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### Will the true labor share stand up?

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### Abstract

We document the consequences of ambiguity in the empirical definition of the macroeconomic labor share. Depending on its definition, the properties of short-run fluctuations, medium-run swings, and long-run stochastic trends of the labor share may vary substantially. Based on a range of historical US time series, we carry out a systematic exploration of discrepancies between the alternative labor share definitions in terms of the observed stochastic trends, shares of short-, medium- and long-run variation in total volatility of the series, degree of persistence, mean-reversion properties, and susceptibility to structural breaks. We conclude that while short-run properties of the labor shares (represented by cyclical variation below 8 years) are relatively consistent across all definitions, their medium-run swings (8-50 years) and long-run trends ( $\geq 50$  years) diverge substantially. As important applications, we document the implications of our findings for growth accounting, the identification of short-run responses of the labor share to technology shocks and for estimating inflation.

**Keywords:** labor share, spectral analysis, persistence, mean reversion, structural breaks

**JEL Codes:** C82, E25, E32

## NON TECHNICAL SUMMARY

We document the consequences of ambiguity in the empirical definition of the macroeconomic labor share. Based on annual US labor share series spanning 1929–2012 and a quarterly series beginning in 1947, we carry out a systematic exploration of discrepancies between the alternative labor share definitions in terms of the observed stochastic trends, shares of short-, medium- and long-run variation in total volatility of the series, degree of persistence, mean-reversion properties, and evidence for structural breaks.

Our results suggest that while short-run properties of the labor shares (represented by cyclical variation below 8 years) are relatively consistent across all definitions, their medium- run swings (8-50 years) and long-run trends (above 50 years) substantially diverge. Having considered the alternatives, we argue that the US series on the share of employees' compensation in GDP, adjusted for proprietors' income is not only most sound theoretically, but also has intuitive, economically interpretable properties. This measure suggests that the US labor share has not only declined after 1970, but also substantially increased before that, exhibiting a hump-shaped pattern over the last 84 years. It corroborates the idea that instead of concentrating on the decline in the labor share since 1970, one could also embrace the larger time span of available data and discuss the (possibly technology-driven) long cycle in this variable.

We then illustrate the possible range of discrepancies in conclusions when certain macroeconomic studies are based upon different measures of the labor share. We concentrate on two interesting applications. The first of them is growth accounting. We conclude that the intertemporal variability of factor shares has a negligible impact on the resulting TFP measures, but differences in average levels of factor shares across various measurements can cumulate over years to about 12% of TFP.

Second, we deal with the question of empirical identification of the impact of technology shocks on the labor share. We find that the identified impulse responses vary largely across labor share definitions. Finally, we examine the sensitivity of the estimation of inflation New Keynesian Phillips curves to different labor share series.

# 1 INTRODUCTION

There has recently been a revival of debates surrounding the sources and consequences of shifts in labor's share of GDP. This revival is probably due to at least two reasons. First, it has been argued that the labor share has exhibited a protracted decline since 1970s (Arpaia, Pérez, and Pichelmann, 2009; Elsby, Hobijn, and Sahin, 2013). Second, it has also been observed that the labor share is subject to substantial countercyclical short-run volatility (Young, 2004; McAdam and Willman, 2013).

A key problem in this debate is the fact that there is no consensus as to how the labor share should be defined. The ambiguity arises from the fact that although total compensation of employees as well as companies' aggregated operational surplus are observable, the labor share is not, because a sizable share of the total value added is generated by the self-employed. This *mixed income* cannot be unambiguously understood as either the remuneration of capital or labor. In consequence, the measured labor share necessarily depends on the assumptions made in relation to the division of mixed income. Another caveat is related to the treatment of taxes on capital and labor incomes which may or may not be included in the computation of factor remuneration and total output (Gollin, 2002).

Although assigning ambiguous income to capital or labor is ultimately a matter of choice, it is rarely appreciated that this conceptual ambiguity has empirical consequences. These consequences still seem not to have been sufficiently researched thus far, further deepening the confusion. For example, it is customary in the business-cycle literature to adjust the labor share by proprietors' income (Young, 2004), whereas the structural analysis econometrics literature prefers to adjust by the fraction of self-employed in total employment (Arpaia, Pérez, and Pichelmann, 2009; Klump, McAdam, and Willman, 2007; Raurich, Sala, and Sorolla, 2012), and neither of these literatures confronts the role of the assumed definition. Thus whilst many papers (for example Gollin's (2002) seminal contribution) promote discussion on how labor shares could be measured, there is none which systematically examines and tries to understand those differences. This paper fills that important gap.

Our contribution then is to provide a systematic exploration of the dynamic properties of a range of alternative labor share measures. Our investigation is based on annual US labor share series spanning 1929–2012 and quarterly series from 1947. We document that these measures are not only divergent in terms of the implied time trends, which are visible to a naked eye, but also differ in terms of their other dynamic properties, such as the shares of short-, medium- and long-run variation in total volatility of the series, degree of persistence, mean-reversion properties, and susceptibility to structural breaks.

Our results point to the general conclusion that while short-run properties of the labor shares (represented by cyclical variation below 8 years) are relatively consistent across all definitions, their medium-run swings (8-50 years) and long-run trends ( $\geq 50$  years) diverge substantially. Therefore, it is indeed important to "get factor shares right", especially if one is interested in the medium and long run.

Having considered the alternatives, we argue that the US series on the share of employees' compensation in GDP, adjusted for proprietors' income following Cooley and Prescott (1995) and Gomme and Rupert (2007) procedure (which we call **PI<sub>2</sub>-GDP**) is probably the most sound theoretically, and also has intuitive, economically interpretable empirical properties. It provides the relatively most consistent message across a range of diverse exercises and applications (discussed below) while remaining in agreement with known "stylized facts" formulated elsewhere in the literature (e.g., it is mean-reverting but highly persistent, countercyclical over the short run, and has recorded a secular decline since 1970). This measure suggests, however, that the US labor share has not only declined after 1970, but also *substantially increased* before that, exhibiting a hump-shaped pattern over the last 84 years. Hence, instead of mostly concentrating the decline in the labor share since the 1970s, one could also embrace the larger time span of available data; the

profile as a whole is suggestive of a long cycle of activity (reminiscent of the work of Kondratieff and Schumpeter).

Having identified the key differences between the respective labor share definitions, we concentrate on three interesting and well-motivated applications. The first is growth accounting, namely the decomposition of output growth into factor accumulation and technical progress. We conclude that the intertemporal variability of factor shares has a relatively small impact on the resulting TFP measures, but differences in average levels of factor shares across various measurements can cumulate over years to about 12% of TFP (which is the difference between  $\mathbf{PI}_2$ -GDP and the “naive” measure, the payroll share, over 1929–2012), or even 40% in the most extreme case.

Second, we deal with the question of empirical identification of the short-run impact of technology shocks (Fernald, 2012) on the labor share. Based on an autoregressive distributed lag specification and a bi-variate VAR model for quarterly data, we find that the identified impulse responses vary largely across labor share definitions. We conclude that although business cycle properties of all these series are relatively consistent when discussed in isolation, their relationships with other macroeconomic variables react sensitively to changes in measurement.

Finally, we re-examine empirical estimates of the popular New Keynesian Phillips curves (NKPC) following Galí and Gertler (1999) and the large subsequent literature (e.g., Mavroeidis, Plagborg-Møller, and Stock, 2014). We find large differences in fixed-price durations and the sensitivity (or “slope”) of inflation when using different labor share measures. The properties of the labor share series used can help shed light on the empirical properties of the associated NKPCs.

The remainder of the paper is structured as follows. Section 2 provides some simple theory background to our study. In Section 3 we construct the time series of the US labor share under a range of its alternative empirical definitions. In Section 4 we discuss their basic dynamic properties, including their degree of persistence and mean reversion properties. In Section 5 we document the evidence for structural breaks. In Section 6 we carry out a spectral decomposition of these series into their short-, medium- and long-run components. The implications of the identified differences between alternate US labor share series are then discussed in three applications. Section 7.1 presents an application of our results to growth accounting. Section 7.2 presents the analysis of labor share responses to technological shocks along the business cycle. Section 7.3 analyses New Keynesian Phillips curves estimation under different labor share definitions. Section 8 concludes.

## 2 SOME SIMPLE THEORY BACKGROUND

Understanding the sources of observed variation in the labor share is a complex matter. Alternate empirical measures of this variable may diverge for many reasons: in the short run, they may be affected by business cycles, factor utilization rates, fiscal and monetary policy changes, etc., whereas in the long run there are technological developments, skill patterns, changes in sectoral composition, economic regulations, changes in tax progressivity, etc. Nonetheless, we can appeal to simple production-function theory to begin our discussion.

Consider the standard CES production function:

$$Y_t = \left[ \alpha \left( \Gamma_t^K \mathbb{K}_t \right)^{\frac{\zeta-1}{\zeta}} + (1-\alpha) \left( \Gamma_t^L \mathbb{L}_t \right)^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}} \quad (1)$$

where  $Y_t$  represents real output and  $\mathbb{K}_t = \kappa_t \times K_t$ ,  $\mathbb{L}_t = \ell_t \times L_t$  where  $K_t$  is the capital stock,  $L_t$  is the labor input,  $\kappa$  and  $\ell$  are measures of factor utilization and  $\zeta \geq 0$  is the elasticity of factor substitution. Function (1) reduces to Cobb Douglas in the limiting case  $\zeta \rightarrow 1$ ; the fixed factor proportions Leontief function when  $\zeta \rightarrow 0$ ; and a linear function (i.e., perfect factor substitution)

when  $\zeta \rightarrow \infty$ . Terms  $\Gamma_t^K$  and  $\Gamma_t^L$  capture capital and labor-augmenting technical progress, respectively. Technical progress can then be Hicks-neutral ( $\Gamma_t^K = \Gamma_t^L > 0$ ), Harrod-neutral ( $\Gamma_t^K = 0, \Gamma_t^L > 0$ ), Solow-neutral ( $\Gamma_t^K > 0, \Gamma_t^L = 0$ ) or factor-augmenting ( $\Gamma_t^K > 0 \neq \Gamma_t^L > 0$ ).<sup>1</sup>

Given this, the relative capital-to-labor income share, given competitive factor markets and profit maximization, can be expressed as

$$\Theta_t = \frac{w_t L_t}{r_t K_t} = \frac{1 - \alpha}{\alpha} \left( \frac{\Gamma_t^K \kappa_t K_t}{\Gamma_t^L \ell_t L_t} \right)^{\frac{1-\zeta}{\zeta}}, \quad (2)$$

where  $r_t$  and  $w_t$  denote the user cost (or marginal productivity) of capital and the real wage, respectively.

For factor incomes shares to be constant,  $d\Theta_t = 0$ , requires:

1.  $\zeta = 1$ . If production is Cobb–Douglas then any trend in the capital–labor ratio, or biased technical change is completely offset such as to maintain stable factor shares.<sup>2</sup>
2. If the bias in technical change exactly offsets accumulation of capital per worker (in growth terms) then factor income shares are stable for any value of  $\zeta$ .

Otherwise, factor income shares are changed by movements in capital per worker or biases in technical change or relative movements in factor utilization. The direction of the effect, however, depends on the value of the substitution elasticity, and, in the case of technology shocks, on their source (i.e., whether they augment capital or labor):

$$\begin{aligned} &< 0 \text{ for } \zeta < 1 \\ \frac{\partial \Theta_t}{\partial (\Gamma_t^L / \Gamma_t^K)}, \frac{\partial \Theta_t}{\partial (\mathbb{K}_t / \mathbb{L}_t)} &= 0 \text{ for } \zeta = 1 \\ &> 0 \text{ for } \zeta > 1 \end{aligned} \quad (3)$$

For instance, it is often suggested that the immediate post-war period was (in many developed countries) a period of rising labor share. If we assume that factors are gross complements,  $\zeta < 1$  (as in Chirinko (2008); Klump, McAdam, and Willman (2012)), then the conditions for such a rise would have to involve either increases in capital per worker and/or capital-saving technical progress:

1.  $\dot{\mathbb{K}}/\mathbb{K} > \dot{\mathbb{L}}/\mathbb{L}$
2.  $\Gamma_K > \Gamma_N$

By contrast, if factors are gross substitutes these two above margins work in the reverse way.

Either of these margins might explain the rise in the labor share, say, up until the 1970s. But whether the labor share continues to rise, falls or stabilizes depends on the evolution of these two inequalities. Rewriting condition (2), the factor ratio (i.e., part 1. of the above) can be shown to be a function of relative factor prices, the direction of technical bias and relative factor utilization:

$$\log \left( \frac{K_t}{L_t} \right) = \phi + \zeta \log \left( \frac{w_t}{r_t} \right) + (1 - \zeta) \log \left( \frac{\Gamma_t^L \ell_t}{\Gamma_t^K \kappa_t} \right) \quad (4)$$

<sup>1</sup>Neutrality concepts associate innovations to related movements in marginal products and factor ratios. An innovation is Harrod-neutral if relative input shares remain unchanged for a given capital-output ratio. This is also called labor-augmenting since technical progress raises production equivalent to an increase in the labor supply.

<sup>2</sup>Although, strictly speaking, technical change (biased or otherwise) cannot be identified in a Cobb–Douglas setting.



where  $\phi$  is a composite constant. Accumulation of capital per worker, thus, occurs if real wages rise relative to the user cost of capital (i.e., factor prices favor capital accumulation) and, assuming gross complements, if technical change is net labor-augmenting, or labor is used more intensively than capital. Under Cobb–Douglas, note, accumulation of capital per worker is driven solely by relative factor prices, one-to-one.

Biased technical change (part 2. above) is difficult to measure given its latent status. While in theoretical models of directed technical change, it depends upon firms’ profit incentives and innovation possibilities (Acemoglu, 2002; Leon-Ledesma and Satchi, 2015), in empirical work, factor-augmenting technical progress it is often proxied by flexible functional forms (see e.g., Klump, McAdam, and Willman, 2007).

Keeping the above discussion in mind, from the next section onwards we shall depart from the above stylized economy with CES production and competitive factor markets, which allowed us to find a direct link between factor shares, factor endowments, and technological progress. Instead, our objective will be to investigate the empirical implications of a range of alternative factor share definitions applied to US data. Nonetheless, the key insight from the above theory is that the magnitude of elasticity of substitution between capital and labor,  $\zeta$ , is key to understanding the findings on factor share cyclicity as well as the results of our three empirical applications (discussed in Section 7) because it affects virtually all relationships between factor shares and other economic variables (Chirinko, 2002; Irmen, 2011; Cantore, León-Ledesma, McAdam, and Willman, 2014).

### 3 ALTERNATIVE MEASURES OF THE LABOR SHARE

Formally, the aggregate labor share is defined as the proportion of total remuneration of the labor force ( $w_t L_t$ ) in aggregate output of the economy (GDP or total value added,  $Y_t$ ):

$$LS_t = \frac{w_t L_t}{Y_t}.$$

While such a definition appears theoretically unambiguous, both the numerator and the denominator of the above ratio can be measured empirically in various ways (Gollin, 2002), with potentially diverging implications.

#### 3.1 EMPIRICAL MEASUREMENT

The simplest, “naive” way to construct an empirical series of the labor share based on this definition is to use total *compensation of employees* ( $CE_t$ ) for the numerator of equation (3). According to the System of National Accounts, compensation of employees contains the sum of both wages and other payments to employees. Thus, to derive the labor share, nominal  $CE_t$  can be simply divided by nominal output  $Y_t$ . Thus one computes a measure which we label “Naive GDP”:

$$\text{Naive-GDP} : LS_t = \frac{CE_t}{Y_t}, \quad (5)$$

where  $Y_t$  is a generic measure of output. Typically it is GDP; however, in sectoral studies gross value added (GVA) is often used (see Bentolila and Saint-Paul, 2003; Young, 2010, 2013).

Although straightforward to compute and easily interpretable, this method (the “payroll share”, cf. Elsby, Hobijn, and Sahin (2013)) has a few crucial empirical disadvantages – the most important of which is that compensation of employees  $CE_t$  does not include *mixed income*, i.e., the ambiguous income earned by the self-employed, which cannot be directly ascribed to capital or labor. Since at least part of mixed income remunerates proprietors’ labor, this leads to a systematic underestimation of the labor share at the aggregate level. There are at least three ways to deal with this

issue: (1) assuming that the self-employed (proprietors) face identical average wage as the non-self-employed; (2) assuming identical labor shares in both groups, and (3) assuming an arbitrary rule of thumb to divide proprietors' income. We elaborate on these options below.

The first approach to include the ambiguous income in the labor share is to use data on the number of self-employed ( $SE_t$ ). The key assumption used in this adjustment is that labor compensation is equal on average for both employees ( $E_t$ ) and self-employed workers ( $SE_t$ ). Then the "naive" labor share is increased by the imputed compensation of the self-employed, as in:

$$\text{SE-GDP} : LS_t = \frac{CE_t}{Y_t} \left( 1 + \frac{SE_t}{E_t} \right). \quad (6)$$

The second way to adjust the labor share refers directly to the concept of mixed income. *Proprietors' income* ( $PI_t$ ), as defined in the System of National Accounts, is the ambiguous part of output which cannot be treated as pure labor or capital income. Labor share estimates can then be adjusted by the means of a simple reduction of the output by  $PI_t$ :

$$\text{PI-GDP} : LS_t = \frac{CE_t}{Y_t - PI_t}. \quad (7)$$

This approach is equivalent to assuming that mixed income is split between labor and capital income in the same proportion as in the rest of the economy.

The third, similar in spirit but more comprehensive approach to dealing with mixed income has been proposed by Cooley and Prescott (1995), and developed by Gomme and Rupert (2007) and Ríos-Rull and Santaeulària-Llopis (2010). Its starting point is a decomposition of total income into two components: ambiguous ( $AI_t$ ) and unambiguous ( $UI_t$ ) income. Ambiguous income  $AI_t$  is the sum of proprietors' income, taxes on production less subsidies, business current transfer payments and statistical discrepancies:

$$AI_t = PI_t + (Tax_t - Sub_t) + BCTP_t + SDIS_t.$$

Neither of these amounts is directly attributable to capital or labor.

Unambiguous income  $UI_t$ , on the other hand, is straightforwardly separated into unambiguous labor and capital income components:

$$UI_t = \underbrace{ULI_t}_{CE_t} + UKI_t,$$

where the latter consists of consisting of rental income, net interests, current surplus of government enterprises, and corporate profits:

$$UKI_t = RI_t + NI_t + GE_t + CP_t.$$

The share of capital in unambiguous income ( $KS_t^U$ ) is obtained as:

$$KS_t^U = 1 - LS_t^U = \frac{UKI_t + DEP_t}{UI_t} = \frac{RI_t + NI_t + GE_t + CP_t + DEP_t}{RI_t + NI_t + GE_t + CP_t + CE_t}$$

where  $DEP$  is the consumption of fixed capital (Table 1.7.5 of NIPA-BEA).

The key assumption underlying the current adjustment method is that the shares of capital and labor in ambiguous income are the same as in unambiguous income,  $AKI_t = KS_t^U AI_t$ . Then, the labor share is computed as follows:

$$\text{PI}_2\text{-GDP} : LS_t = (1 - KS_t) = 1 - \frac{UKI_t + DEP_t + AKI_t}{Y_t}. \quad (8)$$

The theoretical arguments why  $\text{PI}_2\text{-GDP}$  is likely to be a relatively accurate representation of the "true" labor share are as follows. First, it covers the *entire* economy and carefully considers many distinct economic quantities, reported in NIPA, including the ambiguous income.



Hence, from the macroeconomic perspective it should be more robust to structural changes, such as changes in the sectoral or private vs. public composition of value added, than e.g. the corporate labor share (Karabarbounis and Neiman, 2014). Second, its core assumption, that the ambiguous income is split between labor and capital income in the same proportion as in the rest of the economy, makes this measure much more accurate in the case of long-dated series when compared to series assuming that labor compensation is equal on average for both employees and self-employed workers (e.g., **SE-GVA**, see Elsby, Hobijn, and Sahin (2013)): in the early twentieth century in the US, just like in less developed countries today, most self-employed workers were farmers who earned much less than the contemporaneous average wage in industry and services.

Finally, Gollin (2002) proposes also an adjustment where the entire proprietors' income is treated as compensation of labor. Such an approach likely leads to a sharp overestimation of the labor share. Accordingly, Johnson (1954) uses an equally simple rule of thumb: two-thirds of proprietors' income to labor.

Another issue in constructing the labor share is whether aggregate output  $Y_t$  in the denominator is identified with GDP or gross value added (GVA). It turns out that empirically factor shares in value added differ systematically from factor shares in GDP (Valentinyi and Herrendorf, 2008). This argument ought to be borne in mind particularly when GVA is employed in more aggregated frameworks. For instance, Karabarbounis and Neiman (2014) document a global decline in the labor share, using data on corporate gross value added, which accounts for 60% of overall GVA, instead of GDP.<sup>3</sup>

### 3.2 SOURCES OF DISCREPANCY

There are clear-cut theoretical indications under which assumptions the aforementioned labor share measures are equivalent. Failure to meet these conditions is then the reason for their discrepancy. We make four points in that regard.

1. The **naive-GDP** measure could equal any other measure only in the counterfactual case where there were no proprietors' income in the economy. Hence it is *always downward biased*. The difference between the payroll share and adjusted labor share measures is the larger, the greater is the actual share of mixed income in total output.

2. **SE-GDP** coincides with **PI-GDP** if and only if the share of the self-employed in the total labor force is equal to the share of proprietors' income in GDP:

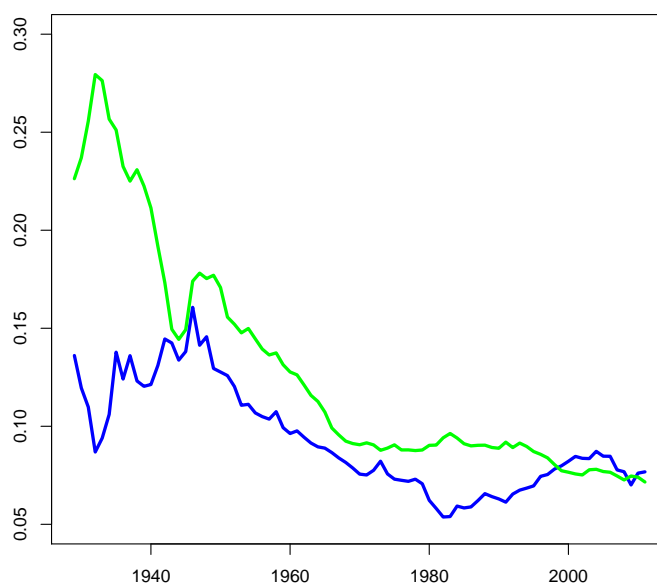
$$\frac{SE_t}{E_t + SE_t} = \frac{PI_t}{Y_t}. \quad (9)$$

Otherwise, the **SE-GDP** labor share measure exceeds **PI-GDP** if and only if, on average, employees obtain a proportionally larger share of output than the self-employed:  $\frac{SE_t}{E_t + SE_t} > \frac{PI_t}{Y_t}$ .

**Figure 1** illustrates that after the peak in the self-employed share during the Great Depression, and a following period of its sharp decline in the 1930s, both sides of equation (9) declined in a roughly parallel way between World War II and the 1970s. The high share of self employed in the pre-war period reflected the importance of Agriculture and the substitution to self-employment during the Great Depression. Thereafter, both as a share of output and employment, Agriculture declined reflecting the rise of Manufacturing with its large scale economies and of the public sector which attracted and absorbed resources from Agriculture.

<sup>3</sup>Measuring the labor share is not limited to aggregate or sectoral data only. Highly disaggregated data are sometimes also used to estimate aggregate factor shares. For example, Young (1995) used census and survey data to match the self-employed and other unpaid workers with employees, cross-tabulated by gender, sector, age and other relevant characteristics. He then imputed implicit labor compensation for the individuals belonging to the labor force groups which are listed as "unpaid" in official statistics. So imputed labor incomes constitute a micro-founded way of adjusting the naive labor share measure. Moreover, the labor share might also gathered from firm-level data (see e.g., Growiec, 2012).

**Figure 1: The Ratio of the Self-Employed to Total Employment and the Share of Mixed Income in GDP.**



**Notes:** The blue line denotes the share of the proprietors' income in GDP ( $PI_t/GDP_t$ ). The green line represents for the ratio of the self-employed to total employment ( $SE_t/(E_t + SE_t)$ ).

From the 1970s onwards, that rapid decline in self employed comes to a halt. This reflected factors such as technological changes which helped reduce operating costs and the importance of scale in favor of smaller-scale enterprises, a greater use of contracting out, demographic shifts, and so on.

The share of proprietors' income has a similar overall dynamic to that of the self-employed share. After 1980, however, the share of proprietors' income in GDP began to rise despite bottoming out of the share of the self-employed in the labor force. Both lines crossed in late 1990s. Now it is the self-employed who earn a proportionally larger share of the GDP than employees (which can be partly due to statistical error, Elsby, Hobijn, and Sahin, 2013) and thus the **PI-GDP** exceeds the **SE-GDP** labor share.

3. **PI<sub>2</sub>-GDP** coincides with **PI-GDP** as long as ambiguous income which is not directly proprietors' income (i.e., taxes on production  $Tax_t$  and business current transfer payments,  $BCTP_t$ ) is positive and attributed fully to capital. If factually this income is also partly generated by labor, however, then **PI<sub>2</sub>-GDP** should be relatively higher while **PI-GDP** (and, by the same token, **SE-GDP**) should be an unambiguously downward-biased measure of the true labor share.

4. The discrepancy between labor share measures based on GDP and GVA follows from the difference between both denominators, driven by taxes on production and imports, minus subsidies.

## 4 PROPERTIES OF THE US LABOR SHARE SERIES

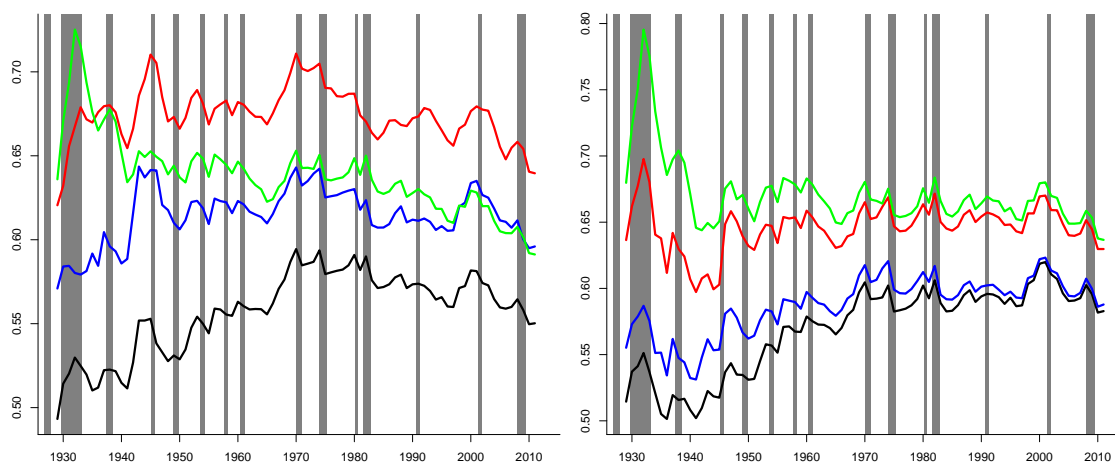
To construct long historical labor share series for the US economy employing all the aforementioned measurement methods, we used annual data from National Income and Product Accounts (NIPA) tables of the Bureau of Economic Analysis (BEA), and quarterly data from the Bureau of Labor Statistics (BLS). This choice of data sources stems from our wish to construct as long series as possible; the annual and quarterly series span 1929–2012 and 1947q1–2013q1 respectively. A detailed description of the constructed series is included in appendix Table A.1.

To recall, the series computed with the naive method, following equation (5), is denoted as **Naive-GDP** (the payroll share of GDP). The next series, **SE-GDP**, includes adjustment for the share of the self-employed (equation (6)). Adjustment by proprietors' income, in the form of a reduction in the considered measure of output (equation (7)), has been used to construct the series **PI-GDP**. A more sophisticated adjustment by proprietors' income (equation (8)) is employed in the series **PI<sub>2</sub>-GDP**. Keeping measures of the labor's share of GDP as our benchmark, we also calculate the "naive" annual labor share in *gross value added* (GVA) in the private sector (**Naive-GVA**, the payroll share of GVA), and in the non-farm private sector (**Naive-GVA-NF**). The next two variants are constructed by adjusting the above series by the number of the self-employed in the corresponding sectors (denoted as **SE-GVA** and **SE-GVA-NF**, respectively).<sup>4</sup> The last version of the labor share used in the current study is taken from the BLS. The **BLS** labor share series is a quarterly index, whose initial level is not determined.

#### 4.1 GRAPHICAL ANALYSIS

**Figures 2 and 3** show the annual and quarterly labor income share time series, respectively. Note first the level differences between the series. For instance, **PI<sub>2</sub>-GDP** exceeds **Naive-GDP** by 11 pp. on average (i.e., almost  $1/5^{th}$  of its level). Systematic differences are substantial also for other pairs of measures. Such differences, as discussed below in Section 7.1, will for instance have implications for growth-accounting exercises and the retrieval of TFP. Eyeballing the historical series suggests that over the long run, differences between the variants often systematically diverge. Thus, the factors which drive a wedge between the series – number of self employed, proprietors' income, taxes and subsidies – are time-varying. This applies in particular to the comparisons between adjusted series and their "naive" counterparts. In Appendix D, we address this issue more systematically by checking stationarity of *differences* between all possible pairs of alternative labor share measures. Our results indicate that stationarity of differences is typically rejected.

**Figure 2: Annual US Labor Share**



**Notes:**

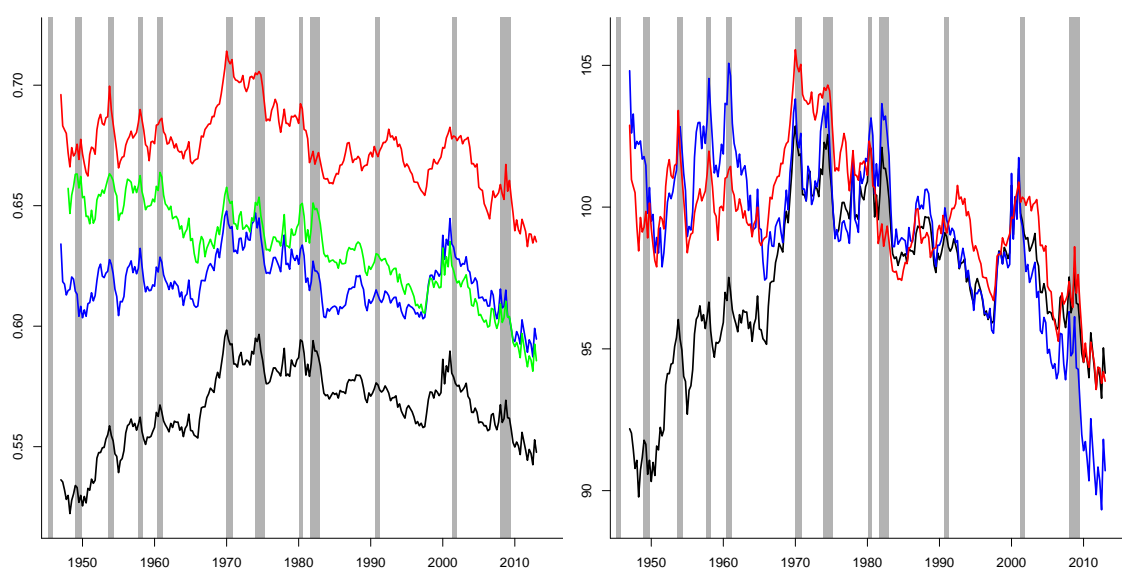
Left panel: Naive-GDP, **PI-GDP**, **PI<sub>2</sub>-GDP**, **SE-GDP**

Right panel: Naive-GVA, **Naive-GVA-NF**, **SE-GVA-NF**, **SE-GVA**. NBER recession periods are overlaid at the appropriate frequency.

Visible discrepancies, however, also relate to dynamics. Most importantly, the "naive" series as well as series adjusted by mixed income exhibit hump-shaped trajectories, whereas the labor share modified by the share of the self-employed records a consistent, strong downward tendency

<sup>4</sup>Note that **SE-GVA** is also the "headline measure" of the US labor share in Elsby, Hobijn, and Sahin (2013).

Figure 3: Quarterly US Labor Share



**Notes:**

Left panel: Naive-GDP, **PI-GDP**, **PI<sub>2</sub>-GDP**, **SE-GDP**

Right panel: Naive-GDP, **PI<sub>2</sub>-GDP**, **BLS**. For comparison all series set at 2000=100.

throughout the period. This particular behavior is likely driven by (a) the sharp fall in the share of the self-employed in total employment until around 1970 (recall Figure 1), and (b) an overestimation of incomes among the self-employed in the immediately following period, identified by Elsby, Hobijn, and Sahin (2013).

Moreover, even the much heralded labor income decline since the 1970s is not universal. Series based on value added have been apparently stable since the 1940s, as has the PI-GDP variant (annual and quarterly). All series, though, do share a steep fall since the 2001 recession.

Notwithstanding, all of these series are meant to measure the same thing: namely the share of US national income that goes to labor. All of them have been widely used in various literatures. Yet, we have little understanding of the properties of these different series: for instance, in line with Kaldor’s stylized facts, can we view the shares as stable or *quasi*-stable (e.g., correcting for structural breaks); how persistent and volatile are they? What, in any given exercise, is the consequence of using one labor share measure rather than another? For instance, if income shares are not stable how would growth accounting exercises (which retrieve TFP) change? And would it alter the importance of TFP in accounting for economic growth? If labor’s share of income cannot be uniquely measured, how would that change debates about income inequality? These are issues we explore.

## 4.2 SUMMARY STATISTICS

Summary statistics (and short-run characteristics) are presented in **Tables 1 and 2** (where  $\tilde{x}$  is the logged then HP filtered series).<sup>5</sup>

We computed the cumulative changes over two subperiods with a breakpoint in 1970, and for most annual series (6 out of 8), the decrease in the labor share after 1970 was smaller than the strong rise from 1929 to 1970. This applies in particular to **PI<sub>2</sub>-GDP**. The series are also mostly

<sup>5</sup>We applied a smoothing parameter equal to 100 and 1600 to annual and quarterly data, respectively. We also examined one-sided HP filtered series as well as used the Ravn and Uhlig (2002) adjustment to the smoothing parameter applied to the annual series, with minimal qualitative differences.

characterized by negative skewness (i.e., by a long tail to the left indicating a few very low values) but with no particular common features in kurtosis (“peakedness”). Likewise the null of Normality is mostly rejected for annual and quarterly series.

The volatility, relative to output, ranges from 23% – 36% for the annual series (the **SE-GVA** series being the most volatile), and 36% – 48% (quarterly) with the **BLS** series more volatile than the rest. The auto-correlation varies between 0.43 – 0.64 (annual) to 0.63 – 0.74 (quarterly). The labor share is generally counter-cyclical (especially in quarterly data and since 1947).<sup>6</sup> However this counter-cyclicality is not especially strong (around -0.2,-0.4 for the quarterly series) and there is in some cases acyclicality (**PI-GDP**). It is further interesting to note that **PI-GDP** and **PI<sub>2</sub>-GDP**, though intended to measure the same aspect (namely labor share corrected for proprietors’ income) have such distinct properties: in annual terms the former is apparently pro-cyclical and the latter a-cyclical; whereas in quarterly data, the former is acyclical and the latter counter-cyclical.

**Table 1: Annual Labor Share: Summary Statistics**

	Naive-GDP	PI-GDP	PI <sub>2</sub> -GDP	SE-GDP	Naive-GVA	Naive-GVA-NF	SE-GVA-NF	SE-GVA
mean	0.557	0.614	0.674	0.639	0.570	0.588	0.646	0.671
median	0.560	0.616	0.674	0.636	0.583	0.593	0.647	0.666
st.d.	0.023	0.016	0.017	0.023	0.032	0.022	0.018	0.026
min	0.555	0.571	0.581	0.591	0.501	0.531	0.597	0.637
occurrence	1929	1929	1929	2011	1936	1941	1941	2011
max	0.594	0.644	0.711	0.725	0.620	0.623	0.698	0.795
occurrence	1970	1943	1970	1932	2001	2001	1932	1932
1970-1929	0.062	0.098	0.115	0.033	0.101	0.017	0.086	0.089
2011-1970	-0.030	-0.063	-0.066	-0.040	-0.044	-0.068	-0.063	-0.063
Skewness	-0.621	-0.423	-0.362	1.034	-0.664	-0.804	-0.514	2.498
Kurtosis	-0.488	-0.242	0.854	2.378	-0.845	0.046	1.241	7.772
Normality	[0.044]	[0.259]	[0.080]	[0.000]	[0.014]	[0.009]	[0.007]	[0.000]
SHORT-RUN CHARACTERISTICS								
$\sigma_{\tilde{l}_s}$	0.013	0.013	0.013	0.015	0.015	0.015	0.018	0.020
$\sigma_{\tilde{l}_s} / \sigma_{\tilde{y}}$	0.229	0.226	0.227	0.258	0.257	0.268	0.316	0.355
$\text{corr}(\tilde{l}_{s_t}, \tilde{l}_{s_{t-1}})$	0.530***	0.528***	0.637***	0.518***	0.434***	0.398***	0.441***	0.540***
$\text{corr}(\tilde{l}_{s_t}, \tilde{y}_t)$	0.174**	0.444***	-0.065	-0.465***	-0.262***	-0.196***	-0.533***	-0.652***

**Note:** Normality test is Jarque-Bera. Superscripts \*\*\*, \*\* and \* stands for rejection of the null about insignificant auto- or cross -correlation at the 1%, 5% and 10% significance level, respectively.

### 4.3 PERSISTENCE

A key property of time series is its persistence. If a time series is subject to a shock, the level of persistence tells us if, and how soon, the series will revert to its mean: the higher the persistence the slower the reversion.

<sup>6</sup>The cyclical co-movement of the labor share with output differs significantly among the constructed variants, though. In particular, as opposed to other labor share measures, the short-run components of annual **Naive-GDP** and **PI-GDP** measures are significantly *positively* correlated with output. This is driven primarily by the strongly pro-cyclical behavior of these series in the beginning of the sample, before World War II. We confirm that these series are again countercyclical when considered for the post-war sub-period only, consistent with their quarterly post-war counterparts.

**Table 2: Quarterly Labor Share: Summary Statistics**

	Naive-GDP	PI-GDP	PI <sub>2</sub> -GDP	SE-GDP	BLS
mean	0.566	0.618	0.674	0.632	104.7
median	0.567	0.617	0.674	0.634	104.9
st.d.	0.016	0.012	0.015	0.019	3.2
min	0.522	0.587	0.633	0.581	94.4
occurrence	1948q2	2012q3	2011q4	2012q3	2012q3
max	0.598	0.648	0.714	0.664	111.0
occurrence	1970q1	1970q1	1970q1	1960q4	1960q4
1970-1947	0.062	0.022	0.026	-0.001	-0.487
2011-1970	-0.047	-0.050	-0.074	-0.066	-12.841
Skewness	-0.454	0.171	-0.105	-0.496	-0.912
Kurtosis	-0.143	-0.094	0.333	-0.432	0.881
Normality	[0.009]	[0.505]	[0.385]	[0.002]	[0.000]
SHORT-RUN CHARACTERISTICS					
$\sigma_{\tilde{l}_s}$	0.008	0.008	0.008	0.008	0.011
$\sigma_{\tilde{l}_s} / \sigma_{\tilde{y}}$	0.370	0.374	0.357	0.359	0.482
$\text{corr}(\tilde{l}_{s_t}, \tilde{l}_{s_{t-1}})$	0.674***	0.637***	0.736***	0.633***	0.658***
$\text{corr}(\tilde{l}_{s_t}, \tilde{y}_t)$	-0.200***	-0.092	-0.232***	-0.385***	-0.275***

**Note:** Changes have been calculated for annual means. See also notes to Table 1.

Assume the labor share is generated by an AR(1) process:<sup>7</sup>

$$LS_t = \mu + \rho LS_{t-1} + \beta_1 t + \beta_2 t^2, \quad (10)$$

This nests three models: (1) only with a constant ( $\beta_1 = \beta_2 = 0$ ); (2) with a linear trend ( $\beta_2 = 0$ ); (3) with a quadratic trend; and model (1<sup>+</sup>) where (1) is re-estimated using the logged and HP filtered labor share series,  $\tilde{l}_s$ , instead of its level.

Models (1) and (1<sup>+</sup>) are consistent with the usual interpretation of the labor share as being stable around its long-run mean,  $\frac{\mu}{1-\rho}$ . In the next two models, the “mean” itself is allowed to shift, reflecting secular trends, long-lasting cycles, structural changes in the economy, and so on. Clearly the  $\rho$  value that emerges from models (2) and (3) captures persistence at the high frequency end since some of the long-run variation is removed by the included trends.

Point estimates of  $\rho$ , although generally high, exhibit substantial heterogeneity (see **Tables 3, 4**). For example, for annual and quarterly Model 1,  $\rho \in [0.751, 0.944]$  and  $\rho \in [0.929, 0.982]$ , respectively, which imply half lives of 2.5 – 12 and 2.5 – 9.5 years. For the annual series, the GVA series are far less persistent. Interestingly, the addition of a linear trend reduces  $\rho$  estimates significantly *only* for the series with GVA as output or adjusted by the self-employed. Models 2 and 3 – as well as the filtered case 1<sup>+</sup> – necessarily contract the persistence and half lives. Extending the autoregressive model by a linear trend limits substantially the persistence only for the **SE-GDP** and **BLS** series, for which the linear trend is statistically significant. The quadratic of Model 3 naturally fits the naive series well given the strong hump-shape in its profile.

#### 4.4 PERSISTENCE AND STOCHASTIC VOLATILITY (SV)

A large and growing literature – starting in finance (Shephard and Andersen, 2008) but recently evolving into macroeconomics – has emerged to analyze the possibility of time-varying stochastic volatility in time series. As Fernández-Villaverde and Rubio-Ramírez (2013) discuss, this appears to be important for aggregate data: periods of high volatility are followed low-volatility periods.

<sup>7</sup>We also tried an AR(2) specification. Despite the fact that, as opposed to the AR(1) model, such a specification is able to capture hump-shaped dynamics with a stationary stochastic process, our results are very similar. The sum of both autoregressive coefficients is generally close to but significantly less than unity. Adding a quadratic trend substantially reduces the estimated persistence. The differences across various labor share specifications are of comparable magnitude. Details are available from the authors upon request.



**Table 3: AR(1) Persistence: Annual Labor Share**

	Naive-GDP	PI-GDP	PI <sub>2</sub> -GDP	SE-GDP	Naive-GVA	Naive-GVA-NF	SE-GVA-NF	SE-GVA
	<u>Model 1</u>							
$\rho$	0.908***	0.840***	0.840***	0.939***	0.944***	0.881***	0.751***	0.872***
	<u>Model 2</u>							
$\rho$	0.909***	0.852***	0.814***	0.726***	0.869***	0.780***	0.740***	0.812***
$\beta_1 \cdot 10^4$	-0.002	-0.029	-0.101***	-0.257***	0.118	0.122*	0.030	-0.130*
	<u>Model 3</u>							
$\rho$	0.728***	0.759***	0.739***	0.716***	0.779***	0.700***	0.738***	0.774***
$\beta_1 \cdot 10^4$	0.727***	0.353*	0.197	-0.440**	0.655***	0.595**	0.164	-0.504*
$\beta_2 \cdot 10^4$	-0.007***	-0.004*	-0.004*	0.002	-0.005**	-0.005**	-0.002	0.004
	<u>Model 1<sup>+</sup></u>							
$\rho$	0.511***	0.528***	0.620***	0.454***	0.417***	0.388***	0.420***	0.485***

**Note:** \*\*\*, \*\* and \* denote the rejection of null of insignificance at the 1%, 5% and 10% significance level, respectively (bootstrapped standard errors used). The estimated constants are omitted for brevity.

**Table 4: AR(1) Persistence: Quarterly Labor Share**

	Naive-GDP	PI-GDP	PI <sub>2</sub> -GDP	SE-GDP	BLS
	<u>Model 1</u>				
$\rho$	0.968***	0.929***	0.975***	0.982***	0.964***
	<u>Model 2</u>				
$\rho$	0.971***	0.920***	0.958***	0.864***	0.913***
$\beta_1 \cdot 10^4$	-0.002	-0.004	-0.006	-0.033***	-2.906***
	<u>Model 3</u>				
$\rho$	0.884***	0.890***	0.920***	0.823***	0.843***
$\beta_1 \cdot 10^4$	0.083***	0.032**	0.033**	-0.004	8.095**
$\beta_2 \cdot 10^4$	-0.003***	-0.001**	-0.002***	-0.001**	-0.496***
	<u>Model 1<sup>+</sup></u>				
$\rho$	0.670***	0.627***	0.722***	0.633***	0.655***

**Note:** See Table 3.

For robustness, therefore, we additionally estimate the SV-AR(1) process:<sup>8</sup>

$$\tilde{l}s_t = \rho_{\tilde{l}s} \tilde{l}s_{t-1} + e^{\sigma_t} v_{1,t}, \quad (10a)$$

$$\sigma_t = (1 - \rho_\sigma) \bar{\sigma} + \rho_\sigma \sigma_{t-1} + \eta v_{2,t}, \quad (10b)$$

where, as before,  $\tilde{l}s$  is the HP-filtered series of the logged series,<sup>9</sup> and  $v_{1,t}, v_{2,t} \sim \mathcal{N}(0, 1)$ . Parameters  $\rho_{\tilde{l}s}$  and  $\rho_\sigma$  represent the persistence of the level and volatility equation, respectively;  $\bar{\sigma}$  is the unconditional mean of the volatility of the process,  $\sigma_t$ ; and  $\eta$  captures the standard deviation of the volatility shocks.

**Table 5** shows that whilst the quarterly point estimates do not differ significantly from the previous (1<sup>+</sup>) case (reproduced in the first row), there is an efficiency gain.<sup>10</sup> The data support moderate time-varying volatility, with persistence similar to that of the labor share series itself. It is estimated that a one standard deviation volatility shock increases the standard deviation of the labor share by around  $e^{(\eta_\sigma - 1)} = 35\%$ .

**Figure 4** retrieves the implied stochastic volatility process. Though differences in the level of stochastic volatility detected, a similar story emerges: namely the increasing volatility built up over 1970s and early 1980s followed by the “great moderation”. This is followed by a peak

<sup>8</sup>We thank Benjamin Born for supplying Matlab code to implement the stochastic volatility estimation from Born and Pfeifer (2014). The Bayesian method used to retrieve these shocks is, for given priors, to evaluate the likelihood using the sequential importance resampling particle filter and Randomized Block Metropolis-Hastings algorithm to maximize the posterior. After filtering, the historical distribution of the volatilities is obtained by a backward-smoothing routine.

<sup>9</sup>As before, we repeated the exercise with a one-sided HP filter with minimal qualitative differences.

<sup>10</sup>We concentrate on quarterly data since that is a frequency often associated with stochastic volatility.

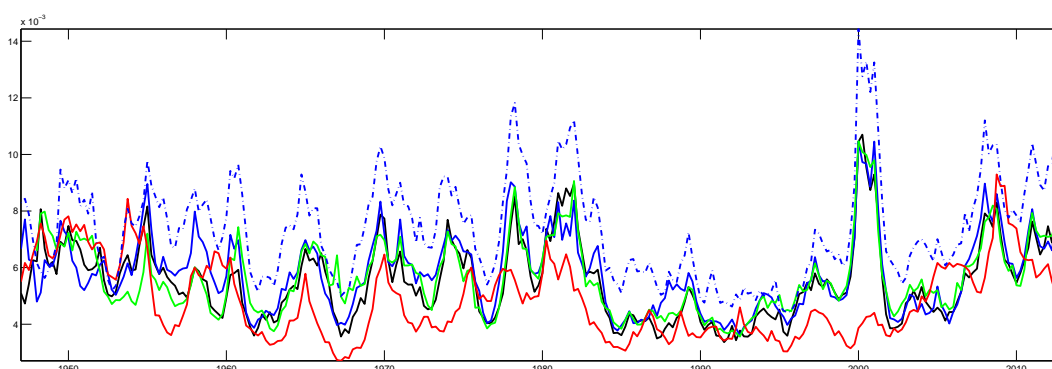
**Table 5: AR(1) and SV-AR(1) Models, Quarterly Labor Share Series**

	naiveGDP	PI-GDP	PI <sub>2</sub> -GDP	SE-GDP	BLS
$\rho$	0.670 {0.581 : 0.759}	0.627 {0.528 : 0.726}	0.722 {0.631 : 0.814}	0.633 {0.539 : 0.726}	0.655 {0.562 : 0.749}
$\rho_{\tilde{I}_s}$	0.705 {0.626 : 0.781}	0.654 {0.578 : 0.732}	0.746 {0.673 : 0.820}	0.685 {0.602 : 0.765}	0.690 {0.614 : 0.766}
$\rho_\sigma$	0.717 {0.513 : 0.885}	0.693 {0.473 : 0.886}	0.806 {0.637 : 0.967}	0.733 {0.550 : 0.889}	0.737 {0.553 : 0.903}
$\bar{\sigma}$	-5.220 {-5.368 : -5.075}	-5.152 {-5.292 : -5.003}	-5.312 {-5.504 : -5.054}	-5.167 {-5.322 : -5.003}	-4.916 {-5.066 : -4.754}
$\eta_\sigma$	0.287 {0.206 : 0.376}	0.293 {0.211 : 0.385}	0.259 {0.179 : 0.349}	0.282 {0.200 : 0.367}	0.281 {0.202 : 0.370}

**Note:** The 95% AR bootstrapped confidence bands used. For the SV-AR(1) the 95% confidence intervals are given below the median estimates.

of volatility around the 2001 recession which rises again thereafter. Interestingly, the **PI<sub>2</sub>-GDP** series projects a far longer moderation period in terms of stochastic volatility and only spikes (and even then only temporarily) around the 2007q4 recession. Accordingly, if the researcher were to examine that series in isolation, she would derive a view of the stability properties of the labor share quite distinct from the alternatives.

**Figure 4: Stochastic Volatility of Labor Share Series**



**Notes:** The plots show  $e^{\sigma t}$ . Sample 1946q1 to 2013q1. Naive-GDP, **PI-GDP**, **PI<sub>2</sub>-GDP**, **SE-GDP**, **BLS** (dot-dashed).

## 5 STRUCTURAL BREAKS

Ultimately the importance of persistence is to gauge whether a series is or is not stationary (around a constant or a linear trend). When persistence takes the form of a unit root, the effect of an innovation is permanent. Since income shares are defined within the unit interval and have not exhibited corner solutions in history, one's prior might be that labor share does not contain a unit root; Tables 3 and 4 suggested as much.

Testing more formally for a unit root, however, is largely inconclusive. We implemented several tests (ADF, PP, ADF-GLS, symmetric and asymmetric ADF-ESTAR, and fractional). Results (see appendix **Tables C.1, C.2**) vary substantially across the series, reflecting the existence of a clear downward trend in some of them (e.g., **SE-GDP**), hump-shaped trends in some others (e.g., **PI<sub>2</sub>-GDP**), and their varying degrees of persistence. Note that some of the aforementioned facts could be a consequence of changes in the sectoral structure of the US economy (see, e.g., Elsby, Hobijn, and Sahin, 2013). Since this goes beyond our remit, an indicative discussion of the role of

the sectoral makeup of the aggregate labor share has been relegated to Appendix B.

Importantly, there is also no systematic evidence for stationarity when a structural break is allowed for.<sup>11</sup> This outcome may have been caused either by complicated dynamics of the considered time series – driven by their large persistence and presence of nonlinear trends – or by the existence of *more than one breakpoint* in the labor share. Accordingly, we complement our analysis by applying a multiple breaks detection procedure proposed by Bai and Perron (2003). As in previous exercises, we consider three assumptions about the deterministic component of the time series: only constant, linear trend, and quadratic trend. For each case we report the *optimal* number of breaks in the data-generating process with corresponding breakpoints. The optimal number of structural changes is chosen with the BIC criterion, restricted to be at most 5.

Results are presented in appendix **Tables C.3-C.4**.<sup>12</sup> This testing procedure allows for changes in the mean and/or slope. Which case one relies upon is largely a matter of judgement. For simplicity and in line with usual interpretations, **Figures 5-6** plot the mean breaks detected.

These results indicate strong evidence in favor of multiple structural breaks. However, the timing of breakpoints varies among different labor share variants. Typically, two to five structural changes might be identified: early 1940s, late 1950s, late 1960s–early 1970s, first half of 1980s, and late 1990s–early 2000s. The first (in annual data only, given the sample), third and fourth of these breaks appear most robust across specifications, and can be identified with World War II, the oil crisis, and the early 1980s recession. Alternatively, the latter two dates might be perceived as a mark of the beginning of the spread of ICT technologies across the US.

To sum up, for each labor share series we find evidence of *multiple* structural breaks, which explains why Zivot–Andrews tests of stationarity subject to a structural break might have had low power. A caveat is some heterogeneity in the dating. For instance, in the case of the **PI<sub>2</sub>-GDP** series, tests suggest either no structural break at all, or three of them: during World War II, in late 1960s, and in early 1980s.

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<sup>11</sup>Our analysis suggests that if there were structural breaks, there have been more than one. This pertains to all the considered series. First, we find that the  $\mathcal{F}$  statistic of the Chow single breakpoint test, based on a simple data-generating process including only deterministic components, is below its critical value at any possible breakpoint. Therefore this test does not allow us to reject the null of no structural break against the alternative of a single break.

<sup>12</sup>Notice the fall in the labor share from 2001 is not always picked up by the tests reflecting the influence of “trimming” at the end of the sample, as well as the fact that we limit the number of break to at most five.

Figure 5: Annual series

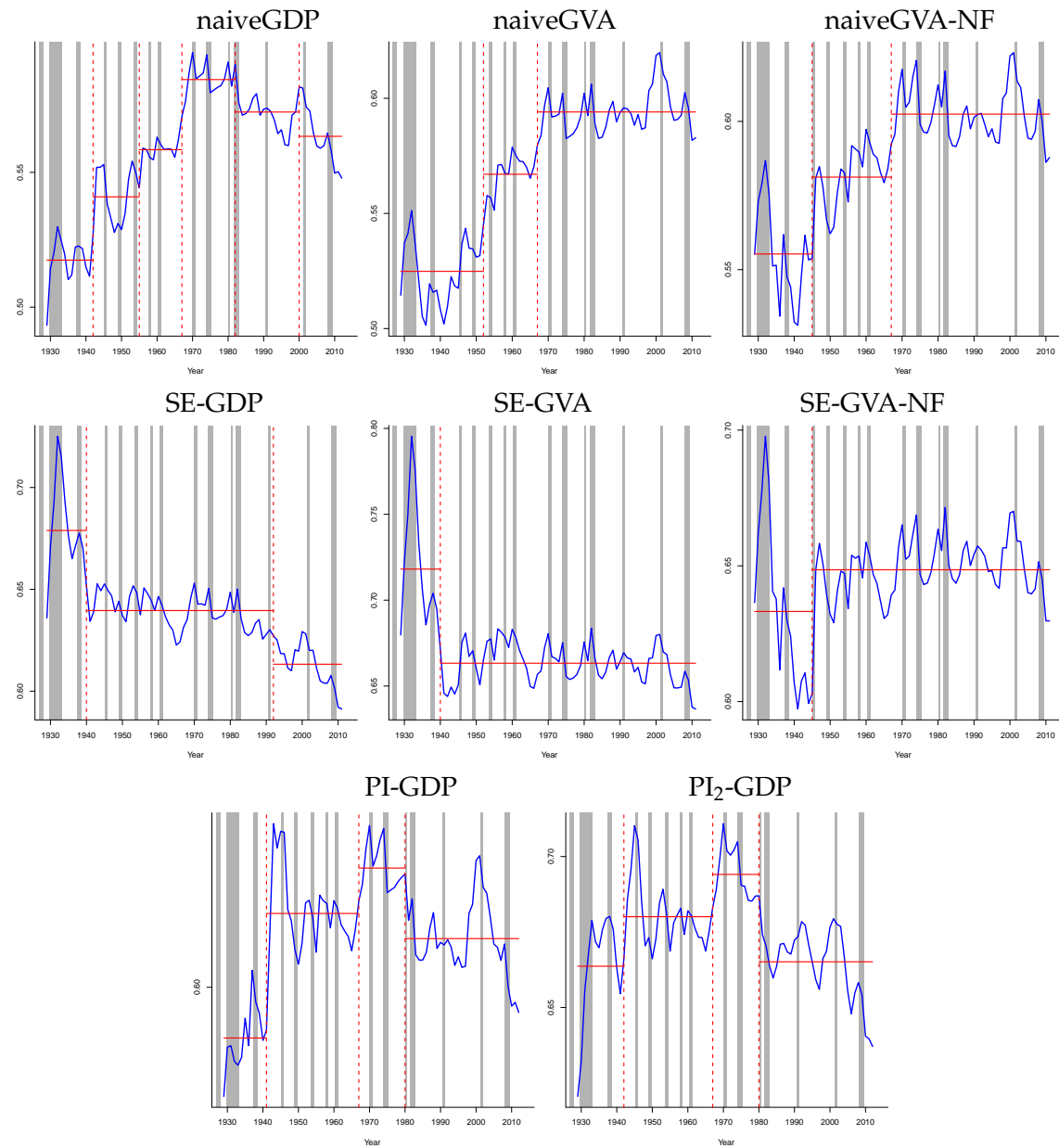
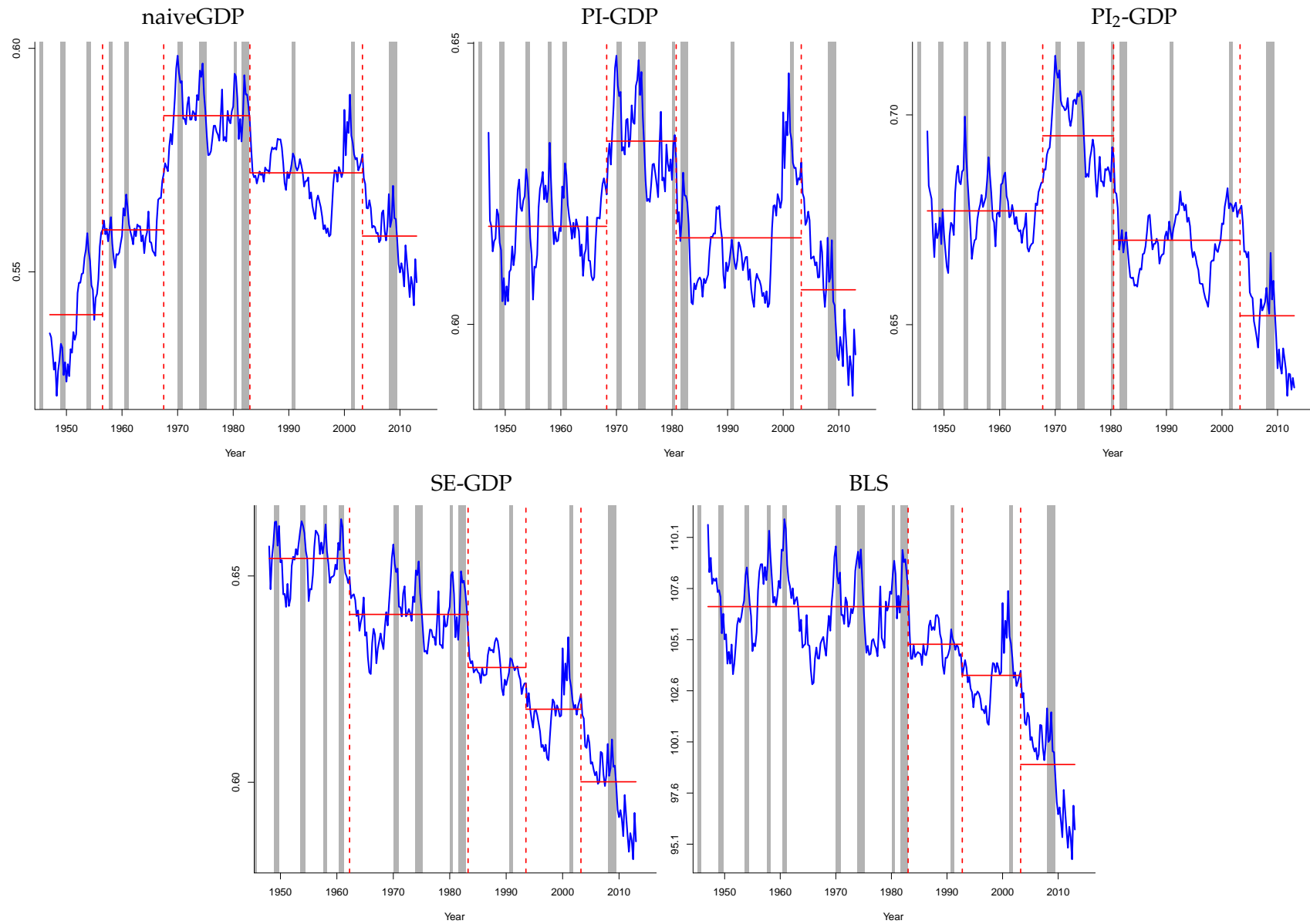


Figure 6: Quarterly series



## 6 SPECTRAL ANALYSIS

### 6.1 SPECTRAL DECOMPOSITION OF US LABOR SHARE SERIES

Ambiguity over stationarity, the presence of structural breaks, and the apparent lack of convergence between the labor share series suggests that it is low-frequency aspects that are most important to understand when comparing these alternate labor share series. We shall investigate this issue using spectral techniques. Our motivation in performing spectral analysis is to assess the importance of fluctuations with given periodicity for the total observed variance of the respective series and to justify whether oscillations of specific frequencies systematically co-move between various definitions of the labor share.

In our exercise we distinguish between the low-, medium- and high- frequency range. High-frequency fluctuations are defined as all oscillations with periodicity below 8 years, interpreted as business-cycle fluctuations (cf. Young, 2004). The second range, the medium-term business cycles, as formulated by Comin and Gertler (2006), includes all fluctuations with periodicity between 8 and 50 years.<sup>13</sup> The longest swings with periodicity higher than 50 years are mapped into the low-frequency component, interpreted as a stochastic trend.

For spectral techniques, the data should not have a unit root.<sup>14</sup> Given the ambiguity in formal unit root testing, we apply three approaches to excluding the deterministic component: removing the mean, linear and quadratic trend from log-levels. Of course a demeaned non-stationary series remains non-stationary, but in the familiar context where the labor share is seen as fluctuating around a constant mean, it provides a natural benchmark.

The estimated shares of specific frequencies in the overall variance are reported in **Tables 6 and 7**. Apart from the **SE-GDP** and **SE-GVA** variants,<sup>15</sup> the role of the low-frequency component is substantial. For the demeaned series, long cycles beyond 50 years (variations in the stochastic trend) are responsible for from 1/4 to almost 2/3 of the overall variance. The contribution of the low-frequency component is significant even if a linear trend is included in data-generating process. For both transformations, the medium-run component is more important than the short-run one in the case of all annual series and 4 out of 5 quarterly series.

**Table 6: Shares of Specific Frequencies in Total Variance (%) – Annual Series**

PERIODICITY (IN YEARS)	DEMEANED			EXCL. LINEAR			EXCL. QUADRATIC		
	≥ 50	8 – 50	≤ 8	≥ 50	8 – 50	≤ 8	≥ 50	8 – 50	≤ 8
Naive-GDP	<b>73.6</b>	20.3	6.1	<b>72.8</b>	17.4	9.9	2.8	<b>65.7</b>	31.5
PI-GDP	25.8	<b>58.3</b>	15.9	28.0	<b>56.5</b>	15.5	3.5	<b>73.8</b>	22.7
PI <sub>2</sub> -GDP	31.5	<b>48.3</b>	20.2	28.3	<b>48.9</b>	22.7	0.2	<b>68.1</b>	31.7
SE-GDP	18.0	<b>56.1</b>	25.9	15.5	<b>46.1</b>	38.4	15.9	<b>45.7</b>	38.4
Naive-GVA	<b>62.4</b>	33.5	4.0	<b>61.0</b>	21.0	17.9	0.9	<b>57.6</b>	41.5
Naive-GVA-NF	<b>54.3</b>	36.8	8.9	<b>46.1</b>	29.5	24.4	0.3	<b>56.1</b>	43.6
SE-GVA-NF	30.1	<b>42.3</b>	27.6	15.4	<b>44.0</b>	40.5	1.6	<b>51.9</b>	46.5
SE-GVA	4.1	<b>53.1</b>	42.8	9.4	<b>49.8</b>	40.8	7.3	<b>49.4</b>	43.3

**Note:** The shares have been calculated based on periodogram estimates. Bold indicates the maximum of frequency share over each respective “stationarizing” processes.

Particularly interesting findings arise when analyzing the series excluding a quadratic trend.

<sup>13</sup>To be more precise, our medium frequency component is equivalent to the Comin and Gertler (2006) “low-frequency subcomponent” of medium-term business cycles.

<sup>14</sup>The shares of given frequency domains in the total variance of a time series have been computed by cumulating raw periodogram values over each desired frequency (low, medium and high), and then dividing by total variance. Such an estimator is only asymptotically consistent, though (for a general overview see Hamilton (1994, chapter 6)), which provides the caveat that its efficiency can be low if the series is short.

<sup>15</sup>The annual **SE-GDP** variant should be treated with caution. Robustness of the long cycle to subtracting a quadratic trend are in the case of this series likely driven by a structural break in the NIPA data on self-employment. To a smaller extent, this break also influences the properties of **SE-GVA**.



**Table 7: Shares of Specific Frequencies in Total Variance (%) – Quarterly Series**

PERIODICITY (IN YEARS)	DEMEANED			EXCL. LINEAR			EXCL. QUADRATIC		
	≥ 50	8 – 50	≤ 8	≥ 50	8 – 50	≤ 8	≥ 50	8 – 50	≤ 8
Naive-GDP	<b>64.8</b>	27.9	7.4	<b>70.4</b>	21.3	8.3	3.8	<b>64.7</b>	31.6
PI-GDP	24.2	<b>51.0</b>	24.8	16.1	<b>60.1</b>	23.8	0.8	<b>69.5</b>	29.7
PI <sub>2</sub> -GDP	<b>42.8</b>	37.9	19.3	29.8	<b>52.4</b>	17.8	0.4	<b>72.6</b>	27.0
SE-GDP	7.0	23.5	<b>69.5</b>	6.9	23.4	<b>69.7</b>	3.1	20.3	<b>76.6</b>
BLS	36.5	<b>36.8</b>	26.7	31.7	<b>35.0</b>	33.3	3.3	44.6	<b>52.2</b>

Naturally, extraction of a quadratic trend from the labor share data series limits the importance of the low-frequency component whose contribution to the overall variance falls below 4%. Secondly, we see that for the “naive” series and for the series adjusted by proprietors’ income, the share of the medium-term component is almost two times higher than of the high-frequency component.

On the other hand, the quarterly **SE-GDP** series (since 1947) seems to be characterized by quite distinct spectral characteristics. Most of its variance is concentrated in short-run frequencies, irrespectively of the data transformation.<sup>16</sup> **PI<sub>2</sub>-GDP**, in contrast, provides a consistent message for both the annual and quarterly frequency: around 80% of its total variability is generated by medium-run cycles and a long-run hump-shaped swing, which can be very well fitted by a quadratic trend.

## 6.2 COHERENCE

Next, we perform cross-spectral analysis by computing coherence. This addresses the question whether the pairs of the different labor share variants systematically co-move within specific frequency ranges.

**Tables 8 and 9** present coherence estimates.<sup>17</sup> Keeping in mind that the annual numbers may be somewhat less reliable due to fewer observations, we find that coherence is always significant in the high-frequency domain. This result corroborates the previously formulated conclusion that labor share series tend to be rather consistent in the short run. Coherence estimates are more ambiguous in the lower frequencies, though. In the medium- and low-frequency domain we identify subgroups for which coherence is very high, reflecting their definitional similarity: annual and quarterly series adjusted by proprietors’ income (**PI-GDP** and **PI<sub>2</sub>-GDP**), all “naive” annual labor share series, all annual series adjusted for self-employment, and a pair consisting of quarterly series **SE-GDP** and **BLS**. Otherwise, the coherence is rather low.

<sup>16</sup>High-frequency fluctuations are also the most important part of the frequency domain for **BLS** series but only when the quadratic trend is extracted from data.

<sup>17</sup>The coherence statistic for a pair of time series  $(x_t, y_t)$  can be understood as the  $R^2$  from  $x_t$  regressed on  $y_t$  as a function of the frequency. Complementarily, one can also compute *dynamic correlation* coefficients, to control the sign of the relationship in each given pair. For all the pairs, dynamic correlation is in line with the general intuition, though: it is positive and significant whenever the coherence for a given pair is significant. In the case of insignificant coherence, the dynamic correlation is not significantly different from zero.

**Table 8: Average Coherence Among the Labor Share Series – Annual Data**

Periodicity	PI-GDP			PI <sub>2</sub> -GDP			SE-GDP			Naive-GVA			Naive-GVA-NF			SE-GVA-NF			SE-GVA		
	≥ 50	8-50	≤ 8	≥ 50	8-50	≤ 8	≥ 50	8-50	≤ 8	≥ 50	8-50	≤ 8	≥ 50	8-50	≤ 8	≥ 50	8-50	≤ 8	≥ 50	8-50	≤ 8
DEMEANED SERIES																					
Naive-GDP	0.52***	0.67***	0.73***	0.09	0.42**	0.72***	0.47	0.19	0.43**	0.86***	0.65***	0.77***	0.89***	0.63***	0.54***	0.42**	0.15	0.31*	0.48***	0.17	0.34*
PI-GDP				0.53***	0.63***	0.63***	0.23	0.10	0.48***	0.26	0.31*	0.62***	0.33**	0.43**	0.71***	0.04	0.22	0.56***	0.61***	0.22	0.43**
PI <sub>2</sub> -GDP							0.05	0.20	0.53***	0.01	0.14	0.46***	0.01	0.19	0.36**	0.03	0.05	0.27*	0.07	0.07	0.37**
SE-GDP										0.65***	0.26	0.35**	0.57***	0.26	0.41**	0.11	0.33**	0.52***	0.70***	0.78***	0.85***
Naive-GVA													0.98***	0.88***	0.81***	0.58***	0.38**	0.58***	0.37**	0.20	0.46***
Naive-GVA-NF																0.64***	0.62***	0.87***	0.34**	0.29	0.55***
SE-GVA-NF																			0.02	0.61***	0.74***
DE-TRENDED SERIES																					
Naive-GDP	0.00	0.42**	0.68***	0.16	0.49***	0.56***	0.46***	0.54***	0.78***	0.37**	0.46***	0.66***	0.41**	0.51***	0.50***	0.19	0.26	0.42**	0.20	0.28	0.66***
PI-GDP				0.60***	0.60***	0.46***	0.31*	0.11	0.54***	0.13	0.11	0.56***	0.04	0.24	0.66***	0.29*	0.21	0.61***	0.49***	0.16	0.53***
PI <sub>2</sub> -GDP							0.01	0.26	0.66	0.04	0.11	0.25	0.01	0.12	0.21	0.13	0.05	0.25	0.15	0.11	0.52***
SE-GDP										0.33**	0.69***	0.56***	0.30*	0.66***	0.44**	0.49***	0.65***	0.50***	0.70***	0.81***	0.91***
Naive-GVA													0.91***	0.91***	0.83***	0.79***	0.84***	0.78***	0.68***	0.81***	0.72***
Naive-GVA-NF																0.79***	0.88***	0.94***	0.55***	0.77***	0.60***
SE-GVA-NF																			0.86***	0.91***	0.71***

**Note:** \*\*\*, \*\* and \* denote rejection of the null of coherence insignificance at 1%, 5% and 10% significance level, respectively. The spectra for a given pair have been estimated using the Parzen kernel.

**Table 9: Average Coherence Among the Labor Share Series – Quarterly Data**

Periodicity	PI-GDP			PI <sub>2</sub> -GDP			SE-GDP			BLS		
	≥ 50	8-50	≤ 8	≥ 50	8-50	≤ 8	≥ 50	8-50	≤ 8	≥ 50	8-50	≤ 8
DEMEANED SERIES												
Naive-GDP	0.33***	0.39***	0.68***	0.21**	0.26**	0.38***	0.01	0.01	0.38***	0.04	0.08	0.43***
PI-GDP				0.82***	0.81***	0.51***	0.30***	0.32***	0.69***	0.44***	0.48***	0.81***
PI <sub>2</sub> -GDP							0.55***	0.54***	0.50***	0.64***	0.62***	0.52***
SE-GDP										0.89***	0.88***	0.89***
DE-TRENDED SERIES												
Naive-GDP	0.78***	0.79***	0.85***	0.64***	0.65***	0.44***	0.18*	0.38***	0.89***	0.11	0.26**	0.77***
PI-GDP				0.83***	0.80***	0.46***	0.24**	0.41***	0.82***	0.22**	0.36***	0.87***
PI <sub>2</sub> -GDP							0.12	0.25**	0.41***	0.08	0.17*	0.40***
SE-GDP										0.89***	0.91***	0.85***

**Note:** \*\*\*, \*\* and \* denote rejection of the null of coherence insignificance at 1%, 5% and 10% significance level, respectively. The spectra for a given pair have been estimated using the Parzen kernel.

The spectral analysis highlights the discrepancies between various labor share series. From the frequency domain perspective it appears that the most outlying are the series adjusted for self-employment, which are characterized by substantially different variance decompositions and insignificant coherence with other variants in the low- and medium-run frequency. For example, (our favored series) **PI<sub>2</sub>-GDP** is generally incoherent with the **SE-GVA** measure, considered the “headline measure” (and subsequently criticized) by Elsby, Hobijn, and Sahin (2013).

Our results obtained so far point to the general conclusion that while short-run properties of the labor shares are relatively consistent across the considered alternative definitions, their medium-run swings and long-run trends diverge substantially. At the same time, ambiguity in unit root test results is likely due to the high persistence and complicated dynamics of the considered series. We can also argue that the **PI<sub>2</sub>-GDP** measure is probably not only the most sound theoretically, but also has intuitive empirical properties: (1) it is mean-reverting but highly persistent, with about 80% of its total variance observed in the medium-to-long run frequency range, (2) it is countercyclical over the short run, (3) it has recorded a secular decline since 1970, and (4) it can be understood as featuring three economically interpretable structural breaks: during World War II, in late 1960s, and in early 1980s.

## 7 THE IMPORTANCE OF THE LABOR SHARE IN ECONOMIC APPLICATIONS

Let us now discuss the potential consequences of using these diverse series interchangeably in empirical applications. We consider three applications: (1) growth accounting, (2) technology-labor share VAR analysis, and (3) estimation of New Keynesian Phillips curves. These three were chosen not only to reflect their broad popular application in various literatures, but also because they help reveal and substantiate some of the differences in the series discussed earlier.

### 7.1 APPLICATION I: GROWTH ACCOUNTING

We begin with growth accounting, which is a widely-used exercise in macroeconomics, development and business-cycle analysis. This exercise decomposes economic growth into that due to factor accumulation, and technical progress (which is derived residually). The standard growth accounting equation can be written as:

$$\Delta \text{tfp}_t = \Delta y_t - \tilde{\alpha}_t \Delta k_t - (1 - \tilde{\alpha}_t) \Delta l_t, \quad (11)$$

where all variables are in logs, and where  $\tilde{\alpha}_t$  and  $\text{tfp}_t$  denote the (potentially time-varying) capital share and log total factor productivity, respectively.

We already know that labor shares are time-varying and have different properties across variants. Accordingly, this should be reflected in how we implement growth accounting. With this in mind, the extraction of TFP can then be done in the following ways:

#### 1. Common Input Factors and Outputs

Derive TFP across different labor share measures based on *common* input factors and *common* inputs:

$Y_t^1$  : GDP in constant USD [NIPA Table 1.6];

$K_t^1$  : Chain-Type Quantity Index for the Net Stock of Fixed Assets [FAT Table 1.2];

$L_t^1$  : Full-Time Equivalent Employees plus Self-Employed in all domestic industries [NIPA Tables 6.5 and 6.7].

#### 2. Common Input Factors But Definitionally Consistent Outputs

Derive TFP across different labor share measures based on common input factors but with output measures related to the specific labor-share measure:

$Y_t^1$  : GDP in constant USD [*NIPA Table 1.6*]

$Y_t^2$  : Real Gross Value Added in the private sector [*NIPA Table 1.3.6*];

$Y_t^3$  : Real Gross Value Added in the non-farm business sector [*NIPA Table 1.3.6*];

$K_t^1$  and  $L_t^1$  as above.

### 3. Definitionally Consistent Factors and Outputs

Derive TFP across different labor share measures based on input factors and output measures related to the specific labor-share measure:

$Y_t^1 - Y_t^3, K_t^1, L_t^1$  : as above;

$K_t^2$  : Chain-Type Quantity Index for the Net Stock of Fixed Assets in the private sector [*FAT Table 1.2*]

$K_t^3$  : Chain-Type Quantity Index for the Net Stock of Fixed Assets in the non-farm business sector [also *FAT Table 1.2*];

$L_t^2$  : Full-Time Equivalent Employees plus Self-Employed in the private sector [*NIPA Tables 6.5 and 6.7*].

$L_t^3$  : Full-Time Equivalent Employees plus Self-Employed in the non-farm business sector [*NIPA Tables 6.5 and 6.7*].

Observe that for **naiveGDP**, **PI-GDP**, **PI<sub>2</sub>-GDP** and **SE-GDP**, all these approaches boil down to the same growth accounting scenario because our reference measures of GDP, capital, and labor are then also definitionally consistent.

Within each of these three cases, we assume that factor shares are time-varying following a Törnquist index:

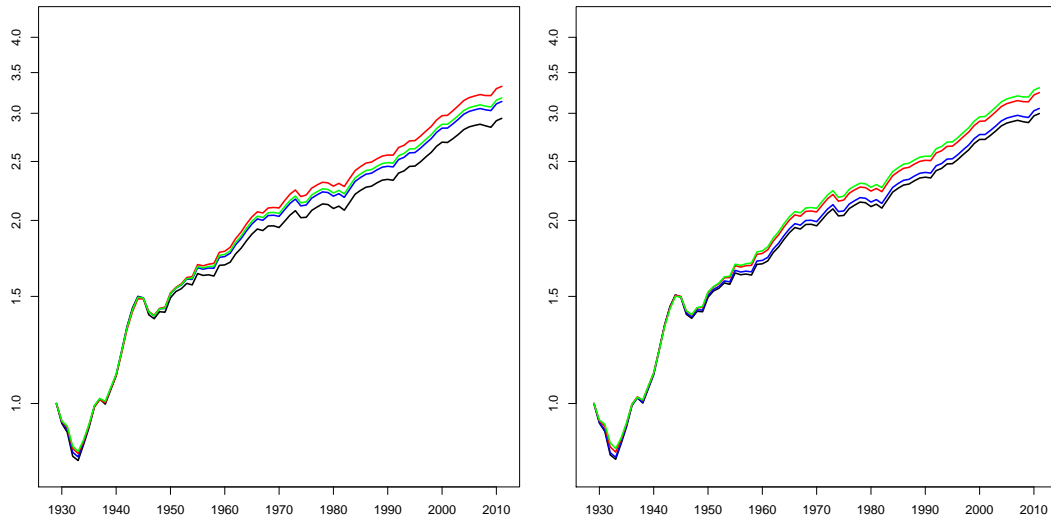
$$\tilde{\alpha}_{j,t} = \frac{\alpha_{j,t} + \alpha_{j,t-1}}{2},$$

where  $j$  denotes the particular labor share variant used (e.g., **naiveGDP**, **PI-GDP**, etc.).

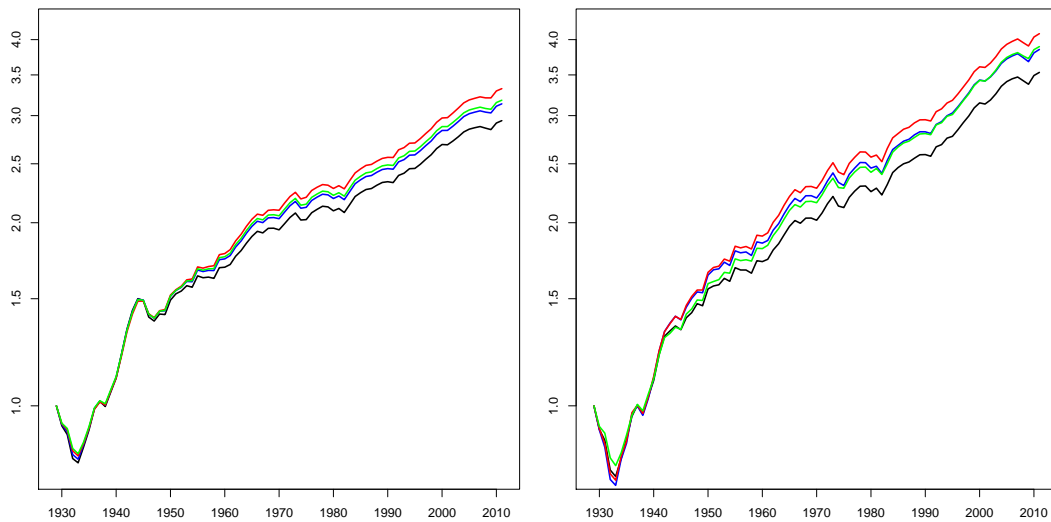
Our purpose therefore is to examine the scope for mis-measurement of TFP (growth and levels) when factor income shares vary and when differences in shares are compounded with those of output and the factors. **Figure 7** shows the cumulated TFP levels for time-varying income shares, where the factors are assumed constant across labor share definitions (the first row), where the output definitions are additionally allowed to change (second row), and where both the inputs and outputs are allowed to change consistent with the underlying labor share definition (final row).

In terms of shape, all series broadly conform to what is commonly understood to be the story behind US TFP (e.g., Fernald (2007), Shackleton (2013)): exceptionally high TFP growth in the mid 1930-1940s, the consolidation of those gains in the decades after WW II, followed by a period of slower residual productivity growth (often dated to the early 1970s), and the acceleration in productivity towards the end of the sample.

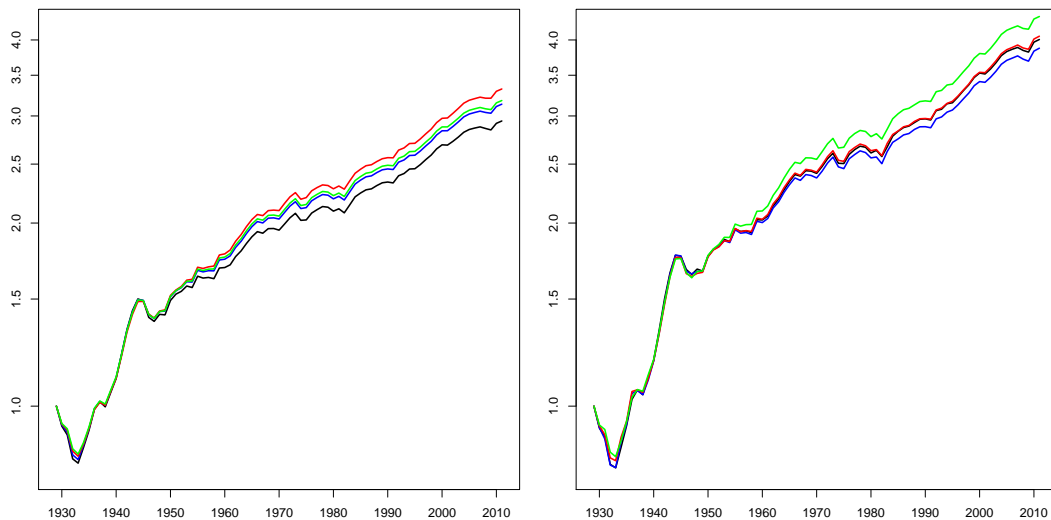
**Figure 7: Cumulative TFP based on Time-Varying Factor Shares (1929=1)  
Common Input Factors and Outputs**



**Common Input Factors, Definitionally Consistent Outputs**



**Definitionally Consistent Input Factors and Outputs**



**Notes:**

Left panel: Naive-GDP, **PI-GDP**, **PI<sub>2</sub>-GDP**, **SE-GDP** ; Right panel: Naive-GVA, **Naive-GVA-NF**, **SE-GVA-NF**, **SE-GVA**  
A log-scale for the level of TFP is used in these graphs for legibility.

**Table 10: Cumulative Change of TFP, Based on Time-Varying Factor Shares (In %)**

	naive-GDP	PI-GDP	PI <sub>2</sub> -GDP	SE-GDP	naive-GVA	naive-GVA-NF	SE-GVA-NF	SE-GVA
	Common Input Factors and Outputs							
1929-2011	108	114	120	116	110	112	118	120
1929-1945	40	40	40	40	40	40	41	40
1945-1970	27	31	34	32	27	28	32	34
1970-2011	41	43	46	44	43	43	45	46
	Common Input Factors, Definitionally Consistent Outputs							
1929-2011	108	114	120	116	126	135	141	136
1929-1945	40	40	40	40	29	33	33	29
1945-1970	27	31	34	32	41	46	50	48
1970-2011	41	43	46	44	56	56	59	59
	Definitionally Consistent Input Factors and Outputs							
1929-2011	108	114	120	116	139	136	140	148
1929-1945	40	40	40	40	57	57	56	56
1945-1970	27	31	34	32	31	30	32	38
1970-2011	41	43	46	44	51	49	51	54

There are, though, certain level differences between the TFPs generated with the use of the alternate factor share series. Some specific discrepancies in dynamics are worth noting too. For example, productivity and TFP growth are often considered to have exhibited a broken trend in the early 1970s (e.g., Fernald, 2007). Whilst this is clearly visible for most of the series, it is less apparent for the GVA series.<sup>18</sup>

**Table 10** shows the cumulative change of TFP based on time-varying factor income shares. There are indeed substantial cumulative discrepancies. To illustrate, whilst **naiveGDP** grew by 108% over the whole sample, **SE-GVA-NF** grew by 140% (in the last accounting scenario).

Interestingly, we also find that the TFP deviations for all labor share specifications against **NaiveGDP** have been gradually increasing since World War II, see **Figure 7** and **Table 11**.<sup>19</sup> This is driven by the fact that the post-war period was characterized by rapid physical capital accumulation, and hence the underestimation of the labor share (equivalently, overestimation of the capital share) in the **naive-GDP** case has systematically led to an overstating of capital's contribution to GDP growth, at the cost of understating the role of TFP. By this logic, it should not be surprising that the relatively highest labor share **PI<sub>2</sub>-GDP** implies also the relatively strongest TFP growth.

## 7.2 APPLICATION II: TECHNOLOGY SHOCKS AND THE LABOR SHARE

Let us now pass to the short-run question of examining the impact of exogenous technology shocks on the labor share. The motivation for undertaking such an exercise is the following. First, it is worthwhile to verify if the apparently consistent short-run properties of all considered labor share measures carry forward to applied econometric studies of the business cycle. The question of the impact of technology shocks seems a reasonable first step in this direction. Second, as argued above the labor share switches from being countercyclical in the short run to being procyclical in the medium run. Ríos-Rull and Santaulàlia-Llopis (2010) have found an overshooting

<sup>18</sup>The absence of a slight hump in the TFP level in the mid 1940s (see middle panel, rhs graph) is caused by the fact that real GDP/GVA grew at slightly different rates. For example, the most spectacular difference was in 1946 when GDP fell by 10% while GVA by less than 1%. Note that this does not reappear in the bottom row rhs graph since then we adjust the inputs consistently with the labor share definition. Note also that it is the period after WWII so there was a substantial shift between sectors (government vs private).

<sup>19</sup>The observed divergence between the GDP-based and GVA-based series in the initial period 1929–1945 in the last two accounting scenarios is due to real GDP/GVA growing at different rates in the 1940s.



**Table 11: Cumulative Deviation from TFP Based on naiveGDP (In %)**

	naive-GDP	PI-GDP	PI <sub>2</sub> -GDP	SE-GDP	naive-GVA	naive-GVA-NF	SE-GVA-NF	SE-GVA
Common Input Factors and Outputs								
1929-2011	0	6	12	8	2	4	10	12
1929-1945	0	0	0	0	0	0	1	0
1945-1970	0	4	8	5	0	2	5	7
1970-2011	0	2	5	2	1	2	4	4
Common Input Factors, Definitionally Consistent Outputs								
1929-2011	0	6	12	8	18	27	33	28
1929-1945	0	0	0	0	-11	-7	-7	-11
1945-1970	0	4	8	5	15	19	23	21
1970-2011	0	2	5	2	15	15	17	18
Definitionally Consistent Input Factors and Outputs								
1929-2011	0	6	12	8	31	28	32	40
1929-1945	0	0	0	0	17	17	16	16
1945-1970	0	4	8	5	5	3	6	11
1970-2011	0	2	5	2	9	8	10	13

response of the labor share to technology shocks, consistently with its short-run countercyclicality and a positive correlation of output with lagged labor shares. We shall verify if this property holds for various labor share definitions.

The current analysis is based on quarterly data spanning 1948q1-2013q1. Our technological shock variable is TFP growth, taken from Fernald (2012). Fernald's TFP measures are superior to the ones derived in the previous section because they distinguish between heterogenous physical capital and labor types, whose unit productivities are inferred from data on relative prices.<sup>20</sup> Another advantage of using Fernald's TFP time series is that they also include TFP adjusted for capacity utilization, which constitutes an important wedge between the available inputs and currently produced output. Capacity utilization (varying machine hours, labor hoarding, etc.) can indeed partially absorb technological shocks before they are transmitted to changes in factor shares.

Let us first, though, elaborate on how, in terms of the earlier theory discussion, we view an "aggregate" TFP shock. To examine this further, we can re-express the CES production function (1) in per-capita log form and apply a Taylor-series expansion around the point  $\zeta = 1$ , we derive (Kmenta, 1967; Klump, McAdam, and Willman, 2012):

$$y_t = \alpha k_t + \Lambda k_t^2 + \underbrace{\alpha \left[ 1 + \frac{2\Lambda}{\alpha} k_t \right] \gamma_{K,t} + (1 - \alpha) \left[ 1 - \frac{2\Lambda}{(1 - \alpha)} k_t \right] \gamma_{N,t} + \Lambda [\gamma_{K,t} - \gamma_{N,t}]^2}_{\Phi = \log(TFP)} \quad (12)$$

where  $y_t = \log[\frac{Y_t}{L_t}]$ ,  $k_t = \log[\frac{K_t}{L_t}]$ ,  $\Lambda = \frac{(\zeta - 1)\alpha(1 - \alpha)}{2\zeta}$  and  $\gamma_{K,t} = \log \Gamma_{K,t}$ ,  $\gamma_{N,t} = \log \Gamma_{N,t}$ .

If  $\zeta \rightarrow 1$  then  $y_t = \alpha \gamma_{K,t} + (1 - \alpha) \gamma_{N,t} + \alpha k_t$ .<sup>21</sup> Otherwise the log of TFP is an average of capital and labor augmenting technologies (with the weights determined by the capital-labor ratio and the income shares). In the absence of careful estimation, we do not observe  $\Phi$  in the factor-augmenting case, for an econometric discussion see León-Ledesma, McAdam, and Willman

<sup>20</sup>In his TFP computations, Fernald (2012) has interpolated annual data on factor shares from the BLS multifactor productivity database, claiming that his "results were little affected in experiments with other reasonable choices, such as using national accounting data". At this, it is reassuring that our findings corroborate the robustness of TFP calculations to changes in factor share definitions, especially in the short run. See also the work of Basu, Fernald, and Kimball (2006).

<sup>21</sup>In other words, the under-identified Cobb–Douglas form.

(2010). But its structure suggests we might think of general TFP shocks as driven mostly by labor-augmenting components.<sup>22</sup> As we know from condition (3), if the elasticity of substitution  $\zeta < 1$ , that shock will reduce the labor share.

### 7.2.1 ARDL MODEL

To assess the impact of exogenous technology shocks on the labor share, we first estimated a range of simple autoregressive distributed lag models:

$$ls_t = \mu + \rho ls_{t-1} + \sum_{i=0}^k \beta_i \Delta tfp_{t-i} + \varepsilon_t \quad (13)$$

where  $ls_t = \log(LS_t)$  and  $\Delta tfp_{t-k}$  is TFP growth (difference in log TFP levels) lagged  $k$  quarters. **Table 12** shows the results.

First, we find a negative contemporaneous correlation between the labor share and technological shocks. The correlation with lagged TFP growth is positive, though. This is suggestive of a non-monotonic, overshooting dynamics of the labor share following a TFP shock.<sup>23</sup>

Second, we find that the effect of the technological shock is highest for the **BLS** and **SE-GDP** series, and lowest for **PI<sub>2</sub>-GDP**, regardless of whether TFP shocks are capacity-adjusted or not.

Third, we extended model (13) to allow for the presence of *asymmetric* effects of technological shocks on the labor share. Thus we have checked whether the labor share reacts differently to positive and negative TFP shocks. Such results were obtained by splitting  $\Delta tfp_{t-1}$  into

$$\Delta tfp_{t-1}^+ = \Delta tfp_{t-1} \mathcal{I}(\Delta tfp_{t-1} > 0)$$

$$\Delta tfp_{t-1}^- = \Delta tfp_{t-1} \mathcal{I}(\Delta tfp_{t-1} < 0)$$

where  $\mathcal{I}$  is the indicator function. Under this specification, it is still estimated that  $\beta_0 < 0$ , i.e., the immediate effect of technology shocks is still to diminish the labor share. On the other hand, we also find that lagged (non-capacity-adjusted) technological shocks are positively correlated with the labor share if they are negative, and essentially uncorrelated if they are positive. This means that negative technological shocks tend to increase the labor share only temporarily (majority of the immediate negative effect disappears after one period), whereas positive technological shocks depress the labor share permanently (or at least for a longer time), and the overshooting dynamic is absent.

This result is largely driven by capacity adjustment, however. We do not find any evidence for asymmetric correlation between TFP shocks adjusted by capacity utilization and the labor share. Hence, it can be concluded that negative TFP shocks may appear as temporary only because they induce substantial declines in capacity utilization. If these declines are properly accounted for, negative TFP shocks tend to increase the labor share permanently as well. The numbers are fairly symmetric across all labor share specifications, albeit again, they are somewhat larger for the **BLS** and **SE-GDP** series, and smaller for **PI<sub>2</sub>-GDP**.

### 7.2.2 VAR ANALYSIS

A more sophisticated approach to assessing the impact of exogenous technology shocks on the labor share requires the researcher to allow for mutual impact of both variables. In its simplest

<sup>22</sup>For instance  $\frac{\partial \Phi / \partial \gamma_L}{\partial \Phi / \partial \gamma_K} > 1$  if  $\alpha + 2(1 + \gamma_K - \gamma_L) \Lambda < \frac{1}{2}$ . For a wide range of values this will typically hold, example for  $\alpha = 0.33, \zeta = 0.6$  and  $\gamma_K \approx \gamma_L$ .

<sup>23</sup>Using rolling window estimation of equation (13) and its counterpart for HP-filtered series we observe a slight decrease in the contemporaneous correlation between the labor share and technological shocks over time.

**Table 12: ARDL Model with TFP**

	Naive-GDP	PI-GDP	PI <sub>2</sub> -GDP	SE-GDP	BLS
NO CAPACITY ADJUSTMENT					
$\hat{\rho}$	0.955***	0.943***	0.987***	0.995***	0.981***
$\hat{\beta}_0$	-0.388***	-0.357***	-0.331***	-0.431***	-0.528***
$\hat{\beta}_1$	0.109**	0.152***	0.0704*	0.098**	0.141**
CAPACITY ADJUSTMENT					
$\hat{\rho}$	0.966***	0.949***	0.987***	0.991***	0.982***
$\hat{\beta}_0$	-0.252***	-0.233***	-0.196***	-0.289***	-0.351***
$\hat{\beta}_1$	0.155***	0.188***	0.140***	0.172***	0.215***
NO CAPACITY ADJUSTMENT + ASYMMETRIC LAG STRUCTURE					
$\hat{\rho}$	0.953***	0.947***	0.993***	1.00***	0.985***
$\hat{\beta}_0$	-0.391***	-0.359***	-0.335***	-0.435***	-0.530***
$\hat{\beta}_{1,-}$	0.324***	0.307***	0.284***	0.267**	0.265*
$\hat{\beta}_{1,+}$	0.006	0.0809	-0.028	0.019	0.084
CAPACITY ADJUSTMENT + ASYMMETRIC LAG STRUCTURE					
$\hat{\rho}$	0.960***	0.956***	0.989***	0.995***	0.985***
$\hat{\beta}_0$	-0.240***	-0.212***	-0.190***	-0.285***	-0.342***
$\hat{\beta}_{1,-}$	-0.079	-0.066	-0.094	-0.116	-0.182
$\hat{\beta}_{1,+}$	0.035	0.105*	0.007	0.007	0.056

**Note:** subscripts \*\*\*, \*\* and \* denote the rejection of null about parameter's insignificance at 1%, 5% and 10% significance level, respectively. The constant estimated is suppressed for brevity.

form, such relationships can be analyzed by the means of a bivariate VAR model:

$$z_t = a_0 + \sum_{i=1}^p \Phi_i z_{t-i} + u_t \quad (14)$$

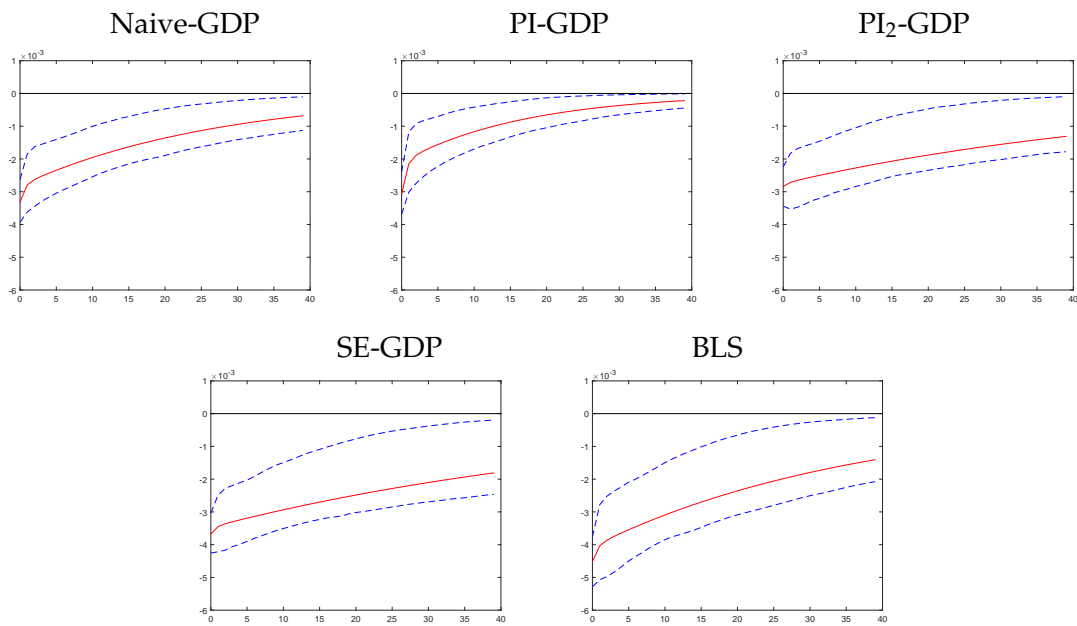
where  $z_t = \begin{bmatrix} \Delta \text{tftp}_t \\ l_{S_t} \end{bmatrix}$  is the vector of jointly determined dependent variables and  $u_t$  is a  $2 \times 1$  vector of disturbances. Lag length  $p$  shall be selected according to the BIC criterion.

To analyze the dynamic response of the labor share to a technological shock we use orthogonal impulse response functions. Since residuals from equations in VAR models can be correlated, the standard IRF analysis does not include such information and, as a result, cannot generate the true trajectories. Therefore, covariance between residuals is taken into consideration via a Cholesky decomposition. Our ordering of variables corresponds to the ARDL approach and, correspondingly,  $\Delta \text{tftp}_t$  is set as the first variable in the system.

We find that in both sets of IRFs (**Figures 8–9**), the effect of a temporary TFP shock is to reduce the labor share. These results are in line with our earlier theoretical reasoning which points out that TFP shocks are typically relatively more labor- than capital-augmenting, and that capital and labor are gross complements,  $\zeta < 1$ . As with the ARDL case, however, we might speculate that some fraction of any technological improvement partly complements the existing capital stock or labor input, and partly raises utilization rates. This latter possibility disguise some of the identification of the technological shock's effect on factor shares and explains why the response to utilization-adjusted TFP shocks is generally much smaller and, at the 95% confidence level, only significant in the first period.

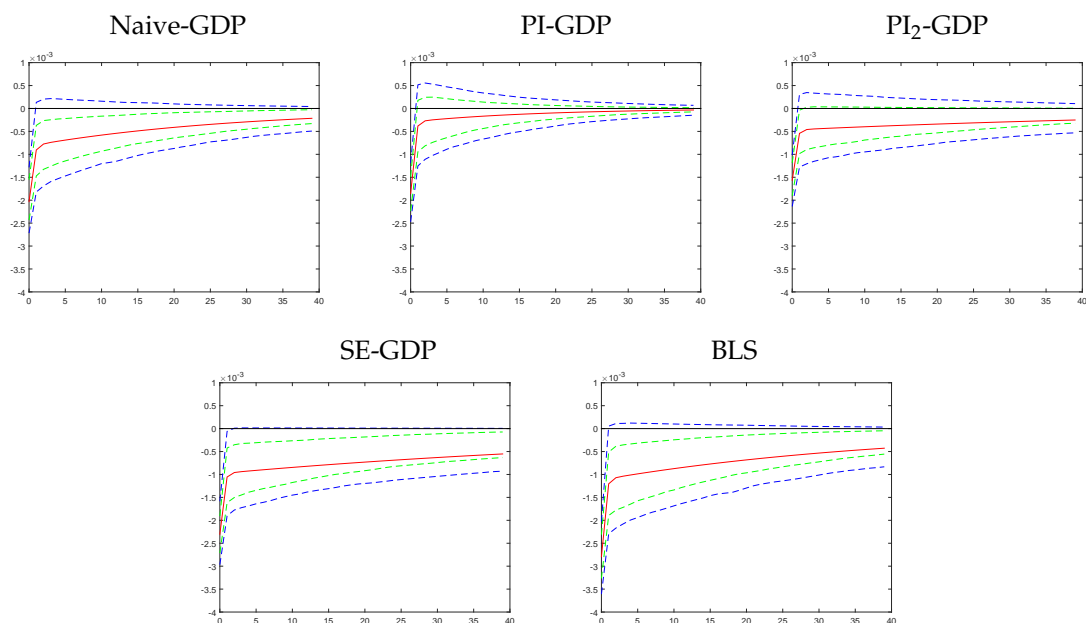
There are also marked differences in the speed of reversion, with **GDPPI** and (largely speaking) **naiveGDP** having returned to their base within a 10 year horizon. For the other series, the effect is highly protracted and stretches into the domain of medium-term business cycles. Clearly in general equilibrium models where the labor share plays a non-trivial role (as for example in labor bargaining models) this differential speed of reversion of income shares from technology shocks will be very important.

**Figure 8: Response of the Labor Share to a Technology Shock, no Capacity Adjustment**



**Notes:** Bootstrapped 95% confidence bands shown in blue dotted lines.

**Figure 9: Response of the Labor Share to a Technology Shock Adjusted for Capacity Utilization**



**Notes:** Bootstrapped 95% (68%) confidence bands shown in blue (green) dotted lines.

### 7.3 APPLICATION III: NEW KEYNESIAN PHILLIPS CURVES

Our final application is in the field of inflation modelling. As in Galí and Gertler (1999) and subsequent literature, the New-Keynesian Phillips Curve literature assumes staggered price setting under imperfect competition, where a fraction  $\theta$  of firms do not change their prices in any given period. The remaining firms set prices optimally as a fixed mark-up,  $\mu$ , on discounted expected

marginal costs. When resetting, firms also take into account that the price may be fixed for many future periods, yielding the optimal reset price  $p_t^*$ ,

$$p_t^* = (1 - \theta\beta) \mathbb{E}_t \sum_{k=0}^{\infty} (\theta\beta)^k [mc_{t+k}^n + \mu] \quad (15)$$

where  $mc^n$  is (the log of) nominal marginal costs,  $\beta$  is a discount factor, and  $\mathbb{E}_t$  is the expectation operator. The overall price level is then a weighted average of lagged and reset prices,  $p_t = \theta p_{t-1} + (1 - \theta) p_t^*$ . Given  $mc_t^r \equiv mc_t^n - p_t$ , and constant marginal costs across firms, the familiar “New Keynesian Phillips Curve” (NKPC) emerges,

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \lambda (mc_t^r + \mu) \quad (16)$$

where  $\pi_t = p_t - p_{t-1}$  is inflation and  $\lambda = \frac{(1-\theta)(1-\theta\beta)}{\theta}$  represents the reduced-form “slope”.

Additionally, it is often assumed that of the  $1 - \theta$  price-re-setting firms a fraction,  $\omega$ , set their price according to lagged inflation. This implies a NKPC with an intrinsic expectations component:

$$\pi_t = \gamma_f \mathbb{E}_t \pi_{t+1} + \gamma_b \pi_{t-1} + \lambda (mc_t^r + \mu) \quad (17)$$

where  $\phi = \theta + \omega [1 - \theta (1 - \beta)]$ ,  $\gamma_f = \frac{\theta\beta}{\phi}$ ,  $\gamma_b = \frac{\omega}{\phi}$ , and  $\lambda = \frac{(1-\omega)(1-\theta)(1-\theta\beta)}{\phi}$ .

Real marginal costs,  $mc^r$ , are difficult to measure, though. An early approach was to proxy them by using the (stationary) deviation of output from a linear/quadratic trend, or a HP-filtered series. Alternatively, Galí and Gertler (1999) and others argued in favor of proxying real marginal costs by average real unit labor costs. Under the special case of a (unitary substitution elasticity) Cobb–Douglas production function, real marginal costs reduce to the labor share; this has tended to be a common (if not the default) choice in the literature. If the elasticity of substitution between capital and labor is not unitary, however, such a proxy can lead to biased estimates.

In the following application, we estimate both NKPC forms (specifications (16) and (17)) over 1960q1-2012q4; the start of the sample is chosen for comparisons with the Galí-Gertler study. Note that the driving variable, i.e., the  $\lambda(\cdot)$  term, whether it contains the output gap or the labor share, should, as befits a (price) gap term, be stationary. Stationarity in this context is simply another way of saying that there is co-integration between the optimal and actual price:  $p_t^* - p_t$ . In the case of a typical non-structural output gap measure that stationarity is assured. As we know, this is less clear for the labor share measures. For instance, revisiting Figure 3, we see (from the 1960s onwards) that **SE-GDP** and **PI<sub>2</sub>-GDP** have exhibited a clear downward trend. The other three series are only borderline stationary in this period. This has a bearing on the success of the resulting estimates.

Outwardly, though, the NKPC estimations work relatively well across labor share types: parameters are correctly signed and tend to be significant (**Table 13**). For example,  $\hat{\beta}$  tends to be around the benchmark region of unity. However, estimates of the duration of price fixedness vary from 8.5 – 13.8 quarters. Although these durations are high (compared, say, to micro price-setting evidence) they are by no means untypical in the literature (see the excellent survey by Mavroudis, Plagborg-Møller, and Stock (2014)).<sup>24</sup>

The slope parameters are of more interest here. To repeat, even though the driving variable should be stationary, at best our labor share series are borderline stationary. Accordingly, the minimization in the estimation algorithm places unusually low weights on the driving variable ( $\lambda \in [0.005, 0.016]$ ). As predicted earlier, the **PI<sub>2</sub>-GDP** and **SE-GDP** variants fare particularly poorly in that regard: the former never supports a statistically significant slope parameter, the latter supports a significant but quantitatively small one. Moreover, both of these specifications produce the most unreasonable price setting durations. The **naiveGDP** and **PI-GDP** variants,

<sup>24</sup>For example, Galí, Gertler, and Lopez-Salido (2001), Gagnon and Khan (2005) and Smets and Wouters (2003) for the euro area.

Table 13: New Keynesian Phillips Curve Estimates

	Naive-GDP	PI-GDP	PI <sub>2</sub> -GDP	SE-GDP	BLS
SPECIFICATION (16)					
$\theta$	0.891***	0.907***	0.928***	0.915***	0.909***
$\beta$	0.980***	1.004***	1.009***	1.009***	1.008***
$\lambda$	0.015***	0.009**	0.005	0.007**	0.008***
$\mathcal{D}$	9.2	10.7	13.8	11.8	11.0
SPECIFICATION (17)					
$\omega$	0.104**	0.086**	0.089	0.065	0.035
$\theta$	0.883***	0.901***	0.924***	0.912***	0.910***
$\beta$	0.961***	0.996***	1.002***	1.002***	1.004***
$\gamma_b$	0.106**	0.087**	0.088	0.066	0.037
$\gamma_f$	0.863***	0.909***	0.914***	0.936***	0.967***
$\lambda$	0.016***	0.009**	0.005	0.007**	0.008***
$\mathcal{D}$	8.5	10.1	13.1	11.4	11.1

**Note:** The covariance matrix was estimated with a 12 lags Newey-West estimator. The list of instruments is the same as in Galí and Gertler (1999): four lags of inflation, the labor share, the output gap, the long-short interest rate spread, wage and commodity price inflation.

by contrast, have the lowest durations, significant slopes and significant parameters across both NKPC forms.

NKPCs are not, naturally, a fool-proof way of gauging inflation movements; there are other modelling approaches. That is not the main issue, though: our main point was that the NKPC literature gave a central explanatory role to the labor share of income. However, arguably this is not what most NKPC paper discuss. Much of the literature has instead become concerned with estimation and identification of dynamics (how much forward and backward-looking price setting there is), which are the best instruments to use, etc. The question of whether results are sensitive to which labor share measure we use has received little attention. In our case, though, we have highlighted that we can tie the success of NKPC estimation to the relative properties of the available labor share variants.

## 8 CONCLUSION

We provided a systematic exploration of the dynamic properties of a range of alternative US labor share measures. We documented that these measures are not only divergent in terms of the implied time trends, which are visible to a naked eye, but also differ in terms of their other dynamic properties, such as the shares of short-, medium- and long-run variation in total volatility of the series, degree of persistence, mean-reversion properties, and evidence for structural breaks.

Our results point to the general conclusion that while short-run properties of the labor shares (represented by cyclical variation below 8 years) are relatively consistent across all definitions, their medium-run swings (8-50 years) and long-run trends ( $\geq 50$  years) substantially diverge.

While we generally recommend caution when designing the empirical labor share measure suited to the given application at hand, we argue that the US series on the share of employees' compensation in GDP, adjusted for proprietors' income (which we call **PI<sub>2</sub>-GDP**) has intuitive, economically interpretable properties, covers the entire economy, and thus might be perceived as the "headline" measure of the US labor share since 1929. This measure, compared to its alternatives, turns out to provide the relatively most consistent message across a range of diverse exercises and applications discussed in this paper while providing implications which remain in accordance with known "stylized facts" formulated in the earlier literature.

This measure suggests that the US labor share has not only declined after 1970, but also *substantially increased* before that, exhibiting a hump-shaped pattern over the last 84 years. It corroborates the idea that instead of concentrating the decline in the labor share since 1970, one could



also embrace the larger time span of available data and discuss the (possibly technology-driven) long cycle in this variable, see (Growiec, McAdam, and Mućk, 2015).

We have also presented three interesting applications of our results. We have documented that while the measurement of factor shares has a modest impact on growth accounting, it can be a central issue for some more sensitive estimations, such as identifying the short-run impact of technology shocks on the labor share, and for gauging inflation properties.

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# APPENDICES

## A DATA DESCRIPTION

**Table A.1: Detailed Description of Data Construction**

Abbreviation	Description	Equation	Frequency
<b>Naive-GDP</b>	Naive method, where $CE_t$ : Compensation of Employees [Table 1.12 NIPA] and $Y_t$ : GDP [Table 1.1.5 NIPA]	5	A&Q
<b>PI-GDP</b>	Adjustment by proprietor's income, where $CE_t$ : Compensation of Employees [Table 1.12 NIPA] and $Y_t$ : GDP [Table 1.1.5 NIPA] and $PI_t$ : Proprietors' income with IVA and CCAdj [Table 1.12 NIPA].	7	A&Q
<b>PI<sub>2</sub>-GDP</b>	Extended adjustment by proprietor's income (see Gomme and Rupert (2007)). Most of the time series were taken from [Table 1.12 NIPA], apart from the GDP [Table 1.1.5 NIPA] and Consumption of fixed capital [Table 1.7.5 NIPA]	8	A&Q
<b>SE-GDP</b>	adjustment by self-employed, where $Y_t$ is GDP [Table 1.1.5 NIPA], $SE_t$ : self-employment in private economy [Table 6.7 NIPA] and $TE_t$ is the sum of self-employment excluded and Full-Time Equivalent Employee [Table 6.5 NIPA]. In order to construct the quarterly labor share we use the date from BLS: Total Employment (sum of private [BLS CES0500000001 Series] and government [BLS CES9000000001 Series]) and Self-employment (sum of non-agriculture [BLS LNS12032192 Series] and agriculture self-employment [BLS LNS12032185 Series]).	6	A&Q
<b>Naive-GVA</b>	Naive method calculated for private sector, where $CE_t$ : Compensation of Employees in private sector [Table 1.12 NIPA] and $Y_t$ : GVA in private sector [Table 1.3.5 NIPA]	5	A
<b>Naive-GVA-NF</b>	Naive method calculated for non-farm private sector, where $CE_t$ : Compensation of Employees in private sector [Table 1.12 NIPA] reduced by the CE for farms [Table 1.12 NIPA] and $Y_t$ : GVA in private sector deduced by farms [Table 1.3.5 NIPA]	5	A
<b>SE-GVA-NF</b>	Adjustment by self-employed, where $Y_t$ is GVA for private economy [Table 1.3.5 NIPA] reduced by GVA in farm sector [Table 1.3.5 NIPA], $SE_t$ : self-employment in private economy reduced by farms [Table 6.7 NIPA] and $TE_t$ is the sum of self-employment excluded by farms sector and Full-Time Equivalent Employee in private sector [Table 6.5 NIPA]	6	A
<b>SE-GVA</b>	Adjustment by self-employed, where $Y_t$ is GVA for private economy [Table 1.3.5 NIPA], $SE_t$ : self-employment in private economy [Table 6.7 NIPA] and $TE_t$ is the sum of self-employment and Full-Time Equivalent Employee in private sector [Table 6.5 NIPA]	6	A
<b>BLS</b>	Labor Share in non-farm business sector [PRS85006173], 2005=100	-	Q

**Note:** All the variables except self-employed data are expressed in current USD. "A" = Annual, "Q" = quarterly frequencies.

## B INDICATIVE SECTORAL ANALYSIS

One of the hypothesized explanations for the labor share decline since 1970s pertains to changes in the sectoral structure of the US economy. As argued, e.g., by Elsby, Hobijn, and Sahin (2013), sectors are subject to various degrees of cross-border integration, and recent decades have witnessed an enormous surge of globalization and offshoring. And when labor-intensive production moves to countries with lower labor costs, one could expect the aggregate labor share to go down. On the other hand, the simultaneous rise of the service sector, and financial services in particular, could have worked in the opposite direction.

However, as presented in Table B.1 and Figure B.1 based on World KLEMS data, factor shares have been far from constant at the sectoral level as well. For example, in Mining and quarrying [C] as well as various branches of manufacturing, the labor share has been systematically falling throughout the period 1947–2010, whereas in numerous other branches, and especially non-market service sectors such as Public administration, defence and compulsory social security [L], Education [M], and Health and social work [N], it has been systematically rising.

Hence, results of shift-share analyses – i.e., contributions of respective sectors to the total change in the aggregate labor share – are going to be driven both by the within- and between-sector component. Figure B.2 illustrates this point. The shares of labor remuneration in manufacturing, as well as agriculture and mining, in total labor remuneration have been systematically falling throughout the entire period 1947–2010, driven both by declining labor shares in these sectors and their declining share of total value added. Market services provide a mirror image of this result. However, the financial sector, whose rise was hypothesized to be one of the drivers of labor share declines, provides a particularly interesting result here. In fact, its share in total labor remuneration has increased in the recent years, as the increase in its share of value added has outrun the labor share decline in this sector.

**Table B.1: Share of US Sectors in Gross Value Added, Labor Share in Sectoral GVA, and Unit Root Tests**

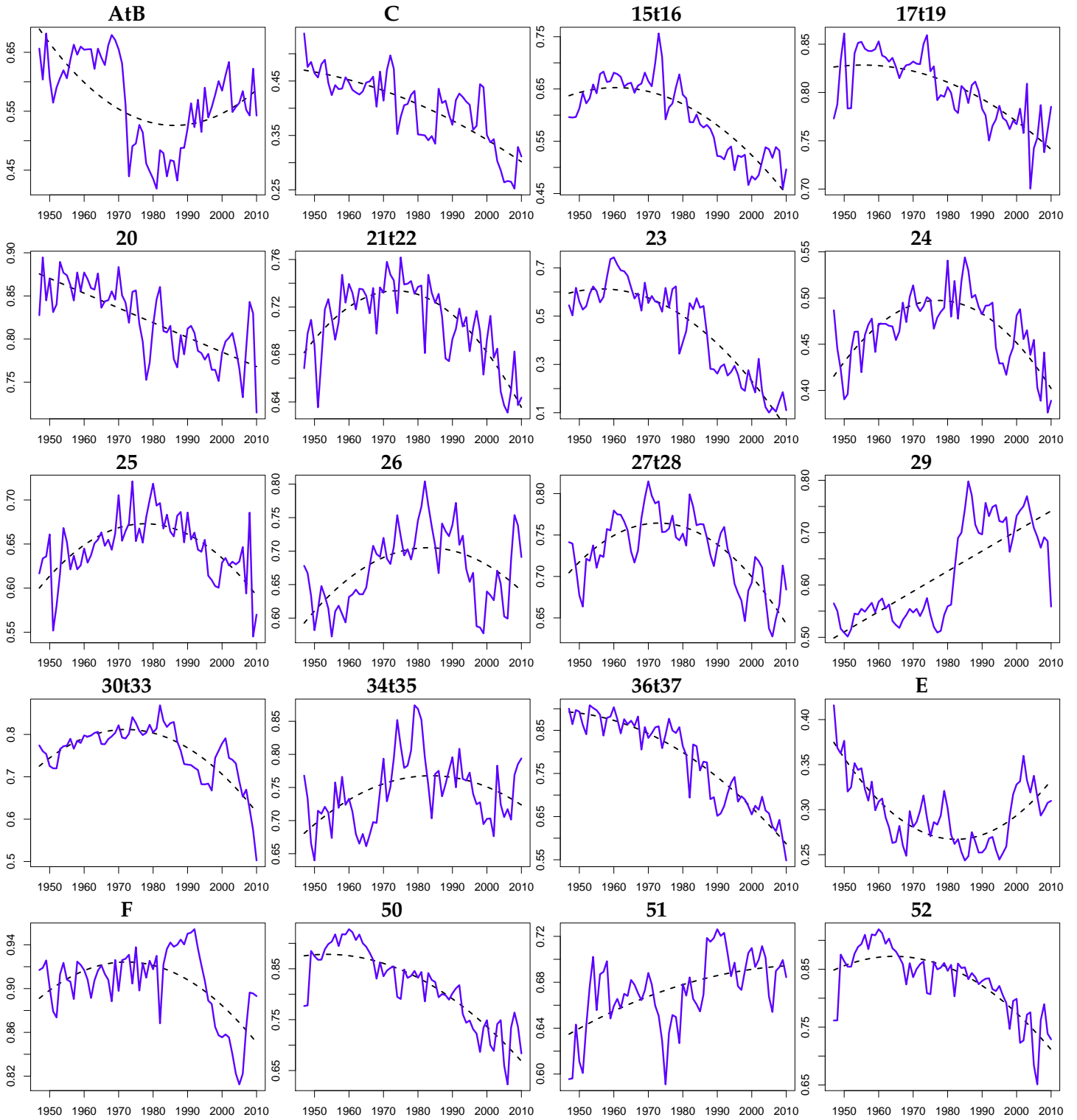
	share in value added ( $w_{i,t}$ )		labor share ( $l_{s_{i,t}}$ )		ADF test		PP test	
	$\bar{w}_i$	$w_{i,2010} - w_{i,1947}$	$\bar{l}_{s_i}$	$l_{s_{i,2010}} - l_{s_{i,1947}}$	const	trend	const	trend
AtB	3.24	-8.54	0.57	-0.11	-1.70	-1.72	-2.22	-2.27
15t16	2.16	-2.21	0.60	-0.10	-1.19	-3.25*	-1.28	-3.16*
17t19	1.43	-3.32	0.80	0.01	-2.25	-5.26***	-2.98**	-5.79***
20	0.83	-0.86	0.82	-0.11	-2.46	-5.32***	-2.53	-5.56***
21t22	2.02	-1.17	0.71	-0.02	-2.09	-2.85	-2.99**	-3.81**
23	0.55	0.51	0.45	-0.43	-0.60	-3.29*	-0.79	-3.26*
24	1.99	-0.08	0.47	-0.10	-1.76	-1.77	-2.58	-2.57
25	0.70	-0.25	0.65	-0.05	-2.14	-2.11	-4.01***	-3.99*
26	0.68	-0.61	0.67	0.01	-2.67*	-2.77	-2.60*	-2.72
27t28	3.06	-2.98	0.73	-0.06	-2.31	-2.76	-2.19	-2.58
29	1.83	-1.05	0.62	-0.01	-1.56	-1.30	-1.39	-1.26
30t33	2.32	0.27	0.76	-0.27	0.73	-0.15	0.98	0.07
34t35	2.58	-1.24	0.74	0.03	-2.67*	-2.83	-3.13**	-3.33*
36t37	0.56	0.00	0.78	-0.35	-0.01	-3.16*	-0.41	-3.87**
50	1.19	-0.29	0.81	-0.09	-1.28	-5.72***	-1.41	-4.80***
51	5.37	-1.11	0.67	0.09	-3.47***	-3.70**	-3.46***	-3.77**
52	5.05	-2.61	0.83	-0.03	-2.01	-5.56***	-2.21	-4.75***
60t63	4.01	-3.37	0.70	0.09	-2.43	-2.10	-2.60*	-2.77
64	2.82	0.93	0.59	-0.34	-1.44	-2.72	-2.02	-3.33*
70	10.75	3.00	0.05	0.01	-1.16	-1.78	-0.91	-1.12
71t74	7.94	11.56	0.63	0.33	-4.37***	-3.70*	-3.81***	-3.24*
C	1.96	-0.95	0.40	-0.23	-2.02	-3.61**	-2.63*	-4.24***
E	2.56	0.22	0.30	-0.11	-2.72*	-2.44	-3.58***	-3.19*
F	4.76	-0.36	0.90	-0.02	-2.00	-2.14	-2.39	-2.59*
H	2.29	0.11	0.80	-0.03	-1.41	-3.29*	-1.71	-3.18*
J	5.13	6.52	0.62	-0.13	-0.85	-2.16	-0.70	-1.80
L	5.42	-3.83	0.64	0.58	-3.14**	-3.55**	-4.81***	-5.96***
M	4.24	3.88	0.66	0.59	-2.99**	-3.08	-4.72***	-5.59***
N	8.53	8.27	0.68	0.60	-2.65*	-2.74	-4.33***	-6.24***
O	3.69	0.36	0.73	-0.03	-2.47	-2.41	-3.06**	-3.00
P	0.31	-0.79	1.00	0.00	-7.06***	-6.98***	-7.77***	-7.70***
TOT	100.00		0.61	0.04	-3.03***	-2.33	-4.29***	-3.74**

**Note:** ADF and PP stand for Augmented Dickey-Fuller and Phillips-Perron test, respectively. Superscripts \*\*\*, \*\* and \* denote the rejection of null about unit root at 1%, 5% and 10% significance level, respectively.

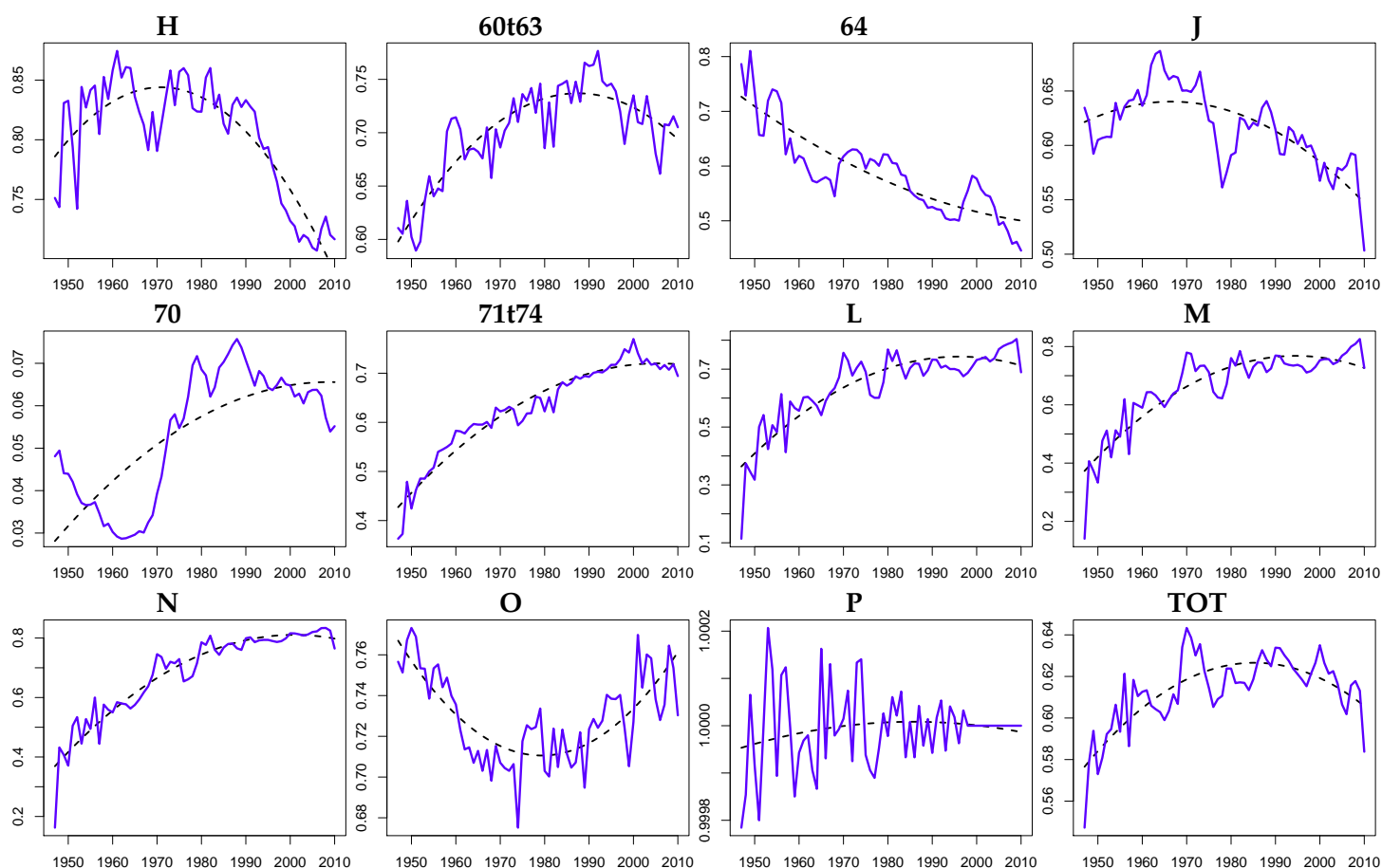
**Sectors:** Total economy [TOT]; Agriculture, hunting, forestry and fishing [AtB]; Mining and quarrying [C]; Manufacture of food products, beverages and tobacco products [15t16]; Manufacture of textiles, wearing apparel, dressing and dyeing of fur, luggage, handbags, saddlery, harness and footwear and tanning and dressing of leather [17t19]; Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials [20]; Manufacture of paper and paper products, publishing, printing and reproduction of recorded media [21t22]; Manufacture of coke, refined petroleum products and nuclear fuel [23]; Manufacture of chemicals and chemical products [24]; Manufacture of rubber and plastics products [25]; Manufacture of other non-metallic mineral products [26]; Manufacture of basic metals, fabricated metal products, except machinery and equipment [27t28]; Manufacture of machinery and equipment n.e.c. [29]; Manufacture of office, accounting, computing machinery, electrical machinery, apparatus, radio, television, communication equipment, medical, precision and optical instruments, watches and clocks [30t33]; Manufacture of motor vehicles, trailers, semi-trailers and other transport equipment [34t35]; Manufacture of furniture, manufacturing n.e.c. and recycling [35t37]; Electricity, gas and water supply [E]; Construction [F]; Sale, maintenance and repair of motor vehicles and motorcycles; retail sale of automotive fuel [50]; Wholesale trade and commission trade, except of motor vehicles and motorcycles [51]; Retail trade, except of motor vehicles and motorcycles; repair of personal and household goods [52]; Hotels and restaurants [H]; Land transport, transport via pipelines, water transport, air transport, supporting, auxiliary transport activities and activities of travel agencies [60t63]; Post and telecommunications [64]; Financial intermediation [J]; Real estate activities [70]; Renting of machinery and equipment without operator and of personal and household goods, computer and related activities, research and development and other business activities [71t74]; Public administration, defence and compulsory social security [L]; Education [M]; Health and social work [N]; Other community, social and personal service activities [O]; Private households with employed persons [P].



Figure B.1: Payroll share in the US sectors



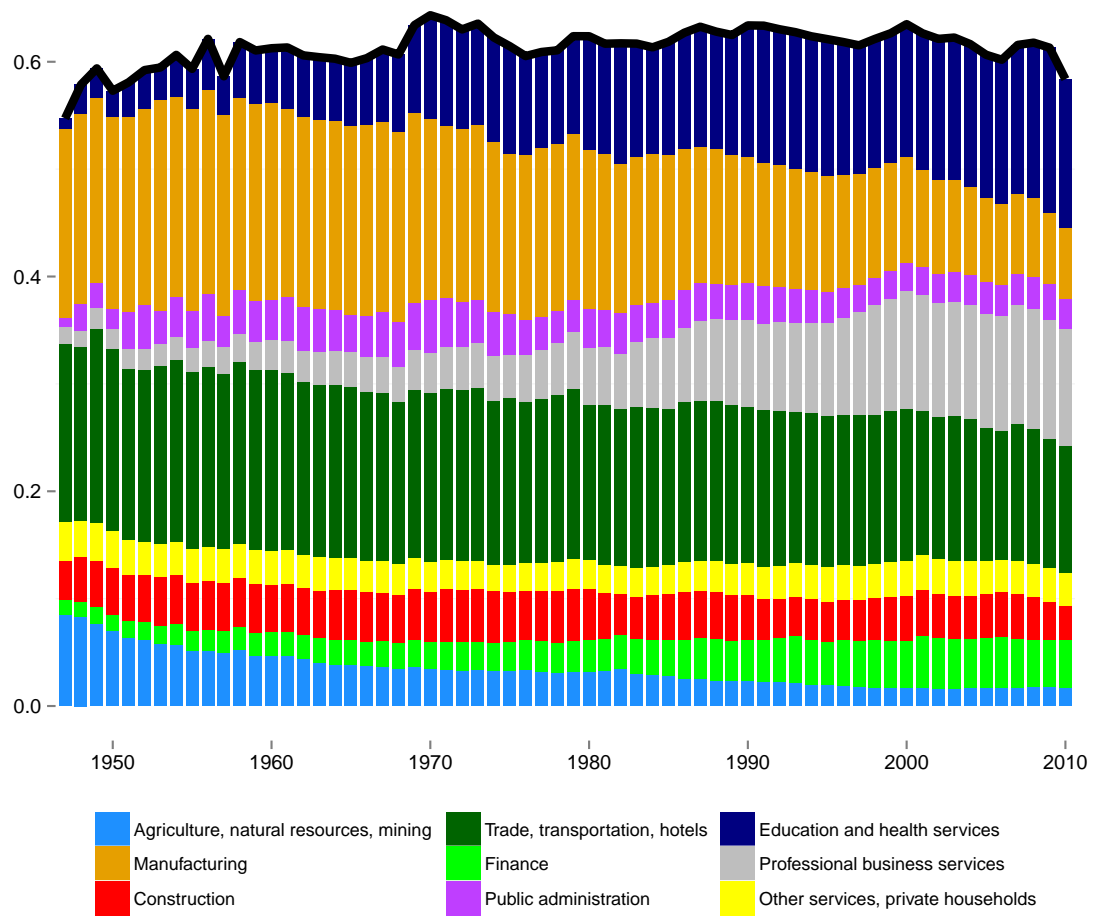




**Note:** solid and dashed lines stand for the payroll share in given sector and its long-run tendency, respectively.

**Sectors:** Total economy [TOT]; Agriculture, hunting, forestry and fishing [AtB]; Mining and quarrying [C]; Manufacture of food products, beverages and tobacco products [15t16]; Manufacture of textiles, wearing apparel, dressing and dyeing of fur, luggage, handbags, saddlery, harness and footwear and tanning and dressing of leather [17t19]; Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials [20]; Manufacture of paper and paper products, publishing, printing and reproduction of recorded media [21t22]; Manufacture of coke, refined petroleum products and nuclear fuel [23]; Manufacture of chemicals and chemical products [24]; Manufacture of rubber and plastics products [25]; Manufacture of other non-metallic mineral products [26]; Manufacture of basic metals, fabricated metal products, except machinery and equipment [27t28]; Manufacture of machinery and equipment n.e.c. [29]; Manufacture of office, accounting, computing machinery, electrical machinery, apparatus, radio, television, communication equipment, medical, precision and optical instruments, watches and clocks [30t33]; Manufacture of motor vehicles, trailers, semi-trailers and other transport equipment [34t35]; Manufacture of furniture, manufacturing n.e.c. and recycling [35t37]; Electricity, gas and water supply [E]; Construction [F]; Sale, maintenance and repair of motor vehicles and motorcycles; retail sale of automotive fuel [50]; Wholesale trade and commission trade, except of motor vehicles and motorcycles [51]; Retail trade, except of motor vehicles and motorcycles; repair of personal and household goods [52]; Hotels and restaurants [H]; Land transport, transport via pipelines, water transport, air transport, supporting, auxiliary transport activities and activities of travel agencies [60t63]; Post and telecommunications [64]; Financial intermediation [J]; Real estate activities [70]; Renting of machinery and equipment without operator and of personal and household goods, computer and related activities, research and development and other business activities [71t74]; Public administration, defence and compulsory social security [L]; Education [M]; Health and social work [N]; Other community, social and personal service activities [O]; Private households with employed persons [P].

Figure B.2: Sectoral Decomposition of the Annual US Labor Share



## C UNIT ROOT TESTS AND STRUCTURAL BREAKS

Table C.1: Unit Roots Tests: Annual

	Naive-GDP	PI-GDP	PI <sub>2</sub> -GDP	SE-GDP	Naive-GVA	Naive-GVA-NF	SE-GVA-NF	SE-GVA
	<u>ADF</u>							
(1)	-2.10	-2.77**	-3.83***	-2.25	-1.45	-2.11	-3.73***	-3.72***
(2)	-1.69	-2.53	-4.36***	-4.35***	-2.37	-3.01	-3.93**	-4.19***
	<u>PP</u>							
(1)	-2.82*	-3.13**	-3.42**	-1.68	-1.83	-2.35	-3.55***	-2.65*
(2)	-2.08	-2.80	-3.79**	-4.39***	-2.26	-3.03	-3.56**	-3.39*
	<u>ADF-GLS</u>							
(1)	-0.58	-1.18	-1.23	-1.02	-0.56	-1.52	-3.31***	-2.37**
(2)	-1.07	-1.69	-1.61	-2.83*	-2.09	-3.32**	-3.86***	-3.56**
	<u>H<sub>0</sub> : ESTAR</u>							
(1)	-2.687*	-2.874*	-4.007***	-2.798*	-1.896	-2.695*	-3.576***	-3.423**
(2)	-2.013	-2.984	-4.256***	-3.634**	-3.307*	-3.426**	-3.329*	-3.804**
	<u>H<sub>0</sub> : asymmetric ESTAR</u>							
(1)	4.450*	5.248**	8.038***	4.275*	2.334	4.634*	6.439**	6.435**
(2)	2.516	4.476	9.190***	10.379***	6.232*	6.928**	6.032*	7.361**
	<u>Fractional</u>							
$\hat{d}$	0.791	0.639	0.421	0.577	0.884	0.696	0.836	0.649
$\hat{d} = 0$	[0.072]	[0.015]	[0.560]	[0.114]	[0.006]	[0.003]	[0.047]	[0.011]
$\hat{d} = 1$	[0.328]	[0.091]	[0.007]	[0.048]	[0.588]	[0.154]	[0.448]	[0.101]

**Note:** \*\*\*, \*\* and \* denote rejection of the null hypothesis of a unit root for all tests at the 1%, 5% and 10% significance level, respectively. Models (1) and (2) incorporate only a constant and a constant and deterministic trend, respectively. The ADF ESTAR and asymmetric ADF ESTAR follow (Kapetanios, Shin, and Snell, 2003) and (Sollis, 2009), respectively.

Table C.2: Unit Roots Tests: Quarterly

	Naive-GDP	PI-GDP	PI <sub>2</sub> -GDP	SE-GDP	BLS
	<u>ADF</u>				
(1)	-2.10	-2.46	-1.32	-0.79	-1.36
(2)	-1.69	-2.78	-2.12	-3.82**	-2.74
	<u>PP</u>				
(1)	-2.16	-2.98**	-1.73	1.01	-1.83
(2)	-1.77	-3.21*	-2.38	-4.19***	-3.27*
	<u>ADF-GLS</u>				
(1)	-0.91	-1.44	-0.51	0.21	-0.11
(2)	-1.14	-3.14**	-2.24	-3.51***	-3.31**
	<u>H<sub>1</sub> : ESTAR</u>				
(1)	-2.237	-3.161**	-1.100	-0.952	-1.038
(2)	-2.532	-3.876**	-2.130	-4.681***	-3.535**
	<u>H<sub>1</sub> : asymmetric ESTAR</u>				
(1)	3.981	4.997**	0.854	1.051	1.350
(2)	3.227	8.058**	3.300	10.951***	7.804**
	<u>Fractional</u>				
$\hat{d}$	1.079	0.866	0.858	0.784	0.854
$\hat{d} = 0$	[0.000]	[0.001]	[0.000]	[0.007]	[0.001]
$\hat{d} = 1$	[0.620]	[0.406]	[0.378]	[0.180]	[0.364]

**Table C.3: Number of Breaks with Corresponding Breakpoints – Annual Series**

	Naive-GDP	PI-GDP	PI <sub>2</sub> -GDP	SE-GDP	Naive-GVA	Naive-GVA-NF	SE-GVA-NF	SE-GVA
WITH MEAN								
<b>1940-1950s</b>	1942 (1940, 1944)	1941 (1940, 1942)	1942 (1938, 1955)	1940 (1939, 1945)		1945 (1943, 1949)	1945 (1942, 1987)	1940 (1939, 1946)
	1955 (1954, 1959)				1952 (1951, 1954)			
<b>1960-1970s</b>	1967 (1966, 1968)	1967 (1964, 1976)	1967 (1963, 1974)		1967 (1965, 1968)	1967 (1964, 1970)		
<b>1980-1990s</b>	1982 (1980, 1984)	1980 (1977, 1981)	1980 (1978, 1981)	1992 (1990, 1993)				
<b>2000s</b>	2000 (1987, 2004)							
WITH LINEAR TREND								
<b>1940-1950s</b>		1942 (1939, 1943)	1946 (1945, 1956)	1940 (1938, 1950)	1942 (1941, 1945)	1941 (1940, 1946)	1945 (1944, 1951)	1945 (1944, 1952)
		1955 (1954, 1968)						
<b>1960-1970s</b>	1968 (1967, 1969)	1968 (1967, 1969)	1968 (1967, 1969)	1968 (1962, 1969)	1974 (1973, 1975)	1968 (1966, 1969)		
<b>1980-1990s</b>		1982 (1977, 1983)	1985 (1984, 1987)					
	1999 (1997, 2000)	1999 (1998, 2000)	1999 (1996, 2000)		1999 (1996, 2000)	1997 (1994, 1998)	1999 (1997, 2000)	
<b>2000s</b>								

**Note:** The breakpoints are calculated in two steps. In the first step, we estimate all the possible models with a number of structural breaks varying from 1 to 5. In the second step, we choose one with the lowest BIC criterion. The years in parentheses are 95% confidence intervals.

**Table C.4: Number of Breaks with Corresponding Breakpoints – Quarterly Series**

	Naive-GDP	PI-GDP	PI <sub>2</sub> -GDP	SE-GDP	BLS
WITH MEAN					
<b>1940-1950s</b>	1956q3 (1956q2, 1957q3)				
<b>1960-1970s</b>	1967q3 (1967q2, 1967q4)	1968q2 (1967q3, 1968q4)	1967q4 (1967q1, 1968q2)	1962q2 (1961q3, 1962q4)	
<b>1980-1990s</b>	1983q1 (1982q2, 1983q4)	1980q4 (1979q4, 1981q2)	1980q3 (1980q2, 1981q1)	1983q2 (1983q1, 1984q2)	1983q1 (1982q3, 1985q4)
				1993q3 (1992q2, 1993q4)	1992q4 (1989q4, 1993q3)
<b>2000s</b>	2003q2 (2002q3, 2003q4)	2003q2 (2000q2, 2005q4)	2003q2 (2002q1, 2003q4)	2003q2 (2002q3, 2003q4)	2003q2 (2002q3, 2003q3)
WITH LINEAR TREND					
<b>1940-1950s</b>	1958q1 (1957q4, 1959q4)			1957q3 (1957q1, 1958q1)	1956q3 (1955q2, 1957q1)
<b>1960-1970s</b>	1968q2 (1967q3, 1968q3)	1968q2 (1967q3, 1968q3)	1969q1 (1968q3, 1969q2)	1969q1 (1968q4, 1969q3)	1969q1 (1968q4, 1969q3)
				1978q4 (1978q1, 1979q4)	1978q4 (1978q2, 1980q1)
<b>1980-1990s</b>	1983q1 (1982q3, 1985q4)	1983q1 (1981q3, 1983q2)	1986q1 (1985q4, 1986q3)		
	1999q4 (1998q4, 2000q1)	1999q4 (1999q3, 2000q1)	1999q4 (1998q4, 2000q1)	1999q4 (1998q4, 200q1)	1999q4 (1998q4, 200q1)
WITH QUADRATIC TREND					
<b>1940-1950s</b>					
<b>1960-1970s</b>	1960q3 (1960q2, 1960q4)	1960q3 (1960q2, 1960q4)	1960q1 (1959q4, 1960q2)	1960q3 (1960q2, 1960q4)	1960q3 (1960q2, 1960q4)
	1970q2 (197q1, 1970q3)	1970q2 (197q1, 1970q3)	1969q4 (1969q3, 1970q1)	1970q2 (197q1, 1970q3)	1970q2 (197q1, 1970q3)
<b>1980-1990s</b>	1983q1 (1982q4, 1983q2)	1986q3 (1986q2, 1986q4)	1981q2 (1980q4, 1981q4)	1983q1 (1982q4, 1983q2)	1983q1 (1982q4, 1983q2)
	1997q4 (1997q3, 1998q1)	1999q4 (1999q3, 200q1)	1992q1 (1991q4, 1992q2)	1997q4 (1997q3, 1998q1)	1997q4 (1997q3, 1998q1)
<b>2000s</b>			2001q4 (2001q3, 2002q1)		

**Note:** The breakpoints are calculated in two steps. In the first step, we estimate all the possible models with a number of structural breaks varying from 1 to 5. In the second step, we choose one with the lowest BIC criterion. The years in parentheses are confidence intervals at 95% significance level.

**Table C.5: Zivot and Andrews (1992) Test for a Unit Root Subject to a Structural Break – Annual Series**

	intercept		trend		intercept and trend	
	$\tau$	$\mathcal{B}$	$\tau$	$\mathcal{B}$	$\tau$	$\mathcal{B}$
Naive-GDP	-3.31	1952	-4.44**	1975	-5.12**	1967
PI-GDP	-4.59*	1942	-3.62	1944	-4.48	1942
PI <sub>2</sub> -GDP	-4.44	2004	-4.35*	2002	-4.63	1967
SE-GDP	-6.33***	1935	-6.30***	1941	-6.09***	1944
Naive-GVA	-3.87	1952	-3.36	1971	-3.86	1952
Naive-GVA-NF	-3.98	1952	-3.93	1974	-4.61	1942
SE-GVA-NF	-5.08***	1934	-4.73**	1936	-6.50***	1946
SE-GVA	-7.27***	1934	-6.73***	1941	-6.85***	1945

Note:  $\tau$  and  $\mathcal{B}$  denote the test statistic in Zivot-Andrews procedure and its breakpoint, respectively. Asterisks \*\*\*, \*\* and \* denote rejection of the null about unit root at 1%, 5% and 10% significance level, respectively.

**Table C.6: Zivot and Andrews (1992) Test for a Unit Root Subject to a Structural Break – Quarterly Series**

	intercept		trend		intercept and trend	
	$\tau$	$\mathcal{B}$	$\tau$	$\mathcal{B}$	$\tau$	$\mathcal{B}$
Naive-GDP	-3.74	1966q2	-4.58**	1972q1	-5.03*	1968q3
PI-GDP	-4.32	1966q2	-4.30*	1970q1	-4.53	1974q4
PI <sub>2</sub> -GDP	-3.74	1966q2	-3.81	1970q2	-4.13	1966q2
SE-GDP	-5.28***	1967q3	-5.26***	2000q2	-5.47**	1999q1
BLS	-4.79**	2009q1	-5.05***	2000q4	-5.11**	2000q1

Note: As in table C.5.

## D STATIONARITY OF LOG-DIFFERENCES BETWEEN ALTERNATE LABOR SHARE VARIANTS

One of the indications that two time series diverge in terms of their dynamics is that their difference is non-stationary. We have checked if this possibility appears among various labor share measures by running a series of unit root tests. Formally, we define the following relationship between two different empirical measures of the (log) labor share:

$$\log LS_{i,t} = \log LS_{j,t} + \eta_{i,j,t} \quad (\text{A.1})$$

For example, if  $LS_{i,t}$  is calculated with the *naive* method then  $\eta_{i,j,t}$  stands for a multiplicative adjustment component, augmenting  $CE_t/Y_t$  in the calculation of  $LS_{j,t}$  (according to some other method). Naturally, when  $\eta_{i,j,t}$  is stationary then we might expect the ratio (and the difference) between both labor share series to be broadly stable over time. Apart from the hypothesis about stability of  $\eta_{i,j,t}$ , we check also whether that difference might be trend-stationary.

**Tables D.7 and D.8** present the results of augmented Dickey-Fuller (ADF) tests for the aforementioned differences. Given that all variables were supposed to measure the same phenomenon, stationarity of the residual component  $\eta_{i,j,t}$  at the 5% significance level is found surprisingly rarely. The only clear evidence is provided for the stability of the difference between two different versions of the labor share adjusted by proprietors' income (in both annual and quarterly data). There is also somewhat more consistency in the case of annual series, especially the ones obtained with GVA instead of GDP, and if one allows for time trends in the differences.

The key message from the unit root tests for the log-differences between the labor share variants is that most of the proposed adjustments to the naive calculation are not constant over time. Please recall that there are two key tendencies which are responsible for that facts (cf. **Figure 1**): (i) a downward trend in the ratio of the self-employed to employees, and (ii) a systematic decrease in the share of ambiguous income in total output.

**Table D.7: Unit Root Test for Differences Between Labor Share Variants – Annual Series**

	PI-GDP	PI <sub>2</sub> -GDP	SE-GDP	Naive-GVA	Naive-GVA-NF	SE-GVA-NF	SE-GVA
<b>Naive-GDP</b>							
(1)	-1.39	-0.82	-2.06	-2.13	-3.70***	-3.15**	-2.75*
(2)	-1.79	-2.86	-2.35	-4.32***	-3.55**	-3.12	-2.88
<b>PI-GDP</b>							
(1)		-2.77*	-2.56	-1.36	-2.39	-4.14***	-3.63**
(2)		-4.63***	-3.03	-3.60**	-3.96**	-4.14***	-3.65**
<b>PI<sub>2</sub>-GDP</b>							
(1)			-3.58***	-0.79	-1.53	-3.42**	-3.96***
(2)			-3.25*	-4.53***	-5.04***	-4.34***	-3.59**
<b>SE-GDP</b>							
(1)				-0.79	-0.83	-1.12	-2.69*
(2)				-2.01	-2.50	-3.84**	-3.73**
<b>Naive-GVA</b>							
(1)					-1.25	-1.49	-1.72
(2)					-2.02	-1.68	-2.06
<b>Naive-GVA-NF</b>							
(1)						-2.51	-1.78
(2)						-3.58**	-2.21
<b>SE-GVA-NF</b>							
(1)							-1.56
(2)							-2.14

**Note:** (1): ADF with intercept. (2): ADF with intercept and a linear trend.

**Table D.8: Unit Root Test for Differences Between Labor Share Variants – Quarterly Series**

	PI-GDP	PI <sub>2</sub> -GDP	SE-GDP	BLS
<b>Naive-GDP</b>				
(1)	-2.75*	-2.56	-3.40**	-2.13
(2)	-1.64	-2.77	-2.15	-2.46
<b>PI-GDP</b>				
(1)		-3.19**	-0.79	-0.67
(2)		-4.27***	-2.42	-1.67
<b>PI<sub>2</sub>-GDP</b>				
(1)			-2.10	-2.30
(2)			-2.45	-2.79
<b>SE-GDP</b>				
(1)				-2.42
(2)				-2.66

**Notes:** See note to table D.7.



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