aspect* on the "maturity type" of the exposure (for a stylised example see Figure 1 below). Each *aspect* can have one or more *elementary layers*. In the interbank market example the *aspect* "maturity type" can for instance have three elementary layers: "short term", "medium term" and "long term"; while the *aspect* "instrument type" could be divided in, say, "credit claims" and "derivatives". A network can have any number of aspects d and one can define the sequence of aspects $\mathbf{L} = \{L_a\}_{a=1}^{d}$, such that to each *aspect* a corresponds a set of elementary layers L_a. The term *layer* is hence reserved for the combination of all *elementary layers* corresponding to all *aspects* of the network.

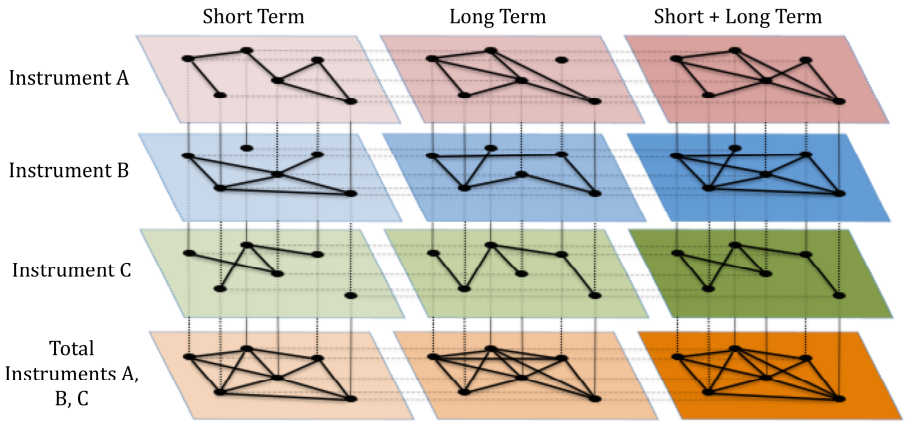

Figure 1: A stylised representation of a multiplex interbank network, where directionality and weights of links have been omitted. Black dots indicate node-aligned banks, and thick lines indicate connections within a given layer. This network displays two aspects, i.e. maturity type and instrument type. The former aspect is composed of two elementary layers (short term and long term), whereas the latter aspect presents three elementary layers (instruments A, B and C). The multiplex network is therefore composed of $2 \times 3 = 6$ layers.

The network in Figure 1 generates 6 aggregated networks depending on the aspect one wishes to emphasise. If the focus is on maturity one can aggregate the networks across instruments to get the overall short term and overall long term networks, whereas if the focus is instead on instrument types one can aggregate across maturity in order to get the network for instrument i (short + long term). Finally, either way one chooses to aggregate one can always get the overall aggregation of exposures (the lower right network).

Generally speaking, in a multi-layer network nodes can be present in all or a subset of the layers, and links can not only exist between a given node and its counterpart in another layer but also between different nodes in different layers. Our setting is in this regard more simple and we are therefore able to abstract from some complications that would require further notation: *(i)* every layer is composed of the exact same set of nodes, and *(ii)* inter-layer links are implicit and given

between a node and its counterpart (i.e. a copy of the same node) in another layer. The first feature implies that our network of interest is *node-aligned* and one can define the set of vertices as $V_M = V \times L_1 \times \ldots \times L_d$ (implying an edge set given by $E_M \subseteq V_M \times V_M$), the second feature implies that the network is *diagonal*.[17] We therefore have that multiplex networks are a subset of multi-layer networks, and we can characterise them by the quadruplet $M = (V_M, E_M, V, \mathbf{L})$.

Having briefly provided some terminology for multiplex networks, we now exemplify the measures presented above with banks' exposures being composed by a variety of instruments or maturities. Of particular interest is the decomposition of the systemic importance index for the aggregated network for a given bank, i.e. a decomposition of the vectors $\mathbf{h}_{(b)}$ and $\mathbf{h}_{(f)}$ introduced above. That is, instead of calculating systemic importance measures for the different layers and for the overall network separately, ideally one would like to see how much of the overall systemic importance score of a given bank can be attributed to each of the different subnetworks that together constitute the aggregated network of connections.

To make the discussion general let us assume that there are $\alpha = 1, \ldots, L$ different layers, such that $\mathbf{X} = \sum_{\alpha=1}^{L} \mathbf{X}_\alpha$. Without loss of generality these could represent the combination of *elementary layers* of different *aspects*: for example one aspect could be instrument type whereas another could be maturity type, as shown in Figure 1 and discussed above. The balance sheet of the banking system would read as:

$$\mathbf{e} + \mathbf{d} + \left(\sum_{\alpha=1}^{L} \mathbf{X}'_\alpha \right) \mathbf{i} = \left(\sum_{\alpha=1}^{L} \mathbf{X}_\alpha \right) \mathbf{i} + \mathbf{l} \tag{11}$$

Focusing on the asset side of the banking system we can perform transformations analogous to those used to arrive to Equation 2 and Equation 3, and we obtain the same expression involving the Leontief inverse, namely $\mathbf{q} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{l} \equiv \mathbf{B}\mathbf{l}$, with $\mathbf{A} = \sum_{\alpha=1}^{L} \mathbf{A}_\alpha$ and $\mathbf{A}_\alpha \equiv \mathbf{X}_\alpha \hat{\mathbf{q}}^{-1}$, where as before each matrix $\mathbf{A}_\alpha$ represents the interbank network for layer α where each *column* is divided by the total assets of the *borrowing* bank.

[17] Alternatively, one can define the multilayer network as a multigraph that can be represented by a weighted adjacency tensor with 3 dimensions (or 4 if we also consider maturity on top of instrument type). Using tensor decompositions one can assign scores to different layers, thereby providing an alternative way of decomposing multiplexity. See Kolda et al. (2005) for a proposal on network centrality using tensor decompositions and Bonacina et al. (2015) for an application to networks of corporate governance in italian firms. While these applications focus on the networks themselves, the approach used here takes a broader look at the balance sheet of the banking system and derives measures of systemic importance within this accounting framework. We thank an anonymous referee for noting the connection to tensor decompositions.

A useful property of the Leontief inverse is that can be expressed as an infinite series:

$$\begin{aligned}\mathbf{B} &= (\mathbf{I}-\mathbf{A})^{-1} \\ &= \mathbf{I}+\mathbf{A}+\mathbf{A}^2+\mathbf{A}^3+\cdots \\ &= \mathbf{I}+\mathbf{A}\left(\mathbf{I}+\mathbf{A}+\mathbf{A}^2+\cdots\right) \\ &= \mathbf{I}+\mathbf{AB}\end{aligned}$$

Using this and noting that $\mathbf{A}=\sum_{\alpha=1}^{L}\mathbf{A}_\alpha$, Equation 3 can be expressed as:

$$\mathbf{q}=\mathbf{Bl}=(\mathbf{I}+\mathbf{AB})\,\mathbf{l}=\left(\mathbf{I}+\sum_{\alpha=1}^{L}\mathbf{H}_\alpha\right)\mathbf{l} \tag{12}$$

where $\mathbf{H}_\alpha \equiv \mathbf{A}_\alpha\mathbf{B}$, $\alpha=1,\ldots,L$.[18] It is apparent from Equation 12 that the balance sheet equation can be decomposed into layers, such that each layer's role in magnifying shocks on the asset's side is evident.

The backward linkage index is still calculated as in Equation 7, but now we are able to attribute to each layer α its contribution to the overall systemic importance index, as measured by the column sum of the $\mathbf{H}_\alpha$ matrices.

To see what the column sums of matrix $\mathbf{H}_\alpha$ (i.e. for an instrument α) look like we can re-express the matrix in vector notation as follows:

$$\mathbf{H}_\alpha=\begin{bmatrix}\mathbf{a}'_{\alpha 1}\mathbf{b}_1 & \cdots & \mathbf{a}'_{\alpha 1}\mathbf{b}_n \\ \vdots & \ddots & \vdots \\ \mathbf{a}'_{\alpha n}\mathbf{b}_1 & \cdots & \mathbf{a}'_{\alpha n}\mathbf{b}_n\end{bmatrix} \tag{13}$$

where $\mathbf{a}'_{\alpha i}$ indicates the ith row of matrix $\mathbf{A}_\alpha$ and $\mathbf{b}_j$ denotes the jth column of matrix $\mathbf{B}$.

Now consider the backward linkage indicator of bank j, which is given by the sum of the elements in column j of matrix $\mathbf{B}$ as shown in Equation 7. The share of this index that can be attributed to layer α is given in turn by the sum of the elements in column j of matrix $\mathbf{H}_\alpha$:

$$\mathbf{i}'\mathbf{H}_\alpha\mathbf{i}_j=\mathbf{a}'_{\alpha 1}\mathbf{b}_j+\cdots+\mathbf{a}'_{\alpha n}\mathbf{b}_j=\left(\mathbf{a}'_{\alpha 1}+\cdots+\mathbf{a}'_{\alpha n}\right)\mathbf{b}_j \tag{14}$$

Note that the overall effect behind the logic of the backward linkage index is captured by the vector $\mathbf{b}_j$, i.e. the jth column of the Leontief inverse $\mathbf{B}$. In the case of a monoplex system the sum of the elements of $\mathbf{b}_j$ would constitute the index of interest. Here we are concerned with the

[18] An alternative way of arriving at Equation 12 is by noting that $\mathbf{q}=\mathbf{Aq}+\mathbf{l}$ and $\mathbf{q}=\mathbf{Bl}$, replacing the second equation in the right-hand side of the first one and using $\mathbf{A}=\sum_{\alpha=1}^{L}\mathbf{A}_\alpha$.

disentanglement of how each layer connecting banks contributes to the overall systemic importance index of banks. This is captured by the $\mathbf{a}_{\alpha i}$, $i = 1, \ldots, n$.

If bank j has exposures on layer α, it follows that this layer will have a bearing on the final index of systemic importance for bank j. On the other hand, bank j might not be exposed to another bank i on layer α, so distress in bank i on layer α will not affect bank j directly. In a monoplex network there exists the possibility that j is nonetheless affected through a third bank k which is exposed to i and to which bank j is itself exposed. The incorporation of this type of channel is indeed a major asset of network analysis. A multiplex structure expands these indirect channels, and these channels can help in assessing the nature and extent of the policy impact.

For the sake of argument, let us assume that bank j has no exposures on layer α, then it will be the case that $\mathbf{a}'_{\alpha j} = \mathbf{0}'$. By Equation 14 it is obvious that this does not imply that layer α is not relevant to account for the overall systemic importance index of bank j, since bank j is exposed to other banks on other layers $\boldsymbol{\beta} \neq \alpha, \boldsymbol{\beta} = 1, \ldots, L$ (as captured by $\mathbf{b}_j$) and these other banks might themselves be exposed in layer α (as captured by $\mathbf{a}'_{\alpha k} \neq \mathbf{0}'$, $k \neq j, k = 1, \ldots, n$).

Likewise, shocks originating in the liabilities are magnified via the different layers via a similar decomposition. As noted above, in the case of forward linkages, interest lies in tracing forward the effect of unitary declines in the primary sources of funding (deposits and equity) of any given financial institution, using the supply-side version of the input output model (see Equation 6). The forward index involves summation along the row dimension of matrix $\mathbf{G} = (\mathbf{I} - \mathbf{O})^{-1}$. In the context of a multiplex network we note that the output matrix $\mathbf{O}$ can be expressed as $\mathbf{O} = \sum_{\alpha=1}^{L} \mathbf{O}_\alpha$, where the output matrix for each layer α ($\alpha = 1, \ldots, L$) is in turn given by $\mathbf{O}_\alpha = \hat{\mathbf{q}}^{-1}\mathbf{X}_\alpha$.

Using the same logic for infinite series as above, we can re-express the Ghosh inverse as:

$$\mathbf{G} = (\mathbf{I} - \mathbf{O})^{-1} = \mathbf{I} + \mathbf{G}\mathbf{O} = \mathbf{I} + \sum_{\alpha=1}^{L} \mathbf{K}_\alpha \tag{15}$$

where $\mathbf{K}_\alpha = \mathbf{G}\mathbf{O}_\alpha$, $\alpha = 1, \ldots, L$. In vector notation such $\mathbf{K}_\alpha$ matrices can be expressed as:

$$\mathbf{K}_\alpha = \begin{bmatrix} \mathbf{g}'_1\mathbf{o}_{\alpha 1} & \cdots & \mathbf{g}'_1\mathbf{o}_{\alpha n} \\ \vdots & \ddots & \vdots \\ \mathbf{g}'_n\mathbf{o}_{\alpha 1} & \cdots & \mathbf{g}'_n\mathbf{o}_{\alpha n} \end{bmatrix} \tag{16}$$

where $\mathbf{g}'_i$ stands for the ith row of $\mathbf{G}$, whereas $\mathbf{o}_{\alpha j}$ denotes the jth column of $\mathbf{O}_\alpha$.

Since the forward linkage indicator is constructed by summing along the row dimension of matrix $\mathbf{G}$, that part of the index for bank i that can be attributed to layer α can be expressed as:

$$\mathbf{i}'_i\mathbf{K}_\alpha\mathbf{i} = \mathbf{g}'_i\mathbf{o}_{\alpha 1} + \cdots + \mathbf{g}'_i\mathbf{o}_{\alpha n} \tag{17}$$

As before, lack of exposure by bank i in layer α implies that $\mathbf{o}_{\alpha i} = \mathbf{0}$, but layer α can still contribute to systemic importance as measured by the forward index via the vectors $\mathbf{o}_{\alpha k}$ ($k \neq i, k = 1, \ldots, L$) and their interactions with the total effect as measured by $\mathbf{g}'_i$.

4 The multilayered network of large European banks

Real-world financial interaction manifests itself through a variety of channels, and the multilayered structure of financial relationships may result in important differences in results obtained by the analysis of any single or aggregate layer alone. As highlighted in section 3 and illustrated in appendix A, operational differences across layers in the multiplex network result in distinct aggregate and network characteristics of different components of a layered network system. However, the distinctions extend beyond aggregate differences highlighted in appendix A, resulting in different landscapes across layers (different configuration and composition of the layers).

These differences are relevant for policy to the extent that the structural differences imply differences in the optimal policy, as is clearly the case for macroprudential policy. Macroprudential policy benefits from this added level of granularity as the latter may clarify broad directions that policy may take in addressing market-specific characteristics, such as structural concentrations in specific layers (maturity or instrument type, for instance). Clearly related to layer-specific issues, macroprudential policy also aims to address structural vulnerabilities of *specific institutions*, such as addressing their systemic importance. Is the choice of a layer for analysis relevant to establishing which institutions are significant? If the answer is positive, *systemic importance* takes a new form. For example, *global* metrics of importance can be decomposed into layer-specific components, as shown in section 3, while the *layer specific* form allows to identify local (e.g. "in the layer") champions. For some policies the difference can be crucial. Policy aimed at a particular issue (say liquidity, market making or structural bank regulations) can profit from a more granular introspection into systemic importance.

This section motivates and builds the analysis of multiplex networks, first by illustrating key different manifestations of layer uniqueness. We illustrate the relevance of multiplex networks for macroprudential policy aimed at addressing *interconnectedness* of financial institutions, as an institution's relative importance in this regards depends on the layer or layers one considers. Finally, the institutions' systemic importance across layers is then formally deconstructed across layers within the multilayered approach outlined in section 3, thus appreciating explicitly, at the level of institutions, how interconnectedness may vary across layers.

Due to space considerations we relegate the detailed description of the data used to appendix A, which also presents a network plot of the multilayer network of large European banks (see Figure A.3). We only shall note at this stage a couple of relevant features. In particular, the dataset presents two *aspects*, namely *instrument* and *maturity* type. The partition of exposures according

to instrument type is given by: *(i) assets (A)*, further subdivided into *credit claims (CC)*, *debt securities (DS)* and *other assets (OA)*, *(ii) derivatives (D)*, and *(iii) off-balance-sheet (OffBS)* exposures. On the other hand, exposures according to maturity type are divided into: *(i)* less than one year including on sight ("short term (S)"), *(ii)* more than one year ("long term (L)"), and *(iii)* a residual of unspecified maturity (U).

4.1 Layer similarity analysis

Network layers' *similarity* or *proximity* can be assessed in a variety of ways, which in essence boil down to measuring how similar are the layer's structures. For instance, looking at the matrix of cross exposures across banks in two different layers should give an idea about how similar the interaction among banks is in the two sub networks. Measuring such "distance", for instance by means of metrics such as the links' distribution or the adjacency matrix of the two layers, is however not trivial.

At a very basic level of comparison, and as noted by Bargigli et al. (2015), it is important to distinguish between topological similarity and point-wise similarity, as one does not necessarily imply the other. For instance, two networks may be very similar in terms of density, degree distribution, etc., but the existence of a link between two nodes in the first network may be irrelevant to explain the existence of an analogous link in the second network. For interbank exposures such differences are particularly relevant, since for two identical distributions of a given characteristic across layers, point-wise dissimilarity would indicate institutional specialisation in the trade of an instrument or within a maturity type, or changes of interbank relations when carrying out the analysis in time.[19] Clearly, structural policy on interconnectedness, including that aimed at institutions like macroprudential regulation, is enriched with the identification of those markets (or layers) that are more relevant, and the institutions that are key in them.

The focus here is on point-wise similarity,[20] in particular using measures designed for binary and weighted networks.[21] A distance metric useful for binary representations of the network layers is the *Jaccard similarity index* (J), capturing the probability of observing a given connection in a network conditional on observing the same link in the other network. For a given pair of vectors $\mathbf{x}$ and $\mathbf{y}$, the index is computed as the quotient between the size of the intersection and size of the union of the two ordered vectors:

[19] Given the lack of a time series of interbank exposures for large European banks we are only able to perform the first type of comparison in the present paper.

[20] Alves et al. (2013) present some measures on topological similarity so we shall not repeat them here. Appendix A presents degree distributions as an additional means of comparing the structure of the different layers.

[21] Binary networks are those indicating only whether ties do or do not exist, in a 0-1 fashion. Weighted networks assign some value to the relationship being modelled (for instance, a monetary value). Both types of measures considered here represent the matrices as ordered vectors which are then compared.

$$J(\mathbf{x},\mathbf{y}) = \frac{|\mathbf{x} \cap \mathbf{y}|}{|\mathbf{x} \cup \mathbf{y}|} \tag{18}$$

For networks with weighted links the *Cosine similarity index* (C) can be used as proximity metric. As indicated by its name, C measures the cosine of the angle formed by the two vectors by means of a normalised dot product between them:[22]

$$C(\mathbf{x},\mathbf{y}) = \frac{\mathbf{x} \cdot \mathbf{y}}{||\mathbf{x}|| \times ||\mathbf{y}||} \tag{19}$$

Table 2 presents results for Jaccard (lower triangle) and Cosine (upper triangle) similarity according to instrument type.[23] While numbers are rarely above 50% (in particular for non-overlapping networks), they are relatively big (compared to, for instance, *Table 7* in Bargigli et al. (2015)), pointing to a relative lack of complementarity between instruments. This is particularly true when comparing *derivatives* with *assets*. *Off-balance-sheet* exposures present lower indices when compared to *assets* and *derivatives*, though values above 40% can still be considered relatively large. When comparing overlapping networks one can see that roughly 80% of the connections that are present in the *total* network are also present in the *assets* network, whereas this percentage drops below 50% when the comparison is between *off-balance-sheet* and *total* exposures. This last number implies, for instance, that almost half of the connections present in the *total* aggregated network are also present in the *off-balance-sheet* network.

As can be seen in the upper triangle of Table 2, similarity computed based on weights rather than on a binary indicator of existence/absence of relationship delivers lower values for the index, but the overall distribution remains unchanged.

[22] See the discussion in Bargigli et al. (2015) regarding the preference for this measure as opposed to the Pearson correlation coefficient for weighted networks. In the case of both similarity metrics used here, we have that $S \in [0,1]$, $S = J, C$. In principle the Cosine similarity index ranges from -1 to 1, but since the network in our application presents non-negative values only, this index only takes non-negative values.

[23] Some of the indices need to be interpreted with caution. For instance, the index comparing the *credit claims* network with the *total assets* network will be necessarily high as one is a subset of the other. While still informative, it should be read in a different light as the index comparing, say, *credit claims* and *derivatives*. We highlight with bold fonts the indices corresponding to "non-overlapping" networks in Tables 2 and 3.

	A-CC	A-DS	A-Other	A-Total	Derivatives	Off BS	Total
A-CC		**0.32**	**0.29**	0.80	**0.33**	**0.18**	0.70
A-DS	**0.50**		**0.08**	0.82	**0.26**	**0.24**	0.71
A-Other	**0.18**	**0.15**		0.29	**0.10**	**0.12**	0.26
A-Total	0.70	0.78	0.16		**0.36**	**0.26**	0.88
Derivatives	**0.50**	**0.46**	**0.15**	**0.53**		**0.13**	0.66
Off BS	**0.44**	**0.37**	**0.16**	**0.41**	**0.41**		0.54
Total	0.57	0.63	0.13	0.81	0.61	0.48	

Table 2: *Jaccard* (lower triangle) and *Cosine* (upper triangle) Similarity Indices, by instrument type. Non-overlapping networks are highlighted in bold fonts. A=Assets, CC=Credit Claims, DS=Debt Securities, OffBS=Off Balance Sheet.

A relative lack of complementarity between different maturities can also be appreciated from the high values reported in Table 3: the long and short term networks share 62% of connections. This number drops to 43% when evaluating Cosine similarity (upper triangle of Table 3, still a high number for a comparison of weighted matrices).[24] Table 4 in appendix B delves more deeply into the combination of instrument and maturity and reinforce the message: highest values of similarity are typically between different maturities for the same type of instrument.

	Long	Short	Total	Unclassified
Long		**0.43**	0.75	**0.03**
Short	**0.62**		0.81	**0.23**
Total	0.69	0.73		0.50
Unclassified	**0.04**	**0.03**	0.16	

Table 3: *Jaccard* (lower triangle) and *Cosine* (upper triangle) Similarity Indices, by maturity type. Non-overlapping networks are highlighted in bold fonts. A=Assets, CC=Credit Claims, DS=Debt Securities, OffBS=Off Balance Sheet.

Layers, therefore, appear *closer* to the extent they represent the same type of business, and to some extent maturity underlying the relation. The differences, however, can be non-negligible across layers of activity types. This is a first indication of the importance of explicitly addressing the multilayer structure of financial networks.

[24]Some matrix comparisons between overlapping networks present a Cosine above the Jaccard similarity: this indicates that when taking monetary values into account the proximity between the matrices is higher than when only considering the existence/absence of relationships.

4.2 Systemic importance across layers of the banking network

Before reaching the point of decomposing systemic importance into layer-specific contributions one can approach the problem of systemic importance in a multilayer system from an additional, complementary perspective. In particular, it is important to evaluate the robustness of institutions' relative systemic importance across layers, as well as gauging how appropriate is the use of an aggregate network representation for supporting policy aimed at addressing a particular type of market failure. For systemic importance one thinks of different institutions' roles in the transmission of shocks or whether an institution belongs to a particular "core".

One could hope that for analytical purposes the coupling of different interactions (or layers) occurs randomly. Yet, in real-world complex systems the specific, non-random structure of multiplexity can result in significant and more systematic differences. The similarity in the transmission across layers has been termed *correlated multiplexity* by Lee et al. (2014).[25] Such feature has been suggested to be central for the unfolding of failure cascades through the system (see Buldyrev et al. (2010)).

In order to evaluate the extent of correlated multiplexity we first look at a number of measures of centrality not allowing for an analytic decomposition between layers, which capture alternative notions of systemic importance and test whether there is evidence of correlated multiplexity for any or all of these measures. When the centrality rankings of banks across layers is *resilient*, then one can speak of the presence of correlated multiplexity, and thus independently of the specific structure of the transmission involved the *central* institutions are likely to be the same (ordered in the same manner). When this is not the case, then multiplexity is not correlated and this centrality is layer-specific. Following the analysis of the robustness of well known systemic importance measures, we illustrate the differences across layers by outlining the contrasting results across layers of a core-periphery analysis.

Correlation of systemic importance across layers For a wider comparison of centrality across layers we explore the correlation between layers of selected measures of local and global connectivity alternative to our analytical concept of systemic importance. We consider degree and strength measures of bank centrality, which for a directed network measure respectively the number of relationships and the weight of such links. A directed version of *PageRank*, the algorithm used by Google to rank webpages within their search engine is used for connectivity at the global level.[26]

The correlation of the measures across layers (see tables 5, 6 and 7) overwhelmingly points to

[25] For instance, it can be expected that a person with many links in the "friendship" layer of a social multiplex network also has a relatively high number of links in other networks by virtue of her being a friendly person. Parshani et al. (2010) show how well connected ports tend to couple with well connected airports. Barigozzi et al. (2010) present similar evidence for the international trade network by comparing commodity-specific sub-networks and the aggregate.

[26] In Aldasoro and Angeloni (2015) it is shown how the input-output measures used here can be mathematically related to degree and eigenvector measures.

the existence of positively correlated multiplexity. Banks that are well connected or important in a network tend also to be well connected in other networks. This is particularly strong when looking at degree centrality for both its incoming and outgoing versions (though in general out-degree centrality tends to present slightly lower correlations than in-degree centrality). When looking at strength centrality the message remains unaltered, though the magnitudes are reduced, in particular for out-strength in the short term derivatives network and the two off-balance-sheet networks. Focusing on node-specific global measures of importance does not alter the takeaway from the analysis: the correlation of PageRank centrality across networks delivers similar results to strength centrality, though slightly stronger than the latter (Table 7).

Two additional centrality measures, *closeness* (in its *in* and *out* versions) and *betweenness* centrality, show similar results.[27] With a couple of exceptions (short versus long term assets and short versus long term derivatives), *in-closeness* does not present strong correlated multiplexity. But for *out-closeness* we again have very strong evidence of positively (and significant) correlated multiplexity. Finally, for betweenness centrality we also have positively correlated multiplexity, significant for all but one combination of networks and with values somewhat smaller than those observed for *PageRank-in* centrality. The results on correlated multiplexity are even stronger when considering Spearman rank correlation instead of the simple Pearson correlation.

Taken together, these results suggest that, for the network considered, centrality is strongly permanent across layers, underlining the usefulness of ranking under limited information in identifying critical institutions. In contrast, using data on the Mexican interbank system Molina-Borboa et al. (2015) find that important actors in the different layers differ[28] and take this as a significant reason to motivate the use of multiplex networks instead of just a single aggregated network. We agree, but argue further that even in the case in which the importance of actors is strongly persistent across layers (as it is the case for the network considered here), there is still significant value in considering all possible layers (i.e. the multiplex network), especially if one is able to decompose systemic importance into layer-specific contributions as our method illustrates. Before substantiating this point in Section 4.3, we look into differences in the core-periphery structure across layers to illustrate further differences.

Core-periphery structure An important avenue for characterising structural features of networks is the core-periphery paradigm, as such structure is ubiquitous in several types of networks.[29]

[27] Results are available upon request from the authors.

[28] They conflate *importance* with *belonging to the core*. While strictly speaking systemic importance and coreness are different (though related) concepts, we find their nomenclature appropriate for their purposes and hence take it at face value.

[29] For applications to interbank networks see: Craig and von Peter (2014) who propose an algorithm for the identification of the core set of banks based on the idea of intermediation and use it with German data, Fricke and Lux (2012) for the case of Italy, van Lelyveld and In't Veld (2012) for the Netherlands, Soramäki et al. (2007) for the FedWire payment system in the U.S. and Langfield et al. (2014) for the U.K., among others. For applications in other sciences see references in Borgatti and Everett (1999).

We focus on two approaches to identify coreness of financial institutions: the *block-modeling*-based approach of Craig and von Peter (2014) and the core-periphery *profile* approach of Della Rossa et al. (2013) (due to space considerations we relegate the details to Appendix C).

Figure C.1 presents the results from the core-periphery analysis based on the algorithm developed by Craig and von Peter (2014).[30] The size of the core is notably large, with exception of the "marginal networks" (i.e. *other assets* and *unspecified maturity*). Important in reading these large figures is noting that the sample of banks in the study is already the core of the european financial system, and thus we are looking at the "core of the core". In particular, banks in the sample are already "national champions", and tend to be well connected and thereby display strong interconnectivity density. Each of the different layers feature more than 20 banks in the core, whereas the overall network presents 31 banks in the core. Within the instrument type division, the off-balance sheet network features the best fitness to the core-periphery model (lowest error score), whereas short term exposures fit the core-periphery model slightly better than long term exposures. In terms of core composition, the instrument dimension shows a bit more heterogeneity: there are 11 banks which are part of the core in all subnetworks (excluding *other assets*). For the maturity dimension this is not the case: out of the 23 banks present in the core of the long term network, 20 are also part of the short term core.

Figure C.2 presents the core-periphery profile by instrument and maturity type against two extreme benchmarks, namely the complete network and the star network. There are some similarities with respect to the insights gained from the block-modelling approach, but also some differences. In particular, the partition according to the instruments' maturity shows more homogeneous results than that of the instruments' type, where differences are more clear.

In addition, the short term network is closer to an idealised core-periphery structure (in the block-modelling analysis this manifested itself in a better fit, see Figure C.1), whereas in the second analysis the core-periphery profile of the long term network is very similar to that of the total network (same number of core banks[31] and very similar centralisation index, see also Figure C.3). There are three banks which are simultaneously present in the core of the short term, long term and total networks.

Regarding results of the type partition, *off-balance sheet* and *derivatives* exposures show more proximity to the star-network ideal (Figure C.2) and a higher centralisation index (just as before they showed a lower error score, see Figure C.1). Different from the the block-modelling approach, the *debt securities* network shows the most proximity to the total aggregated network in terms of its core-periphery profile. There is only one bank that belongs to the core in all networks according to instrument type (excluding *other assets*). At the same time, all of the banks belonging to the core in the *derivatives* and *off-balance sheet* networks are part of the core in the aggregated network,

[30] We thank Ben Craig for making available the code needed for the computation.
[31] We take as core banks those that have an α-coreness above 0.5. See Appendix C for details.

while for the *debt securities* network this number is only 4 out of 8. Similar core-periphery profiles are therefore no guarantee of similar composition of the core.

Finally, we note that at the very top of the coreness ranking we never find the same bank for any pair of networks when the focus is on either instrument or maturity type. That said, the bank with the highest coreness in the *short term* network is also the same bank sitting at the top of the *debt securities* coreness ranking (also for the aggregated network), whereas the top bank in terms of coreness for the *long term* network coincides with that of the *derivatives* network. This is critical for policy aimed at core banks: depending on whether the layer used for assessing the importance of a bank is representative of the externality to be addressed by policy can lead to different and possibly erroneous institution-specific regulations.

Even though the stability of these results through time cannot be asserted with the information available, they already indicate the importance of some banks across and within layers may not be as universal as suggested by the monoplex approach. In order to better understand what differences may be evident in the data that escape the broader correlation analysis and which may motivate differences in the core-periphery boundary across measures, we now look in more detail at the alternative and more detailed specification of decomposition of the layer structures.

4.3 Identifying systemic importance across layers numerically

Motivated by the results on the varying composition of the core, we look into more detail on the identification of systemic institutions. For this purpose, we look at an institution's importance in the input and output mechanisms of shock transmission, as these illustrates its criticality for spreading external shocks affecting banks' balance sheets (lending and/or equity/deposit).

The decomposition of global (monoplex) systemic importance into layer-specific contributions developed in section 3.2 provides an estimate of the contribution of a bank's activity within each layer to the overall importance of that bank. The decomposition of backward and forward indices for the top 10 banks according to instrument and maturity type is illustrated in figures D.1 and D.2 in Appendix D . There is no reason to expect that the ranking of top 10 banks for each indicator would coincide, and indeed this is not the case. That said, 5 out of the top 10 banks are shared by the two indicators.

For the analysis of the decomposition of systemic importance indicators into the contributions by each layer we make use of the classification outlined in Table 1 and focus on the set of systemic banks, i.e. those that present both normalised backward and forward indices above one. Figure 2 presents the normalised indices for all banks for the two indicators considered; systemic banks are those in the upper right quadrant of the figure (i.e. those that show a score above the mean of the system in both indicators). Some banks highly ranked in one of the indicators may not be considered systemically important because they rank poorly in the other indicator: this is the case

most notably for the third ranking bank according to the forward index (bank 27). Additionally, there is one bank that is not in the top 10 banks in either indicator but still is systemically important. Not surprisingly, five banks feature in both lists of top ten banks.

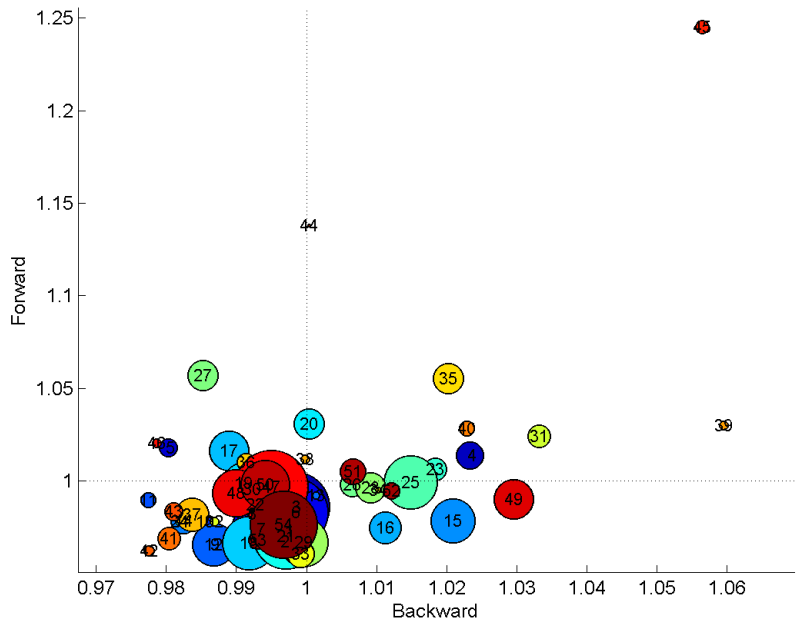

Figure 2: Normalized backward and forward indices. Bubble size indicates (scaled) total assets.

Figure 3 presents the backward and forward indices for systemically important banks by instrument type. For the backward index a significant portion of the contribution to the overall systemic importance of the most important banks is given by the two main *asset* exposure sub-categories, namely *credit claims* and *debt securities*. This is broadly in line with the composition of exposures by instrument (see Figure A.1), in particular for the two top banks for which also *off balance sheet* exposures account for a non-negligible share of their critical transmission role.

Despite accounting for slightly more than 25% of exposures, the *derivatives* network does not contribute much to the ranking of systemically important banks, pointing to one important insight of our analysis: importance in terms of interconnectivity is driven by more than size.

The backward index focuses on the importance of the transmission of shocks to the borrowing side of the balance sheet, by summing along the column dimension of the transformed interbank matrix. Conversely, by summing along the row dimension, the forward index illustrates the transmission of shocks on the lending side of the balance sheet. In the case of such asset shocks, the outline of importance is quite different: the top bank (bank 45) stands out notably against the rest, and an overwhelming majority of this difference is attributable to the *off-balance sheet* net-

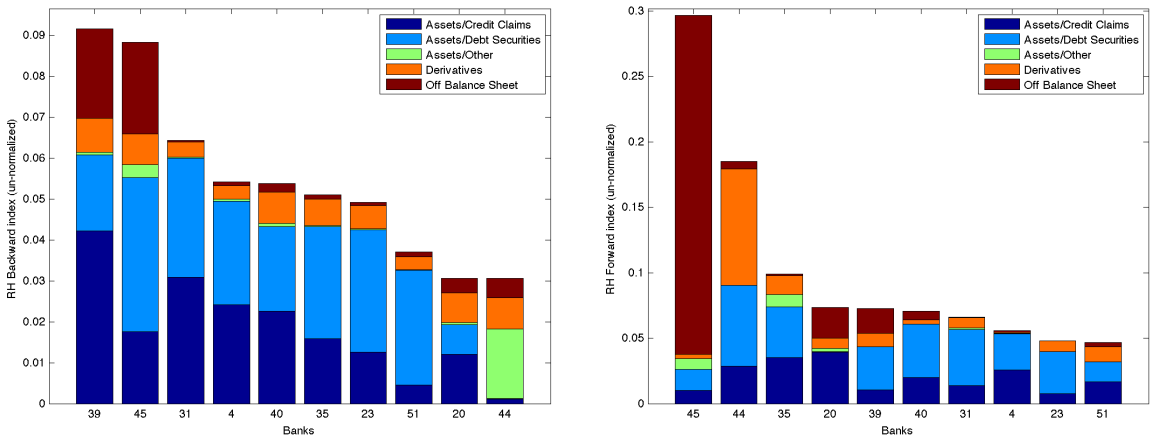

Figure 3: RH backward (left) and forward (right) indices, for systemic banks by instrument type.

work. Naturally, bank 45 is an important player in terms of interconnectedness in this particular network,[32] but still it only accounts for roughly 1/5 of all exposures and it is not in fact the top bank in this regard. A network that has a rather minor share of overall exposures (the *off-balance sheet* network accounts for 1/7 of exposures, see Appendix A) can nonetheless be a major driver of the systemic importance score of specific banks. While such a result might seem obvious at the intuitive level, our measures provide a clear-cut way of quantifying this. Such insights go unnoticed in a layer specific analysis of centrality.

The second ranking bank (bank 44) has on the other hand a significant share of its score accounted for by the derivatives network. Interestingly, four out of the ten systemically important banks do not form part of the core of any network (according to Craig and von Peter (2014)'s algorithm). Systemic importance measured in terms of a bank's ability to transmit shocks then also needs to be qualified by the nature of the shock affecting the system, in addition to the market through which the transmission takes place.

The comparison of a bank's forward and backward transmission role indicates that the drivers of systemic importance can vary substantially depending on the criterion of emphasis, and there is value in the consideration of both dimensions simultaneously.[33] Figure D.3 presents a comparison of the contribution of the different layers to the systemic importance score of all banks, by instrument type. It can be seen that while the contribution of the *off balance sheet* network to the forward score of bank 45 is substantial, this is not so for the backward index (nor for other layers like derivatives

[32] In fact, this bank belongs to the core of only two networks: the *other assets* (a rather "marginal" network) and the *off-balance sheet* networks.

[33] Furthermore, the methodology presented in Section 3.2 allows for further decompositions, though for ease of exposition we do not pursue this here. For instance, we could partition the indices according to *both* instrument and maturity type, allowing for a finer distinction of the sources of systemic importance. For illustrative purposes, we focus on a specific partition by instrument and maturity type, and a specific subset of banks.

for the same bank). In these charts, a layer contributes more to systemic importance as captured by one type of indicator vis-à-vis the other to the extent that more bubbles lie on one side of the 45 degree line.

Figure 4 displays the partition of systemic importance indices by maturity type for the 10 more systemic banks. *Long* and *short term* exposures account for roughly 4/5 of the total Figure A.1), in almost equal proportions. For the backward index we see that with a couple of exceptions *long term* exposures account for the largest share. Most notable within the exceptions is the second highest ranking bank, for which half of the index is explained by *short* term exposures with the other half shared equally between *long term* exposures and those of *unspecified* maturity.

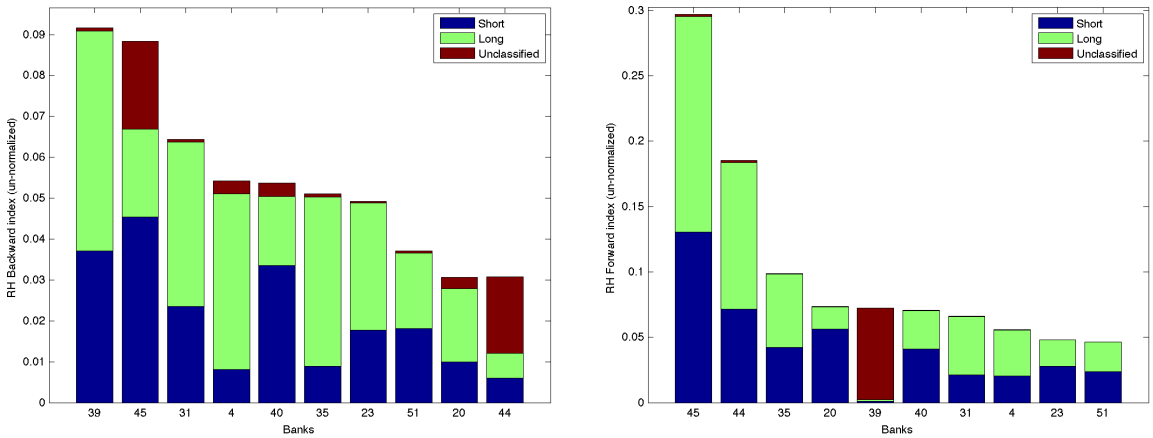

Figure 4: RH backward (left) and forward (right) indices, for systemic banks by maturity type.

When it comes to the forward index we see that for the three top systemically important banks at least half of the score is accounted for by the *long term* network. With the exception of one bank, the network of *unspecified* maturity does not play a role (the entire forward transmission score of the exceptional bank is attributable to this network). It could in principle be the case that a bank has a great deal of its systemic importance score explained by *off-balance sheet* instruments of *unspecified* maturity. The ability to detect such opacity as the driver of the systemic importance ranking of a bank is another potential interesting insight from the analysis we present. In particular, such a case might be in the interest of supervisors/regulators to identify. Similarly to Figure D.3, Figure D.4 presents a comparison of contributions by the different layers to backward and forward indices by maturity type.

Based on the algorithm by Craig and von Peter (2014) five out of the ten systemically important banks belong to the core of the *short term* network, whereas three out of this five also belong to the core of the *long term* network. Based on the alternative algorithm for identification of a core-periphery profile by Della Rossa et al. (2013) , only one systemic bank belongs to the core of the

long term network (bank 35) and none to the *short term*. There are no banks that are present in the core of most networks regardless of the method used to identify the core. When the method used is that of Craig and von Peter (2014), bank 35 is present in all cores except that of the *off-balance sheet* network. Identification of systemic importance is far from being a synonym with being in the core, neither for the aggregate network nor for its different layers, and the choice among the different implementations matter for policy makers.

The banks identified in the figures above pose a risk to systemic stability, and the method proposed here allows for a decomposition and description of the sources of such systemic importance from a holistic perspective, starting from a matrix representation of the balance sheet of the entire banking system. Even when the derivation of the measures is grounded in theory and involves some matrix algebra, the ultimate motivation of our endeavour is practical in nature. We posit that such measures can be of use for bank regulators and supervisors. In this respect, we view their simplicity and straightforward implementation as an important asset.

5 Concluding remarks

The recent financial crisis brought to the fore the relevance of interconnectedness in general, and in particular in interbank markets. Of critical importance in this context is the identification of the key players in the financial network. While early contributions on the topic have focused on aggregated exposures, it is now increasingly recognised that the web of reciprocal exposures linking bank balance sheets is more intricate and complex. Interbank networks are better characterised as multiplex networks, featuring connections at multiple levels.

In the present paper we analyse the multiplex structure of the network of large European banks, making use of a detailed dataset presenting exposures partitioned according to maturity and instrument type. We find a high level of similarity between the different layers (either by instrument or maturity), a core periphery structure which comprises a large core (relative to studies using country-specific datasets), and positively correlated multiplexity. The results suggest that an institution's role in the channel of transmission is critical in determining the global importance of such institution, that the notion of importance may not be related to the traditionally studied core-periphery role of the institution, and that nevertheless key forms of non-decomposable (across layers) centrality measures are resilient across layers, indicating that global forms of centrality are robust to the absence of granular data availability.

Importantly, we develop a measure of systemic importance suited to the case in which banks are connected through an arbitrary number of different layers. This allows us to compute systemic importance indicators and decompose them into the contributions of the different layers, providing a holistic analysis that truly incorporates the mutiplex structure of the network (instead of doing separate analyses for the different layers and the aggregate network) and is built directly from a

consistent accounting representation of the system. Previous literature has justified the need to use granular data by noting that banks can rank differently in different layers in terms of systemic importance and thereby the focus on a single layer can be misleading. We note that even when centrality is persistent across layers, there is still important information to be obtained from granular data, in particular if one is able to decompose global systemic importance into layer-specific contributions. We illustrate our measures with the dataset on exposures between large European banks. Our approach builds on the logic that drives the policy process of assessing banks' importance at the Financial Stability Board (FSB) in its recognition of the importance of interconnectedness, but delves deeper into this aspect by considering the different layers in an integrated accounting framework. The insights from the measures proposed can be of policy relevance for supervisors and regulators.

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A Overview of the data

We make use of a unique dataset of anonymised interbank exposures between large European banks, originally presented in Alves et al. (2013). The preparation of the dataset was part of a collective effort undertaken by the *European Systemic Risk Board*, the *European Banking Authority* and national supervisory authorities. In particular, the dataset presents bilateral exposures between 53 large European banks as of end 2011, with breakdown according to both maturity and instrument type.[34] Banks report their exposures at the level of the banking group, the implication being that non-financial subsidiaries and insurance are not included in the reporting exercise, and exposures to counterparties were aggregated using an accounting scope of consolidation (hence including all subsidiaries of that counterparty). The exposure data represents a directed and weighted network, in which each node is represented by a banking group and each link going from node A to node B accounts for an exposure of the former to the latter. The multiplex structure of the dataset stems from the level of disaggregation that the dataset contains, namely along the instrument and maturity dimensions.

The partition of exposures according to instrument type is as follows: *(i)* assets, further subdivided into credit claims, debt securities and other assets, *(ii)* derivatives, and *(iii)* off-balance-sheet exposures. On the other hand, exposures according to maturity type are structured according to: *(i)* less than one year including on sight ("short term"), *(ii)* more than one year ("long term"), and *(iii)* a residual of unspecified maturity.[35] Coming back to the terminology introduced in section 3, the interbank exposure dataset used presents two *aspects*, namely *instrument* type and *maturity* type. The former has either 3 or 5 *elementary layers* (depending on whether we consider the subdivision of assets or not) whereas the latter presents 3 *elementary layers*. As a consequence we can have either 9 or 15 *layers*.[36]

The combination of the three types of asset exposures (namely credit claims, debt securities and other assets) accounts for almost 2/3 of overall exposures (see Figure A.1). Derivatives explain 27% of exposures, whereas off-balance-sheet items account for the smallest share at 17%. Furthermore, almost all banks report exposures on credit claims and debt securities and therefore the shares are more evenly distributed among banks: to reach 80% of total exposures for each of these categories one needs 18 banks. While also a significant amount of banks report derivatives exposures, these are more concentrated as it takes 12 banks to reach the 80% share of total exposures in this category. Off-balance-sheet exposures, on the other hand, are characterised by fewer banks reporting and

[34] There are 53 reporting banks, while there is a bank which did not report exposures to other institutions. Hence, there are 53 banks reporting exposures to 54 banks, and the matrices we work with are of dimension 54-by-54.

[35] The original dataset presents further subdivisions of interbank credit claims and off-balance-sheet items. We keep the partition mentioned above as it is enough for our illustrative purposes. But it should be noted that the method presented here can be applied to any arbitrary number of sub-divisions.

[36] As noted earlier, one can focus attention on a given aspect of the network by aggregating across the other aspect , i.e. in order to focus on maturity type, aggregate across instrument type to have the "short term", "long term" and "unspecified maturity" networks.

much more concentration: the first 6 banks account for 80% of exposures in this category. Regarding maturity type, the distinction between exposures is not different as it is for instruments: short and long exposures account for roughly the same share of overall exposures and both present a similar number of reporting banks.[37]

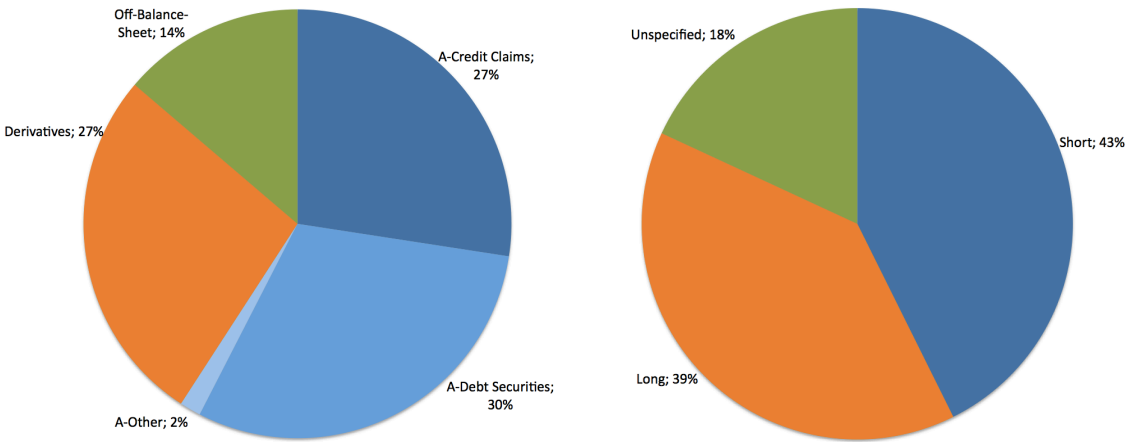

Figure A.1: Composition of exposures by instrument and maturity (left and right panel respectively).

Exposures of unspecified maturity behave quite differently, with just 7 banks reporting and with the first 3 accounting for 75% of exposures in this category. When combining the maturity and instrument type dimensions, one sees that each instrument presents its own nuances in terms of maturity breakdown (see Figure A.2). While asset exposures are marginally tilted toward shorter maturities, derivatives present more diversity and are slightly leaning towards longer term exposures. Off-balance-sheet exposures, on the other hand present a good deal of dispersion for short term but not so much for long term exposures, which present a very small median. Furthermore, for this instrument type there is a non-negligible share of exposures of unspecified maturity, which speaks to the well-known opacity of these types of instruments.

It should be noted that the network built with the data comprises the exposures *between* the group of banks which participated in the exercise, hence exposures of this group of banks to other EU banks are not considered in the analysis. That said, the bilateral exposures reported by banks represent slightly more than half of total exposures to EU banks, thereby capturing a good piece of the action, in particular between the larger banks.[38]

The level of connectivity of the network of large European banks is large relative to other

[37] It should be noted that in the case of long term exposures there is one bank which stands far above the rest, accounting for almost 20% of exposures in this category.

[38] The share of exposures to large EU banks relative to total exposure to EU banks varies depending on instrument type and bank, with the median for total exposures being 60%. For details see Alves et al. (2013), in particular *Chart 2*.

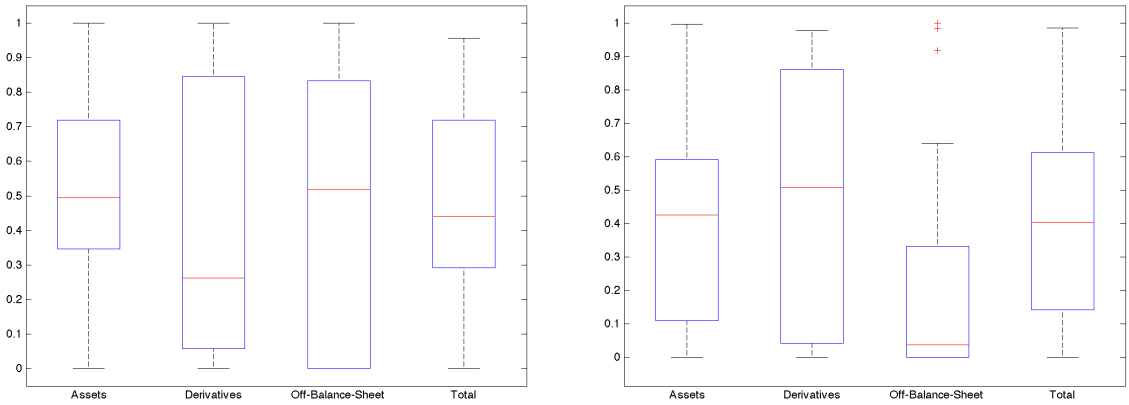

Figure A.2: Share of short and long term exposures (left and right panels respectively) in total exposure by instrument.

studies that focus on national banking systems, as reflected in the relatively high density of the network: 60% of all possible connections are actually present for total exposures. While the density is naturally lower in composing layers (48%, 36% and 29% for assets, derivatives and off-balance-sheet respectively), values still remain well above those encountered in previous studies focusing on national banking systems.[39] This can be appreciated visually on Figure A.3, which provides the real data counterpart to the stylized multilayer chart presented in the main body of the paper.

Figure A.4 and Figure A.5 present the degree distributions according to instrument and maturity type respectively. While there is not much of a difference between the degree distributions of the long and short term networks, the layers corresponding to different instruments present more diversity. The layer corresponding to the assets network is closer to the overall network for both in- and out-degree distributions, in line with the share of these type of exposures in overall exposures (see Figure A.1). On the other hand, the off-balance-sheet layer is farthest from the total network, with the derivatives layer lying somewhere in between. The distinction between layers becomes less clear-cut for the out-degree distribution.

Further properties of the network are its high level of reciprocity (especially for higher levels of aggregation of exposures) and its disassortative behaviour. Higher reciprocity implies that a high share of exposures are reciprocated by a corresponding counter-exposure, suggesting that systemic risk might be lower for high levels of aggregation since some exposures might be netted. A network is said to be assortative (with respect to its nodes' degrees) if high (low) degree nodes tend to be connected to other high (low) degree nodes. If instead high degree nodes tend to be connected to low degree nodes the network is said to be disassortative. Interbank networks are typically found to

[39] See among others Craig and von Peter (2014), van Lelyveld and In't Veld (2012), Fricke and Lux (2012), Soramäki et al. (2007) for the cases of Germany, the Netherlands, Italy and the U.S. respectively.

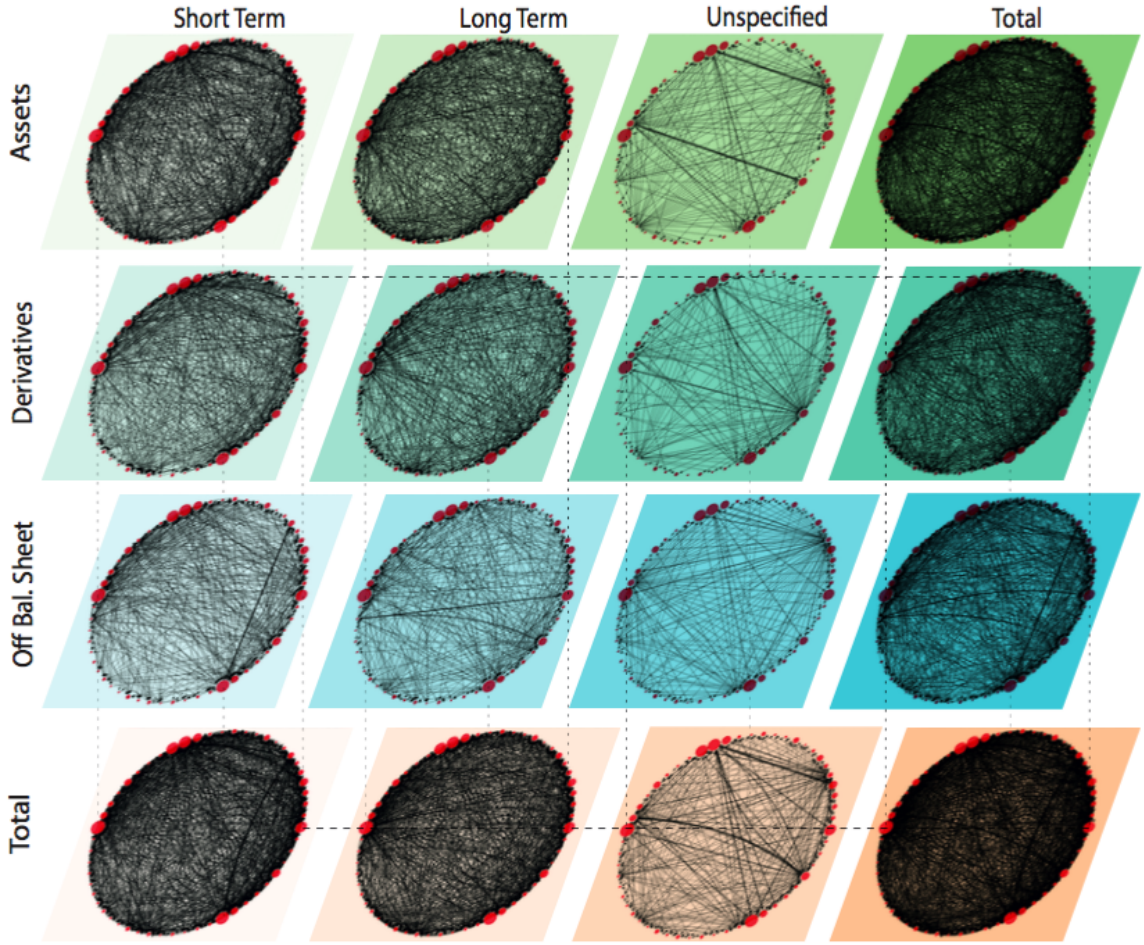

Figure A.3: The multilayer network of large European banks. Node size indicates total assets, links width indicate size of exposure, whereas link direction goes from lender to borrower bank.

be disassortative, and the network of large European banks is no exception. This feature is closely associated to the core-periphery structure of interbank networks found in the extant literature, and it reflects efficient specialisation between the actors in the network. In the European interbank network this specialisation is more apparent in granular instrument and maturity types. For further details on the network and some of its topological properties we refer the interested reader to Section 3 in Alves et al. (2013).

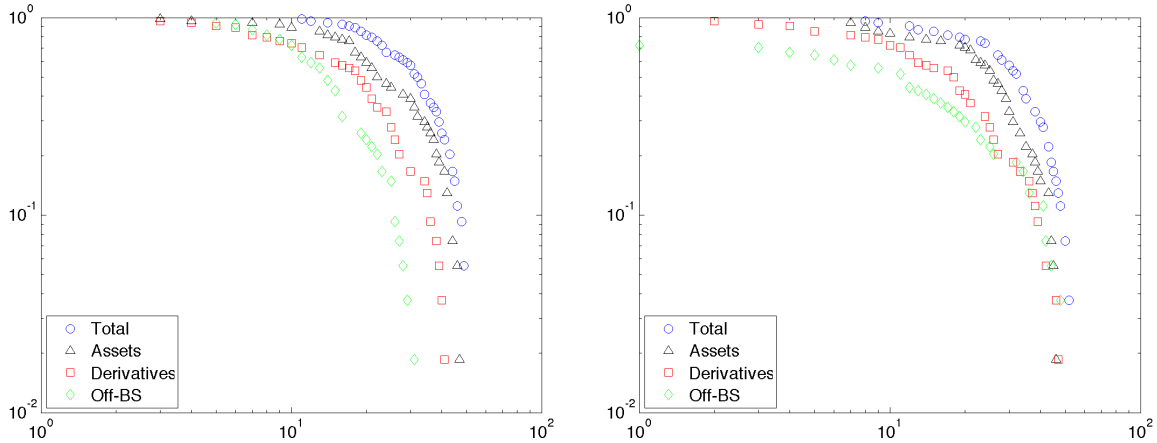

Figure A.4: In- and out-degree distributions - left and right panel respectively - by instrument type (in log-log scale).

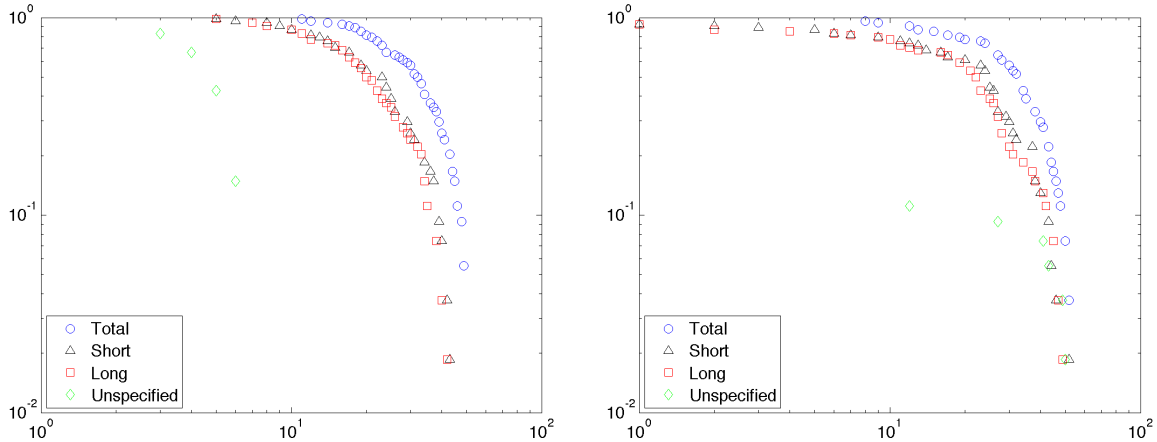

Figure A.5: In- and out-degree distributions - left and right panel respectively - by maturity type (in log-log scale).

B Additional Tables

	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*
(1) **Assets-CC L**		0.23	0.33	0.14	0.30	0.14	0.35	0.06	0.08	0.11
(2) **Assets-CC S**	0.33		0.22	0.13	0.08	0.15	0.24	0.15	0.04	0.12
(3) **Assets-DS L**	0.31	0.43		0.31	0.07	0.07	0.24	0.09	0.04	0.23
(4) **Assets-DS S**	0.25	0.37	0.42		0.01	0.05	0.11	0.04	0.05	0.28
(5) **Assets-Other L**	0.16	0.09	0.09	0.08		0.09	0.13	0.02	0.01	0.01
(6) **Assets-Other S**	0.13	0.11	0.10	0.11	0.18		0.07	0.03	0.15	0.12
(7) **Derivatives L**	0.27	0.35	0.35	0.27	0.11	0.11		0.18	0.07	0.11
(8) **Derivatives S**	0.27	0.40	0.33	0.24	0.09	0.09	0.45		0.02	0.06
(9) **OffBS L**	0.35	0.25	0.26	0.23	0.11	0.09	0.23	0.23		0.36
(10) **OffBS S**	0.32	0.38	0.27	0.26	0.11	0.10	0.29	0.34	0.31	

Table 4: *Jaccard* (lower triangle) and *Cosine* (upper triangle) Similarity Indices, by instrument and maturity type. CC stands for Credit Claims, DS stands for Debt Securities, and L (S) stands for Long (Short) Term.

	Assets-L	**Assets-S**	**Deriv.-L**	**Deriv.-S**	**OffBS-L**	**OffBS-S**
Assets-L		0.77***	0.64***	0.52***	0.57***	0.44***
Assets-S	0.88***		0.71***	0.58***	0.39***	0.60***
Deriv.-L	0.69***	0.80***		0.72***	0.44***	0.53***
Deriv.-S	0.78***	0.89***	0.90***		0.40***	0.63***
OffBS-L	0.83***	0.87***	0.73***	0.79***		0.52***
OffBS-S	0.84***	0.91***	0.79***	0.86***	0.92***	

Table 5: Correlation indices for *in-degree* (lower triangle) and *out-degree* (upper triangle) centrality. L (S) stands for Long (Short) Term. *** (**,*) denotes statistical significance at the 1% (5%, 10%) level.

	Assets-L	**Assets-S**	**Deriv.-L**	**Deriv.-S**	**OffBS-L**	**OffBS-S**
Assets-L		0.62***	0.83***	0.15	0.14	0.25*
Assets-S	0.50***		0.49***	0.18	0.08	0.26*
Deriv.-L	0.46***	0.64***		0.11	0.17	0.17
Deriv.-S	0.37***	0.70***	0.85***		-0.00	0.09
OffBS-L	0.26*	0.67***	0.48***	0.55***		0.69***
OffBS-S	0.40***	0.79***	0.55***	0.55***	0.52***	

Table 6: Correlation indices for *in-strength* (lower triangle) and *out-strength* (upper triangle) centrality. L (S) stands for Long (Short) Term. *** (**,*) denotes statistical significance at the 1% (5%, 10%) level.

	Assets-L	Assets-S	Deriv.-L	Deriv.-S	OffBS-L	OffBS-S
Assets-L		0.62***	0.78***	0.14	0.18	0.39***
Assets-S	0.60***		0.45***	0.18	0.07	0.22
Deriv.-L	0.52***	0.71***		0.15	0.16	0.19
Deriv.-S	0.46***	0.75***	0.87***		-0.00	0.16
OffBS-L	0.33**	0.65***	0.52***	0.53***		0.73***
OffBS-S	0.50***	0.80***	0.61***	0.61***	0.56***	

Table 7: Correlation indices for *PageRank in* (lower triangle) and *out* (upper triangle) centrality. L (S) stands for Long (Short) Term. *** (**,*) denotes statistical significance at the 1% (5%, 10%) level.

C Core-periphery Analysis and Figures

In the wake of the seminal contribution by Borgatti and Everett (1999), different methods to assess the core-periphery structure of networks have been proposed. Borgatti and Everett (1999) propose two types of core-periphery analyses, namely a discrete and a continuous version. The former builds on block-modelling and proposes a two-class partition of nodes, in which some nodes form part of the densely connected core (a *1-block* in block-modelling terminology) while the remaining nodes make up the periphery, loosely connected to the core and not connected within itself (a *0-block*). A stylised representation in matrix notation would be as follows:

$$\mathbf{M} = \begin{bmatrix} \text{Core-Core} & \text{Core-Periphery} \\ \text{Periphery-Core} & \text{Periphery-Periphery} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & ? \\ ? & \mathbf{0} \end{bmatrix} \tag{20}$$

where the off-diagonal blocks are left to be specified by the researcher depending on the application at hand. The idea is then to a find a fit (optimal in some sense) between a given empirical network and this idealised structure, such that the optimal core size is found in the process.

The continuous model of core-periphery analysis assigns a level of "coreness" to each node, without the need to partition the network into two (or more) classes of nodes ex ante. The notion of *closeness* plays an important role in this method, as the strength of the connection between a pair of nodes will be a function of their closeness to the centre.

For the case of interbank networks, the seminal contribution by Craig and von Peter (2014) builds on the block-modelling approach and motivates the choice of a specific structure for the off-diagonal elements of the block matrix. In particular, building on the concept of intermediation, they suggest that core banks should lend to and borrow from at least one bank in the periphery. This translates into two restrictions: the *Core-Periphery* block should be row regular (i.e. each row has at least one 1) and the *Periphery-Core* block should be column regular (i.e. each column has at least one 1). According to this approach, core banks are a strict subset of intermediaries.

A more flexible definition of the core used by Della Rossa et al. (2013) avoids the explicit

(and often artificial) partition in subnetworks of block-modelling and is more natural for weighted networks, in contrast to block-modelling which is better suited for binary networks.

Della Rossa et al. (2013) associate to each network a *core-periphery profile*, which is a discrete and non-decreasing function $\alpha_1, \alpha_2, \ldots, \alpha_n$ that: (i) assigns a *coreness* value to each node, (ii) provides a graphical representation of the network structure, (iii) and allows for the computation of a centralisation indicator which measures the distance to an idealised core-periphery structure (the "star" network). Furthermore, the method allows for the definition of the α-periphery which collects all nodes with a coreness value below a given threshold α,[40] and identifies the set of p-nodes, which are those that constitute the periphery in a strict sense, i.e. those nodes that together form a *0-block*.[41]

Below we present figures summarizing the results from the two approaches as applied to our data.

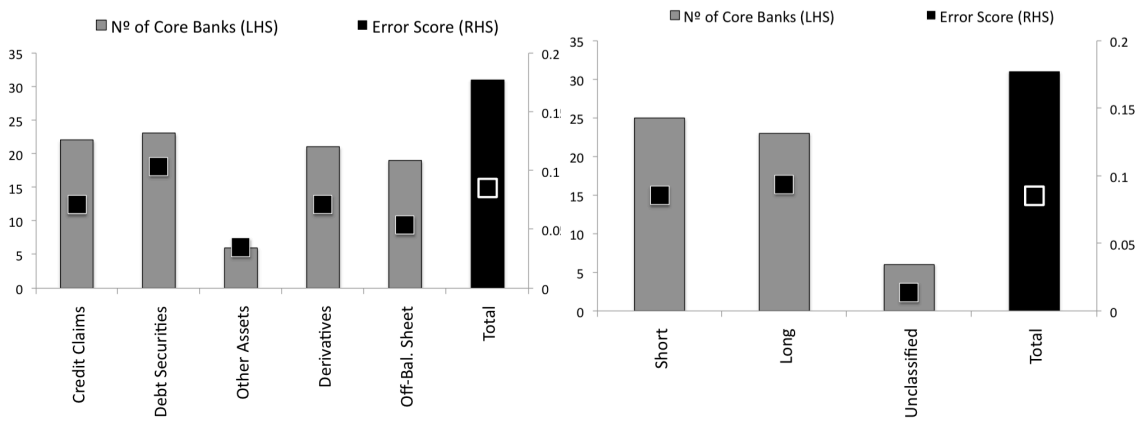

Figure C.1: Core banks and error score based on Craig and von Peter (2014) algorithm, by instrument and maturity (left and right panel respectively). The error score is expressed as a share of all possible connections.

[40] Alternatively, one can think of the α-core as the complement of the α-periphery.

[41] The code for the computation of the *core-periphery profile* is available from the authors' website.

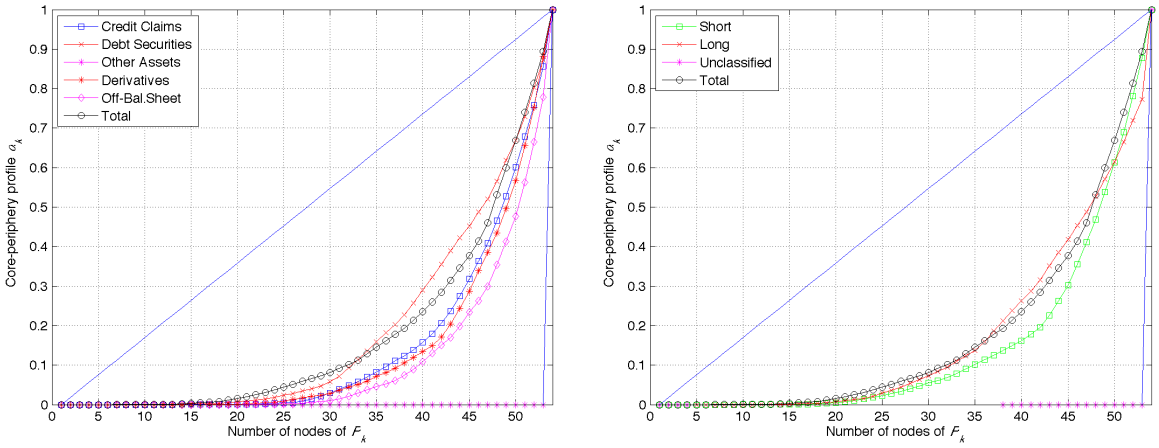

Figure C.2: Core-periphery profile by instrument and maturity (left and right panel respectively), based on the method by Della Rossa et al. (2013). Blue straight lines indicate the complete (diagonal) and star ("inverted L") networks as benchmarks.

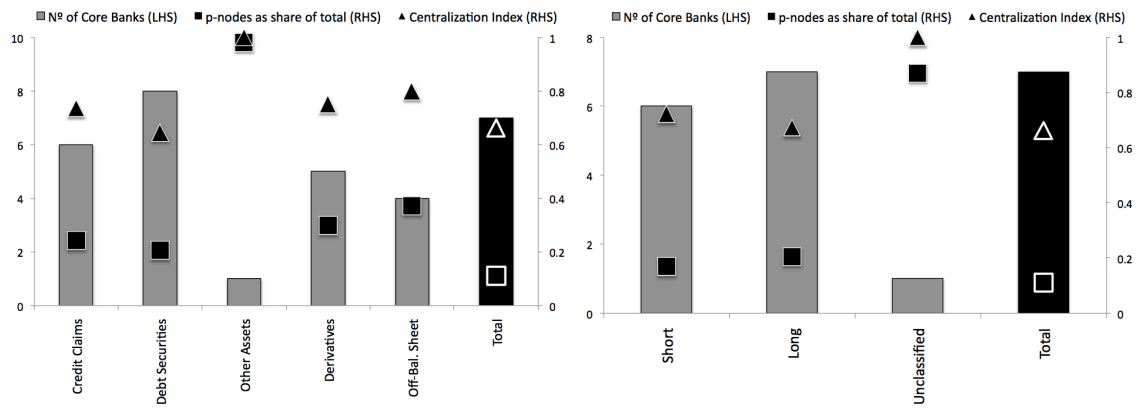

Figure C.3: Core banks, p-nodes and centralisation by instrument and maturity (left and right panel respectively), based on the method by Della Rossa et al. (2013). Core banks are those with $\alpha_k > 0.5$; p-nodes are periphery nodes in the strict sense ($\alpha_k = 0$).

D Additional Figures

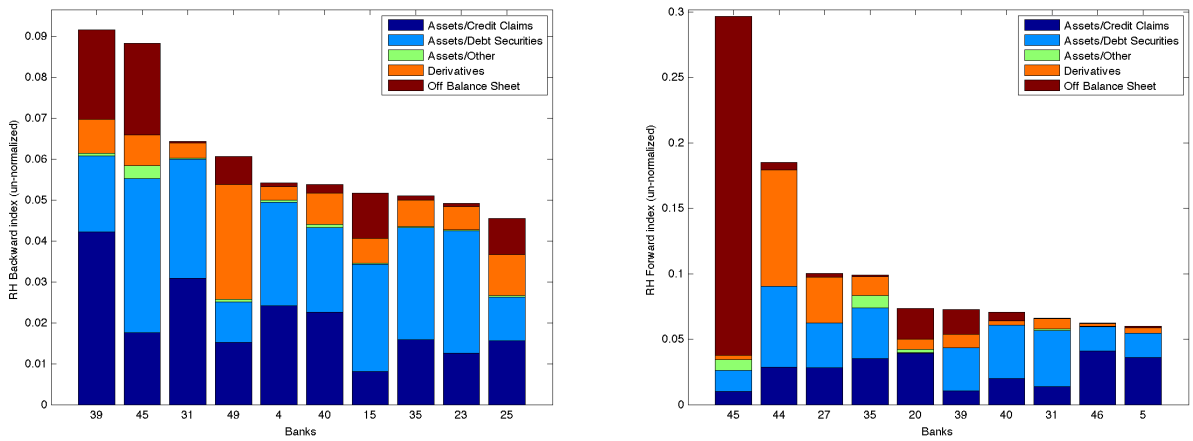

Figure D.1: RH backward (left) and forward (right) indices, for top 10 banks by instrument type.

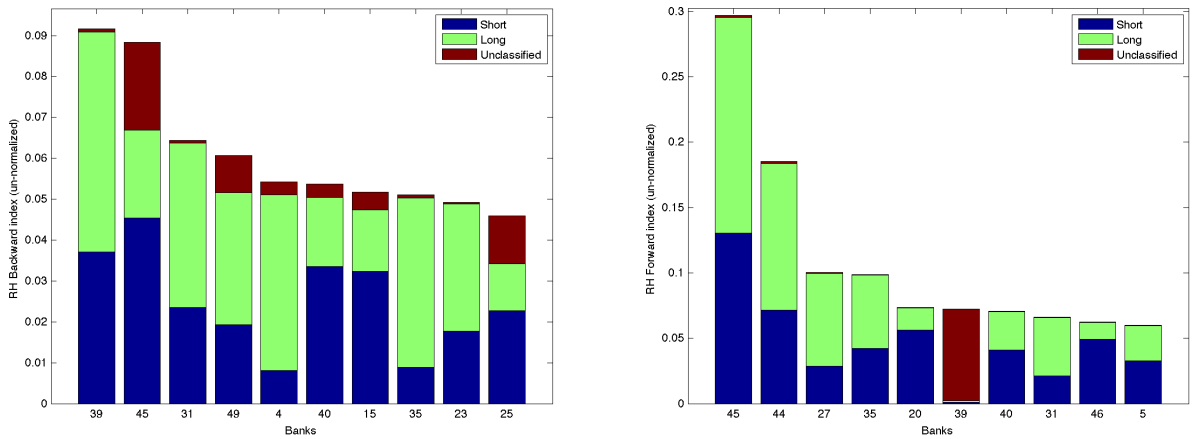

Figure D.2: RH backward (left) and forward (right) indices, for top 10 banks by maturity type.

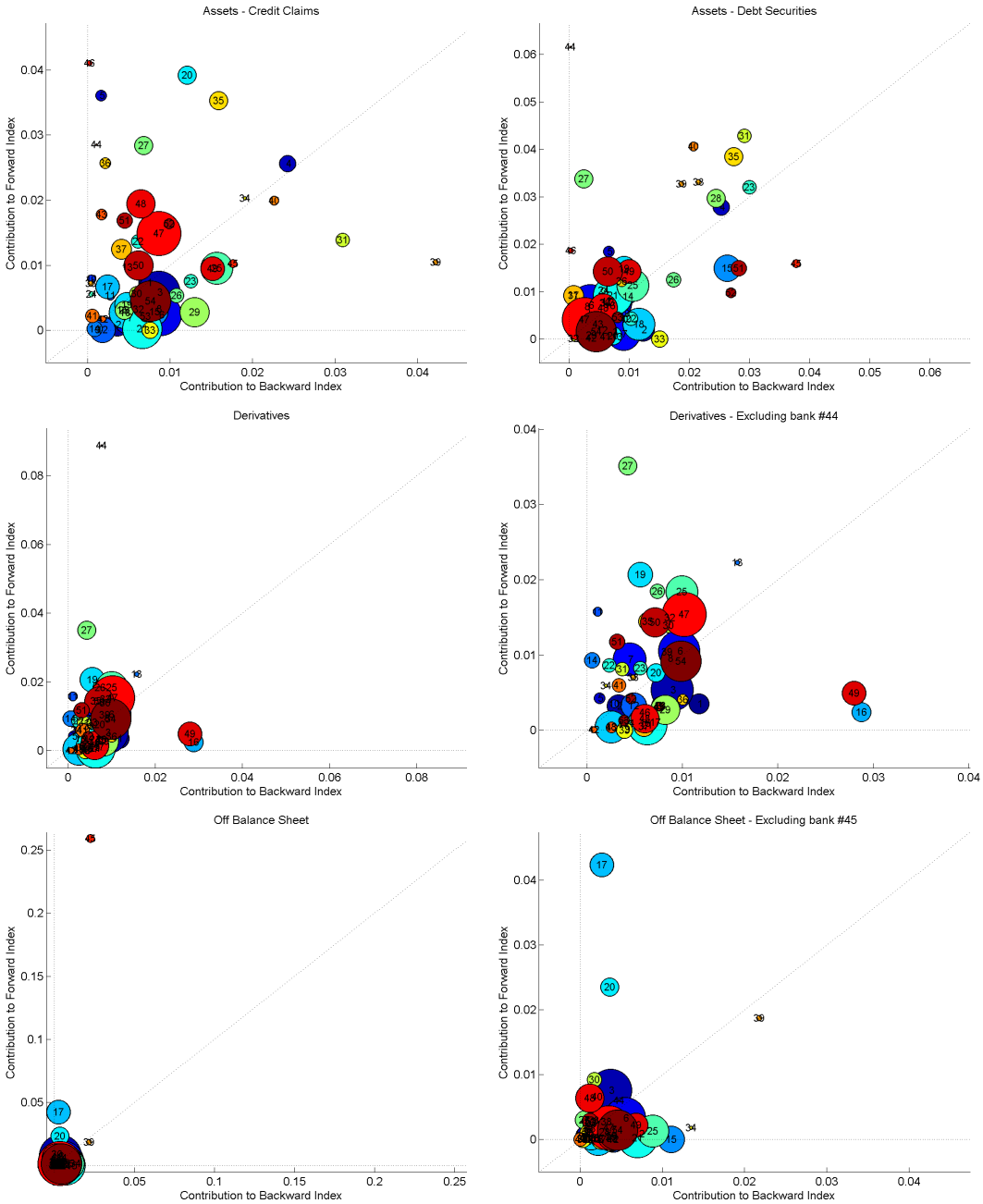

Figure D.3: Contribution to backward and forward indices by layer of instrument type. Bubble size indicates (scaled) total assets.

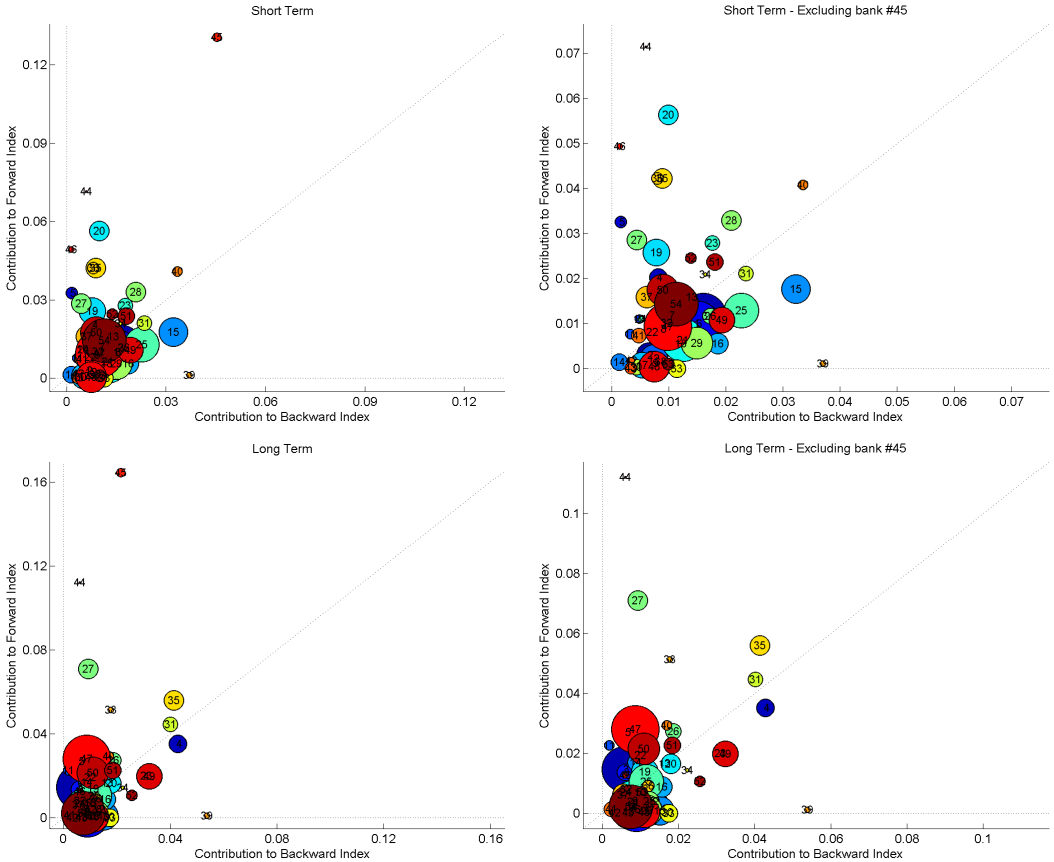

Figure D.4: Contribution to backward and forward indices by layer of maturity type. Bubble size indicates (scaled) total assets.

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