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OPTIMAL ASSET STRUCTURE OF A BANK

BANK REACTIONS TO STRESSFUL MARKET CONDITIONS

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Abstract

The aim of the paper is to propose a model of banks' asset portfolios to account for the strategic and optimising behavior of banks under adverse economic conditions. In the proposed modelling framework, banks are assumed to respond in an optimising manner to changes in their economic environment (e.g. interest rate and credit risk shocks, funding disruptions, etc.). The modelling approach is based on the risk-return optimal program in which banks aim at a particular composition of their assets to maximise risk-adjusted returns while taking into account regulatory capital and liquidity constraints. The approach is designed for applications in banks' stress testing context, as an alternative to the typical static balance sheet assumption. The stress testing applications are illustrated for a large sample of European banks.
Keywords: Portfolio optimisation; banking; stress-testing

Non-technical summary

Optimal structure of banks' assets is an interesting problem both from banks' management and regulatory perspective. Banks may want to benchmark their asset structure by a theoretically optimal, model-based one which may be a convenient way to quantify complexity of relationship between different balance sheet categories. From the macro-prudential perspective, it is crucial to predict how banks may respond to some adverse macro-financial scenarios considered by central banks and supervision authorities. The optimisation-based analysis of banks' asset structures is so far not present in the stress testing tools which are one of the most important components of the macro-prudential toolkit. The proposed model allows for analysing theoretically optimal behavior of banks in adverse market conditions, is a novelty in this respect. This modeling approach gives theoretical background for studying macroeconomic developments. One important aspect of banks' responses is the deleveraging risk implied by unfavorable developments in banks' economic environments.

The results of the simulations imply some interesting policy implications. The proposed model can shed light on the general depth of the reduction of the loan supply from microeconomic perspective. By applying the well established optimal portfolio theory, the approach may positively contribute to understanding of the strength of the credit channel of the monetary policy transmission mechanism.

Notably, optimisation of banks' asset structure is a difficult task since it is a multi-factor and multi objective problem. As evidenced in the literature, banks are not only strongly dependent on the macroeconomic environment but their behavior is a function of the regulatory regime (especially via capital and liquidity constraints) and is subject to many form of agency problems. The later is related to different interest of banks' creditors (depositors) and shareholders. Moreover, banks are highly leveraged institutions with part of their debt guaranteed by the states. For stress testing applications it is important that a model of banks' behavior is comprehensive enough to allow for analysing various macro-financial scenarios but not at a cost of losing computational tractability.

The proposed framework is based on a one-period model based on the risk adjusted net income optimisation accounting for the risk constraints imposed by banking regulators. The approach is based on an extension of Markowitz portfolio optimisation program to account for risky funding sources and risk and liquidity constraints. Multi-period setting would allow accounting for some important intertemporal effects present in investment decision-making but it would easily get computationally intractable. Notably, banks predominantly use one-period settings in their asset an liability management systems as well.

The model is taken against the real stress testing scenarios defined by ECB staff for the financial stability assessment of EU banking system, the outcomes of which are presented in June 2012 Financial Stability Review. Theoretically optimal reaction of a sample of the European banks to interest rate shifts, deterioration of credit quality of their loan portfolios, adverse changes in sovereign risk and funding constraints are considered in the paper. The initial balance sheet structures of banks in the sample are taken from banks' financial reports (broken down by some aggregate maturity and product categories). The risk and cost and return parameters of the model are estimated based on publicly available data (i.e. bond yields across various maturities, CDS spreads attached to wholesale funding sources) or taken from ECB databases (in particular interests paid by retail loans and deposits). Various robustness checks are provided concerning some key assumptions in the framework (e.g. risk tolerance, default probability of new loans or capital requirements). Since two year scenarios are considered, the one-period models are applied in a sequence, whereby the resulting asset structure of the first period of the optimisation is used as a starting level in the second period.

The outcomes of the developed model show strong relationship between some particular directions in asset restructuring and measures of banks' income and risk potential. First, the risk and return parameters at the outset of the projection determine the sensitivity of banks responses to the stress testing scenarios. For instance, the lower the expected return from loans or the weighted average cost of capital, the higher the deleveraging on retail loan portfolios. The implication of that is the heterogeneity of theoretically optimal adjustments made by banks to their asset structures. Second, the assumed depth of deterioration of the macro-financial conditions in the scenario is reflected into the magnitude of assets' restructuring (or even reduction). This statement is most evidently supported by the influence that risk parameters exert on securities and retail loan portfolios.

1 Introduction

For decades researchers have been trying to describe how banks decide about their asset and liability structures in order to optimally meet objectives of shareholders and management. There are various motivations for understanding this decision making process. From a bank perspective it is crucial to benchmark its asset and liability structure in an automated, algorithmic process, even though the ultimately applied strategy is an outcome of the board level debate. It can be part of the decision support system. Our perspective is a macroprudential one. We aim to find an appropriate description of banks' optimal policies in order to study the relationship between funding structure on one side and lending activity and securities portfolio restructuring on the other side. Within such a framework, possible reactions of banks to changing market conditions can be modeled. The optimisation of asset structures is incorporated into macro stress testing toolkits employed by central banks and supervisory authorities.

The primary goal of this research is to propose a model of optimal asset structure useful in context of stress testing of banks. It allows for relaxing of the standard static balance sheet assumptions taken in most of the top-down stress tests, thereby reducing the arbitrariness of assumptions about evolution of banks' balance sheet. This, ideally, should help in analyzing phenomena such as delveraging, esp. in the loan portfolios, in response to a general aggravation of credit risk in the economy or to funding strains. Our ambition is to develop a flexible tool that allows for analysing implications of complex and comprehensive macroeconomic stress testing scenarios. They usually involve large number of parameters and the framework has to incorporate at least the most important ones, such as various types of yield curves, credit quality measures (default probabilities and CDS spreads) and term and product structure of funding sources.

A rather complicated picture of the bank decision making process that emerges from the literature review indicates that the model of asset portfolio choice of a bank can easily become intractable.[1] It is particularly acute if the model covers a multiperiod decision horizon, endogenous lending margins or bankruptcy risk. Therefore, a reasonable set of conditions is specified in order to obtain, on one hand, a comprehensive model and, on the other hand a tractable one. In order to analyse banks' behaviors in adverse market conditions and sensitivity of asset portfolio restructuring to various financial parameters, a simplistic model in terms of decision making process is solved but with quite detailed (still aggregated) balance sheet structures of banks. It means:

- Reducing the dynamics to just one period[2];
- Restricting the asset and liability classes as much as possible but allowing for capturing stylised market, credit and funding risk;
- Defining the goal function in a standard linear-quadratic optimisation framework, i.e. the least complicated form of the portfolio problem capturing risk / reward trade-offs in any decision making process about the optimal portfolio choice.

[1] The detailed review of the relevant literature is postponed to the next subsection 2.

[2] As Adam (2008) explains, although multi-period, dynamic asset allocation can in theory capture some intertemporal effects of investment strategies and better addresses the financial risk development, it is still much more practical to use the one period models since "*computation times are too important to solve the optimization programme*".

The proposed model of banks' asset structures introduces some important novel approaches and solutions to the bank stress testing framework. It is a tool to analyse the theoretically optimal responses of banks to changing market conditions. Notably, it accounts for the optimising behavior of banks and for a complexity of asset and liability structures and interdependence among balance sheet items but still offers tractability and flexibility. Most importantly, the approach allows for relaxing some arbitrary, static balance sheet assumptions taken usually in stress-testing exercises and in this way introduces optimisation of asset structure to the top-down stress-testing framework. The sequential algorithm in which the one period models were applied to project the optimal asset structures in a longer horizon gives a convenient way to pass through the model various macroeconomic and financial scenarios. In this way, stress testing tools are improved by diversified, banks-specific, balance sheet based response functions to the external shock.

The modeling framework for banks' asset structures borrows both from the optimal portfolio theory and asset and liability management practice. First, banks' optimal responses are founded on the ground of the risk-return portfolio choice. Second, risk and asset and liability management techniques are introduced to the stress-testing analysis of banking system.

Simulations performed with the optimising tool for a large sample of European banks lead to quite intuitive results. On the one hand, banks react to the assumed market shocks in a predictable way. The direction of balance sheet restructuring can be explained by market conditions in a given country and strength of funding sources; theoretically optimally behaving banks deleverage less (meaning: they dispose less volume of loans) in countries that proved to be more resilient to the recent crises. On the other hand, more funding risk or cost implies higher reduction of loan portfolios if market conditions are assumed to deteriorate further. Nevertheless, the results shed light on the directions and depth rather than the exact amounts in a possible deleveraging process.

The model is taken against a set of real stress testing scenarios defined by the ECB staff in its June 2012 edition of the Financial Stability Review. The model suggests deep reduction of loan portfolio volume for most of the banks in the sample. The magnitude of the theoretically optimal asset restructuring can be explained by some intuitive ratios derived from banks' balance sheets, i.e. weighted average cost of capital (WACC), changes in loan probabilities of default (PD), cost of capital, country CDS spreads (and changes within horizon) or duration of funding.

The rest of the paper is structured as follows: section 2 presents a short literature review. In section 3 a stylised balance sheet structure of a bank is presented assuming that it consists of just a few broad categories both on the asset and liability side. Subsection 3.1 provides a description of the return and cost generated by these particular categories linking them with market parameters. In this way risk associated with these items is defined (variance of return and cost). Subsections 3.2 and 3.3 are devoted to the formulation of the optimal program of a bank. Section 4 proposes some techniques allowing for more robust results. Section 5 provides details of the data sample used to parametrise the model. Section 6 contains results of tests verifying the model performance for a group of 54 European banks assuming some simple and isolated adverse scenarios of loan portfolio quality and funding conditions. An application of some consistent stress testing scenarios designed for June 2012 Financial Stability Report of the European Central Bank is presented in section 7. Finally, section 8 concludes.

2 Literature review

An optimization based approach to a problem of determining banks' asset and liability structures faces serious modeling obstacles attributable to a technical complexity. First, it may be perceived as a multi-objective optimisation problem where profit maximisation plays a central role.[3] A very practical implementation presented by Kruger (2011) shows complexity even in the linear setup. It results from a mixture of many potential goals that has to be addressed and multiple constraints reflecting the limitations imposed by regulators, shareholders and funding markets as well. Second, risk taken by banks is one of the main determinants of banks' decision making process, if not the most important one. It is not only a constraint imposed on the set of the admissible strategies. Banks, being investment companies operate in a risk-reward balancing regime described in a pioneering article by Markowitz (1952). According to the principles of the asset and liability management (ALM) (Adam, 2008), banks try to maximize risk-adjusted return on capital (or on risk-adjusted capital) within market and regulatory constraints. Choosing the right investment path involves a lot of risk management tools and expert judgement.[4] This notwithstanding, mathematical modelling of this process may serve as a benchmark for banks' decision making process. Furthermore, it may improve the right understanding of how impulses to banks' balance sheets could propagate to the wider economy. It can thus be used as a complementary tool in assessing the banking channel of monetary policy (e.g. loan supply shocks related to deleveraging). Notably, it can also be integrated in the "what if?" scenario analyses carried by central banks and market regulators trying to understand the implications of their actions.

An adequate description of banks' decision-making process in a modeling framework is complicated since it is not fully transparent how bank managers respond to the constraints imposed by shareholders, how creditors perceive the true financial standing of the bank and how all these parties trust each other. A more theoretical strand of research in financial economics provides some ideas about what features such a framework should posses. Notably, it depicts some particular features of banking domain which hampers the adequate design of mechanisms in banks' decision making process. One important concern is related to debt and capital mixture of funding. It is extensively described by many authors in corporate finance and can be found under a common label of asymmetric information. The agency problems stemming from this asymmetry can be dealt with a requirement for banks to hold sufficient equity capital. It reduces risk shifting and incentivises banks to keep appropriate volume of uninsured deposits. The optimal level of capital is related to competing forces of disciplining effects of funding via debt issuance and risk-shifting consequences of excessive leverage. Higher level of debt financing induces the uninsured creditors to closely monitor the management actions, i.e. credit quality of loans and securities in their portfolios (Calomiris and Kahn, 1991; Calomiris, 1999). Moreover, equity funding sources are costly due to necessity of loan monitoring and this justifies an increase of leveraging (Gale and Hellwig, 1985). Opposed to that, insufficient level of capital may be an incentive for managers to take excessive risk that in turn may increase the value of the equity option with the underlying bank's assets. This

[3] Kosmidou and Zopounidis (2008) present an overview of stochastic linear programming techniques applied for asset and liability management purposes ranging from deterministic linear programming models (Chambers and Charnes, 1961), stochastic programming (see for instance a very comprehensive one proposed by Kusy and Ziemba (1986)).

[4] It means, market intelligence, internal policies stimulating sustainable business development (e.g. funding transfer pricing system), hedging and accounting practices (Adam, 2008).

observation was made by Jensen and Meckling (1976); Brewer (1989). In the presence of the deposit insurance (and other form of safety nets) the risk-shifting is even more pronounced as banks tend to become more leveraged (see Miles et al. (2011)). All in all, the risk-shifting phenomenon necessitates an introduction of minimal capital requirements. Some authors, e.g. Allen et al. (2006), indicate that banks anyway retain capital clear above regulatory minimum since sound loss absorption capacity may have positive signaling effects for borrowers. The authors argue that higher capital increases the level of monitoring of borrowers and this is good for borrowers since it increases return from their projects financed by banks. Mehran (2011) notice that well capitalised banks have high survival probability which induces better monitoring and improves the credit quality of loan portfolios. These game-theoretical problems seriously complicate the right specification of banks objectives and capital constraints in a tractable mathematical form.

The complexity of balance sheet management of a bank is also related to the fact that it is a multi-criterial problem and it is not obvious how shareholders and managers weigh the importance of various criteria. Should it really be maximisation of the shareholders' value understood as the call option on the value of the assets with the debt volume as the strike of the option like in the classical Merton (1974) article. Perhaps, it should be maximisation of income (in short- or long-run) as proposed eg. by Thakor et al. (2010); Yilmaz (2009) or of risk-adjusted profit (Stoughton and Zechner, 2007) or even risk-constrained profit (Danielsson et al., 2008). Or how should the profit maximisation also account for liquidity risk minimisation; targeting acceptable level of liquidity or minimising the chance of liquidity shortfall? Last but not least, how should the bankruptcy risk be treated? In some models (eg. in the classical Leland and Toft (1996) or in Yilmaz (2009)) bankruptcy occurs if asset value hits a given target level (usually related to the amount of debt) but there are also examples of a different approach where the firm does not default but pays a penalty cost of being in a state of default and in the optimum these costs prevent it from entering the bankruptcy region (see eg. Browne (1995); Halaj (2008)). The definition of the goal function is then a serious problem that has to be overcome.

Optimisation based approach to banks' balance sheet structure can be applied in various theoretical and practical contexts. It can be embedded into an ALM process as a benchmark for managements' decisions (Kusy and Ziemba, 1986; Adam, 2008). Notably, many DSGE models integrate banks' optimisation of income into their structure (Gerali et al., 2010; Pariès et al., 2011). As Borio et al. (2012) notice, *some [stress test] models allow for the possibility that banks adjust their balance sheets in response to the shocks, although so far only through mechanical rules of thumb.* Usually, the response is in a form of a fire-sale of predefined asset classes with an assumed pecking order (Aikman et al., 2009). Drehmann et al. (2008) consider banks as passive investors whose assets evolve according to estimated default probabilities (PD) and losses given default (LGD). Notably, they also model dynamics of banks' liabilities. Alessandri et al. (2009) describe the "rule of thumb" embedded into the Bank of England stress testing model (the so-called RAMSI model) which allows for changes in asset composition when banks make profits. They assume that banks target their leverage ratio and try to maintain the product structure of their assets. Some methodologies do not consider any adjustments to the asset structure and changes in funding sources (e.g. deposit outflow or roll-over risk of the wholesale funding) (Bunn et al.; Čihák, 2007). For bottom-up stress tests static balance sheet assumption makes sense, since it allows for avoiding the risk of banks' manipulation with parameters that may imply overly optimistic outcomes, but less so for top-down approach.

Anyway, the scope of the top-down stress tests is usually limited by the very aggregate data on banks' balance sheets which additionally complicate the adequate modeling of balance sheet dynamics (see IMF (2011) for comparison of bottom- and top-down stress tests for UK).

3 The model

A bank is assumed to operate in one period $\mathbb{T}_0 : = [t_0, t_1]$. Let us assume that $t_0 = 0$ and $t_1 = 1$, and it can be interpreted as a 1Y horizon within which the bank decides about the composition of their assets to maximise shareholders' value. In this way, the model can be perceived as a *static* one. It also *non-stochastic* means that the development of market conditions (market scenario), of the interest rates in particular, is known at t_0. More specifically, each bank knows at t_0 the expected return from the possible investment, the associated risk (i.e. variance) and the dependence structure (i.e. correlation) between all balance sheet categories. These three parameters related to each of the asset categories from the admissible investment set drive the optimal investment behavior. This general setting allows for attaching any macroeconomic scenario to the risk and return parameters and analysing its impact on the theoretically optimal investment strategies of banks.

Since the model is meant to be embedded into a stress testing framework, which is usually multi-period, the one period module will be used sequentially in such a way that the end of period structure is assumed to be the starting structure for the next period with other relevant market parameters following the assumed macroeconomic or financial scenario. The details are outlined in section 7.

3.1 Balance sheet items

The model of asset and liability categories is rather complex even in the most aggregated setting. Still, it accounts for the fundamental differences of how balance sheet items perform and contribute to the overall income and expenses of banks. With this in mind, before delving into detailed description of the models with all the intricacies, just an outline of the approach is presented.

Outline

Each bank is assumed to have a particular asset and liability structure at the beginning of the investment horizon. Within the following period it decides about restructuring of part of the asset composition. Not all the categories are allowed for liquidation - loan portfolio is kept until maturity, so the reduction of the volume can only result from partial reinvestment of the maturing part. The total balance sheet sum of banks follows the exogenously determined sum of liabilities and equity which are beyond banks' discretion. The sum can change if only liabilities are not entirely rolled-over or the capital base shrinks.

Decision of a bank to restructure composition of its assets is based on the optimisation of risk-adjusted income and takes into account several parameters defining performance of asset and liability classes. The focus is on a particular list of aggregate categories (yet comprehensive enough to reflect the some important complexities of banks' decision making process related to asset and liability management). The assumed simplified bank balance sheet comprises of customer loans (L), securities (S) and the interbank placements (I) on the asset side and customer deposits (D) interbank funding (F) and own debt issued (DB) on the liability side. For the sake of clarity of the presentation, no other residual categories like other assets and other liabilities are assumed, so the capital equals the difference $(L+S+I)-(D+F+DB)$.[5]

In order to get a comprehensive description of banks' balance sheets, several parameters of a usual bank's balance sheet are introduced to the model. The details are explained in the reminder of this subsection (subsection 3.1). In general, these parameters determine expected risk and return related to particular balance sheet categories, and also correlation between them. On the asset side, customer loan portfolio is described by the average interest of the outstanding volume (weighted by product and maturity structure), on the new origination and by default probability and average maturing stock within the analysed period of investment. The later two parameters influence the effective rate of return from the loan book. On the risk side, the volatility of the return is determined by the historical volatility of loan interest rates. The second group of assets – securities – is presented in some aggregate maturity buckets, each of which gets its expected interest payment and risk related to historical performance of yield curves. Apart from the interest payments, the mark-to-market securities (non-HTM portfolios) can be revalued if the curves shift which translates into the effective rate of return from the investment. The risk measure of securities relates, on one hand directly to the volatility of yield curves but on the other hand to the duration of the mark-to-market portfolios. Finally, the third group is the interbank lending characterised by the interbank market rates (and volatilities).

The funding side of a bank is broken down by customer deposits, interbank lending, own debt issued and equity. Deposits, interbank funding and own debt are presented in aggregate maturity buckets to which cost rates (and risk parameters) are assigned accordingly. Additionally, in order to capture potential disruptions on the funding market, all the liability categories have their rollover rates allowing for analysis of situations in which only part of the maturing funding can return to banks. The cost of interbank funding and debt issued relates to banks' credit quality perceived by the market. The cost (and its risk) of capital is established in the CAPM framework. The liability side is static in the model meaning that is not subject to the optimisation of it structure. However, its structure can be arbitrarily chosen by an analyst in order to reflect the changing (potentially adverse) market conditions (e.g. funding roll-over risk).

3.1.1 Loans (L)

Volume Loan portfolio is assumed to be a homogeneous portfolio with no term structure[6] The aggregation is consistent with the available average loan portfolio default probabilities. Let us assume additionally that a fraction a of the portfolio matures (or is prepaid) in $\mathbb{T}_0$. Later, the amount aL_0 is added to the potentially reinvested volumes of assets. Implicitly, it is then assumed that the outstanding volume of loans that matures after t_1 cannot be sold or securitised.[7]

[5] Other assets and other liabilities can be easily introduced as non-interest bearing components.

[6] However, in the stress testing applications of the model, the average interest rate paid by the loan portfolio is a weighted average by the product structure. It is easy to extend the setting to account for the term structure meaning that the portfolio of loans comprises of eg. 3M, 1Y, 5Y and 20Y loans priced according to their maturities.

[7] For simplicity any securitisation of the outstanding volume of loans is excluded. These transactions are

The decision of a bank is to grant loans of volume π_0^L out of the reinvested volume of assets. At time 1 the expected volume of the portfolio is equal to

$$L_1 = (1-a)L_0 + \pi_0^L. \tag{1}$$

Income The average nominal interest rate paid by loan stock at t_0 is constant and denoted r^L. Any shock to the interest rate can be introduced via a loan/deposit multiplier gauging the elasticity of the loan rates to the market rates, like EURIBOR 3M.[8] The new loans π_0^L are granted in new market conditions, therefore they are supposed to pay a different rate r^{New}.

The income earned on the loan portfolio is affected by the credit quality. Credit quality of the loan portfolio is described by a parameter ρ, $0 \leq \rho \leq 1$, interpreted as the probability of default of the portfolio within $\mathbb{T}_0$.[9] More specifically, for some Loss Given Default (LGD) parameter, a loan portfolio pays r^L interest rate with probability $1-\rho$ and $(1-\text{LGD})r^L$ with probability ρ, which gives an average rate of $(1-\text{LGD}\rho)r^L$ on the outstanding volume of loans.[10] Usually, the credit quality of new loans is better and in order to account for this fact a separate default probability ρ^{New} is considered for this fraction of the loan portfolio. Consequently, the average rate on the portfolio of new loans is $(1-\text{LGD}\rho^{New})r^{New}$.

The loan portfolio generates income between t_0 and t_1 which is defined by the average interest rate on new and the old loans: $\text{Inc}_1^L = (1-\text{LGD}\rho)r^L\frac{L_0+(1-1)L_0}{2} + (1-\text{LGD}\rho^{New})r^{New}\frac{\pi_0^L}{2}$.

Risk The risk associated with loans is assumed to be solely driven by the volatility of the market interest rate to which the lending rates are indexed and by the default risk ρ, independent of the volatility of the indexing market rate.[11] All in all, it means that the standard deviation of the interests paid by loans is equal to

$$\sigma^L = \sqrt{\left((\sigma^{r_L})^2(1-\text{LGD}\rho) + r_L^2(1-\text{LGD}\rho) - r_L^2(1-\text{LGD}\rho)^2\right)},$$

where σ^{r_L} is the volatility of loan interest rates.

3.1.2 Securities (S)

Volume The portfolio of securities at the starting point t_0 consists of N classes of securities S^n, $n \in \{1,\ldots,N\}$, with different maturities (eg. 3M, 6M, 1Y, 2Y, 5Y, 10Y). Part of the portfolio

costly not only in terms of the expenses incurred for marketing of a selected tranche but in terms of the time needed to complete the securitisation project as well. That is why they are infrequent and, therefore, excluded from the set of banks' admissible investment strategies in the proposed optimisation program.

[8] The method developed for estimation of the coefficients was presented in Box 7 of the December Financial Stability Report of the ECB.

[9] In reality, the credit quality of the loan portfolio is not that homogenous but the assessment hinges on the availability of time series of PD by product categories, i.e. if it is possible to assign average default rates in the mortgage, consumer or investment segment of the loan market.

[10] The loss given default (LGD) is assumed to be equal to 50%. Some sensitivity analysis of this assumption is provided in section 6.4.

[11] This assumption may be far-fetched (a drawback also pointed out by a discussant of the conference version of the paper). If the volatility of the interest rates rises then it may be a reflection of the worsening macroeconomic conditions. In such the circumstances, the credit quality is also likely to deteriorate. However, there is no good proxy for the relationship at hand.

comprises of mark-to-market bonds and their yields refer to the corresponding points on the bond yield curve and changes in valuation directly impact profit and loss accounts (P&L).[12] However, banks usually keep also HTM portfolios which have much less volatile impact on P&L (through constant to maturity effective yield pricing). In order to calculate valuations and risk measures, the securities are assumed to be purchased at par at t_0. Their nominal value at t_0 is equal to the market value and the book value as well.

It is assumed that the whole volume of securities can be reinvested at t_0 and the reinvestment volumes $\pi_0^{S,n}$ retains the structure of the accounting portfolios observed at t_0 in a given maturity bucket. It means that the share of HTM volume in the portfolio of securities is kept constant through the investment horizon. Normally, HTM portfolios are not liquidated long enough before maturity but this assumption mainly simplifies the notation and presentation of the model. We, anyway measure the risk and return of portfolios according to their accounting features.

Income and pricing The impact of securities portfolio on banks' profits is twofold. The first component consist of the coupons paid by the securities. Second, the valuation changes (resulting from the curve shifts) are included. All the securities held by the banks are assumed to be the coupon paying bonds.[13] Each class n pays annual interest rate of r_n^S. Coupons correspond to the particular maturities on the current (t_0) yield curve. The yields are subject only to some parallel shifts.

At time 1, value of the portfolio is described by

$$S_1^n = \pi_0^{n,S} + \pi_0^{n,S} \cdot (YTM_1^{(m^n)} - YTM_0^{(m^n)}) \cdot \text{uBPV}_0^{(m_n)}, \tag{2}$$

where $\text{uBPV}_0^{(m_n)}$ is the sensitivity measure (Basis Point Value measure) of the unite nominal bond value following a parallel movements of the yield curve and typically is negative. Basis point value is calculated as duration of the portfolio times its market value. Notably, we use the convention that $\text{uBPV}_0^{(m_n)}$ for HTM portfolios is equal to 0. This is basically the mechanism that allows for introducing shocks to the curve.

Namely, the yield $\text{YTM}_k^{m_n}$ of the nth bond issued at k and maturing at m_n can be transformed to the equivalent coupon paid annually to maturity. Any shock to the curve would then be reflected in the valuation of this bond being the discounted (along the new curve) flow of the coupons and the nominal. So assuming the value S_0^n of the security (bought at par at t_0) at time 0 with yield to maturity equal to $\text{YTM}_0^{m_n}$ we get coupon c_0,

$$c_0 = \left(S_0^n - S_0^n (d_{0,n}^f)^{m_n}\right) \frac{1 - d_{0,n}^f}{1 - (d_{0,n}^f)^{m_n - 1}},$$

where $d_{k,n}^f = (1 + \text{YTM}_k^{m_n})^{-1}$. Hence, period 1 the value S_1^n is the discounted future coupons c_0 plus the nominal. But the discounting is done by means of a new (potentially different)

[12] This may be an important source of income variability. However, banks always tend to hedge the related interest rate risk (Fabozzi et al., 2003).

[13] Banks hold also zero coupon bonds, bonds with imbedded options, asset backed securities. Our simplifying assumption about the homogenous composition of securities portfolio is particularly reasonable in the context of macro stress testing performed usually on a very aggregate data.

yield $\text{YTM}_1^{m_n-1}$, so the price (per 100) of the bond at 1 is given by:

$$p_1^{S,n} = 100\frac{\sum_{i=1}^{m_n-1} c_0 \cdot (d_{1,n}^f)^i + S_0(d_{1,n}^f)^i}{S_0}. \tag{3}$$

The risk measure $\text{uBPV}_0^{(m_n)}$ is calculated as a variation of $p_0^{S,n}$ to the one basis point shifts of the yield curves. Let us put $d_{k,n}^f(x) :\ = (1 + \text{YTM}_k^{m_n} + x)^{-1}$, $x \neq -(1 + \text{YTM}_k^{m_n})$, for x being a basis point shift of $\text{YTM}_k^{m_n}$. Moreover,

$$p_0^{S,n}(x) :\ = 100\frac{\sum_{i=1}^{m_n-1} c_0 \cdot (d_{0,n}^f(x))^i + S_0(d_{0,n}^f(x))^i}{S_0}.$$

Consequently, we define $\text{uBPV}_0^{(m_n)}$ as the average change of unit volume bond price in response of the upward and downward parallel shift of the yield curve by 1 basis point:

$$\text{uBPV}_0^{(m_n)} = \frac{1}{2}\left(p_0^{S,n}(1) - p_0^{S,n}(0) + p_0^{S,n}(0) - p_0^{S,n}(-1)\right)/100. \tag{4}$$

The averaging accounts for the convexity of bond prices.[14]

Risk The risk of of the securities portfolio is measured by the sum the volatility of the historical yields in the HTM portfolio and standard deviation of market prices of some reference bonds (corresponding to the maturity buckets) in the case of non-HTM portfolios. Implicitly, the riskiness of securities is only related to the volatility of the interest rates without any considerations of the credit risk other than that already reflected into the yield curve changes.

The equation 2 paves the way for calculation of the standard deviations of bond values once the volatility of the yield curves is inserted into it. Therefore, risk of non-HTM portfolios is gauged by the standard deviation of historical YTM of bonds maturing in m_n multiplied by $\text{uBPV}_0^{(m_n)}$.

The risk of the HTM portfolio is measured by the historical volatility of the corresponding yields. The resultant risk of the whole securities portfolio is the weighted risk of mark-to-market and HTM portfolios.

3.1.3 Interbank placements (I)

Volume In general, this category refers to all the wholesale market exposures on the asset side. It is broken down by N_2 maturity buckets, generally much shorter than in securities portfolio (and customer loans portfolio, which is however presented in aggregate terms).

Income They pay interest rate r_n^I, $n \in \{1, \ldots, N_2\}$, indexed to the interbank market interest rates. The bank decides about the accepted volume $\pi_0^{I,n}$ of the interbank placements which

[14] Another option to define $\text{uBPV}_0^{(m_n)}$ and to capture convexity would be to calculate the derivative of $p_0^{S,n}(x)$ with respect to x at $x = 0$.

level is maintained till t_1 (so, $\pi_0^{I,n} = \pi_1^{I,n}$) but the bank is the price taker as far as the applied interest rate is concerned. Therefore, the generated income amounts to $\text{Inc}_1^I = r^I I_1 = r^I \pi_0^I$.

Risk The risk is measured by the observed, historical volatility of the underlying interest rates.

3.1.4 Cash (C)

Reinvestment means collecting maturing loans and interbank placements and selling part of securities. It brings their cash equivalent C_1 which is riskless (eventually can be utilised to pay back part of the debt). It is assumed that at 0 $C_0 = 0$ and cash may appear in the process of balance sheet restructuring until t_1. This "cash" concept is used in case the initial risk and liquidity constraints are not satisfied already at the outset of the horizon (see formula 13).

3.1.5 Deposits (D)

Volume This balance sheet category refers to the retail and corporate deposits but excluding the wholesale market and can be referred to as *customer deposits*. Deposits have term structure: there are K classes of deposits maturing at d_k, $k \in \{1, \ldots, K\}$. The set of maturities is expected to be different than those of loans. Maturities of deposits are usually shorter. This inherently leads to the interest rate risk of maturity mismatch.

Bank are assumed to have no influence on the stock of deposits. Maturing deposits are rolled-over or replenished on the wholesale market. However, their volume may follow an exogenous path potentially reflecting deposit outflow. In that case, $D_1^k < D_0^k$. It is a simple way to relax the assumption of the unchanging balance sheet sum.

Cost Customer deposits pay interest rates r_k^D. Two submodels can be considered for testing the influence of the retail funding conditions on banks' behavior:

1. Deposits are rolled over at the same cost r_k^D (or adjusted by the shift in the market interest rates relevant in terms of the maturity) once they mature or
2. If maturity of a category k of deposits falls into the interval (t_0, t_1) then these deposits are renewed on the wholesale market, possibly at a higher, stressful time cost r_k^F given by the stress-test scenario.[15]

The deposits incur cost of interest paid to depositors, in our framework defined as

$$\text{ad 1.} \qquad \text{Cost}_1^D = \sum_{k=1}^{K} r_k^D \frac{D_0^k + D_1^k}{2}$$

or

$$\text{ad 2.} \qquad \text{Cost}_1^D = \sum_{k=1}^{K} \left[\mathbb{I}_{\{m_k^d < 1\}} (r_k^D m_n^d + r_k^F (1 - m_n^d)) + \mathbb{I}_{\{m_k^d \geq 1\}} r_k^D \right] r_k^D \frac{D_0^k + D_1^k}{2}.^{16}$$

[15] This functionality has not yet been used in the applications of the model.
[16] $\mathbb{I}_B$ is the indicator function of set B

Since $t_1 = 1$, the interests on deposits rolled over on the wholesale market are weighted by the fraction of time (m_n^d).

Risk The risk of customer deposit costs is measured as the standard deviation of the interest rates paid. For a given maturity k it is volatility of r_k^d in case 1 or $\mathbb{I}_{\{m_k^d<1\}}(r_k^D m_n^d + r_k^F(1 - m_n^d)) + \mathbb{I}_{\{m_k^d\geq 1\}} r_k^D$ in case 2.

3.1.6 Interbank funding (F)

Volume This part of funding sources is assumed to have its own maturity structure, usually very short one (ON, 1M, 3M) but longer term funding lines are also common. We assume M maturity buckets.

Having in mind the serious funding market tensions observed in recent years it is worth mentioning that there is some modeling space in the framework to account for the possible dry-up / stall of that funding sources. It may therefore be reasonable to assume that the interbank funding cannot be rolled over. It means that part of funding maturing within $[t_0, t_1]$ cannot be replenished with new amounts. With a slight abuse of the notation it translates to $F_0 = \sum_{k>1\text{Y}} F_0^k + f_{rollover} \sum_{k<1\text{Y}} F_0^k$, and $f_{rollover} < 1$ reflects limited possibilities of rolling over of this type of funding. This would in turn lead to adjusting reactions on the asset side it would trigger.

Cost The interest paid on the interbank funding may increase substantially above the retail market rates should the funding condition on the market deteriorate. The volatility of the interest rates, most likely positively correlated with the interest rates applied to deposits D, is higher than that of client deposits, following volatile liquidity supply and demand on that market. These interbank deposits pay interests according to the interest rates r_k^F and the cost Cost_1^F associated with them is calculated in the analogous way to the customer deposits D.

Risk Similar to the already described categories, risk is computed as the standard deviation of the interest rates. Notably, the roll-over parameters does not influence these calculations.

3.1.7 Own issuance (DB)

Volume Debt instruments issued on the financial market are another source of the external funding, usually in form of the public placement of the corporate bonds, rather unsecured.

Cost The cost of the issued debt depends on the bond yield curves. Apart from the level of the market yield curves, their cost is closely related to the investors' perception of the bank credibility to pay back the debt. The risk is priced into the issued bonds by the increased coupon (or discount) which is typically related to the bank's CDS spread and the maturity of the tranche. Assuming that bank issued debt $\text{DB}_0 = \sum_{m=1}^{M_2} \text{DB}_0^m$ with M_2 maturities and corresponding coupons r_m^{DB}, $k \in \{1, \ldots, M_2\}$, the cost is equal to $\text{Cost}_1^{DB} = \sum_{m=1}^{M_2} r_m^{DB} \text{DB}_0^m$.

Risk The associated risk is the volatility of the historical government debt yield curves adjusted by the term structure of the bank CDS spread or the peer bank's spread if the default swaps are not quoted for a given bank or the quotations are illiquid.

3.1.8 Capital (E)

Volume Capital is the "residual" of assets after paying back the debt, so in general at the outset of the analysed horizon $E_0 = A_0 - D_0$. At time t_1, the capital is assumed to remain unchanged, i.e. $E_1 = E_0$.

Cost The cost of capital is assumed to be related to the excess return and banks' beta estimates derived from CAPM through the formula:

$$r^E = r^{risk-free} + \beta * \text{risk premium},$$

where $r^{risk-free}$ is a reference interest rate swap rate (possibly close to the risk-free interest rate) and β is the sensitivity of the cost of capital to changes in the excess return. Coefficient β is obtained from the estimation of the following regression:

$$(r_t^E - r_t^{risk-free}) = \alpha + \beta * (r_t^{index} - r_t^{risk-free}) + \epsilon_t, \tag{5}$$

where $(r_t^E - r_t^{risk-free})$ is excess return from the investment in shares of a given bank (eg. monthly excess return) and r^{index} is the return on investment in the (sufficiently) broad stock market index. α is usually close to 0.

Several approaches to find the right proxy for the *risk premium* were tested. The applied one is based on the average across the stock market of a given country, long-term price/earnings (P/E) ratios and assuming that the mature US market excess return is known. The approach follows Damodaran (2009) who proposes to calculate the country specific (average) premium relative to the US market premium.[17] Then the excess country premium r^c over the US excess return could be defined as an additional rate of return that equates the average P/Es of the US stock market and a given country assuming long term estimates of earnings on US market and on the given country market. More specifically, for US excess return denoted p^{US} and the risk-free rate r^{US}, it means that

$$\frac{P/E^{US}}{P/E^c} = \frac{\sum_{i=1}^{\infty} \frac{\bar{\delta}_{US}}{(1+r^{US}+p^{US})^i}}{\sum_{i=1}^{\infty} \frac{\bar{\delta}_c}{(1+r^{US}+p^c)^i}}, \tag{6}$$

where $\bar{\delta}_{US}$ and $\bar{\delta}_c$ are the dividend levels in US and country c respectively. Premium p^c should in this way account for differences in the risk level between US and the given country. Let

[17] Damodaran (2009) collects various methods to proxy the premium and analyses their advantages and shortcomings. The extensive survey made by Damodaran (2009) suggests that the proper estimation of the equity premium is cumbersome. The simple solution is preferable since we do not have sufficiently long time series for classical CAPM estimation or forward looking income (dividend) growth estimates. Moreover, the cost of capital is not the main purpose of the paper. See also Brandt and Wang (2003); Lettau et al. (2008)

us suppose that the development of $\bar{\delta}_{US}$ and $\bar{\delta}_c$ follow the long term dividend growth model with growth ratios g^{US} and g^c respectively with some common starting level (which cancels out in the ratio 6). The country specific growth rates were estimated by e.g. Ritter (2005). Therefore, the relationship of price-earnings ratios can be rewritten in a form:

$$\frac{P/E^{US}}{P/E^c} = \frac{\sum_{i=1}^{\infty} \frac{(1+g^{US})^i}{(1+r^{US}+p^{US})^i}}{\sum_{i=1}^{\infty} \frac{(1+g^c)^i}{(1+r^{US}+p^c)^i}},$$

which after elementary transformations leads to the premium estimate:

$$p^c = (1 - \frac{P/E^{US}}{P/E^c})(r^{US} - g^c)\left(\frac{P/E^{US}}{P/E^c}\frac{r^{US} - g^{US}}{1 + r^{US} + p^{US}} - 1\right)^{-1}.$$

There are several other methods to calculate the risk premium. One – classical – way to gauge the premium is to calculate the long term, average excess return of the market portfolio of stocks over the risk free rate, so applying the same scope of data as in the case of β. However, this relationship proves to be meaningful in the deep, liquid stock markets of the economies in which firms finance themselves mainly through the capital markets. Practically, this approach is limited to the US market. The other way – potentially applicable in Europe – could be based on the credit default swap spread approach. Namely,

$$\text{risk premium}_C :\ = \text{risk premium}_{US} + \frac{\sigma_C^{stock\ market}}{\sigma_C^{gvt\ bond\ market}} \text{CDS}_C^{spread},$$

where:

risk premium$_C$ – country specific risk premium;
risk premium$_{US}$ – risk premium in US stock market;
CDS_C^{spread} – difference between country CDS spread and US CDS spread;
$\sigma_C^{stock\ market}$ – country stock market volatility;
$\sigma_C^{gvt\ bond\ market}$ – country government (most liquid) long-term bond volatility. Therefore, the cost of capital is correlated with other balance sheet items through the applied reference bond yield mixed with the CDS spread scaled by the relationship between the stock market and bond market volatility and the market interest rates to which loans and deposits are indexed. However, the estimates obtained in this way explode at the second half of 2011 owing to the very limited liquidity of the CDS markets. The limited liquidity considerably distorts the picture of the cost of capital.

Summing up, the cost Cost_1^E generated by the capital E_0 is

$$\text{Cost}_1^E = r^E E_0.$$

Risk The risk of the cost of capital is gauged by the volatility of the calculated cost rates r^E which are based on the formula 5 with estimated parameters α and β, and the country specific risk premium p^c.

3.1.9 Interdependence

All flows of income and cost generated by the balance sheet items are correlated mostly due to correlation among interest rates paid by categories of assets and liabilities. It is measured by the covariance structure in the balance sheet. The covariance matrix of the asset and liability categories results from the correlation (CM) of interests paid by loans and deposits, which are assumed to be indexed to the market interest rates.

In order to make the measurement of the interdependence least prone to the estimation error in the correlation, the insignificant[18] entries of the correlation matrix were set to 0. Moreover, section 4.2 presents a method that helps to increase robustness of the optimal asset structures with respect to estimated correlation.

3.2 The goal function

It should be noticed that there is no stochasticity in the baseline model. The resultant optimization problem is a version of the Markowitz-type portfolio choice, extended by the funding side and risk constraint in form of the capital adequacy ratio.

The goal function is to maximize the return on equity adjusted by the risk borne in the balance sheet. The adjustment should take into account the reward required by the shareholders for the risk taking while investing in the bank.[19]

The net income (NI) of the bank is defined as

$$\mathrm{NI}_1 = \mathrm{Inc}_1^L + \mathrm{Inc}_1^S + \mathrm{Inc}_1^I - (\mathrm{Cost}_1^D + \mathrm{Cost}_1^F + \mathrm{Cost}_1^{DB} + \mathrm{Cost}_1^E) \tag{7}$$

and is already *known* at t_0 given the market scenario of interest rates that is going to realize in $[t_0, t_1]$. We deliberately do not call it *net interest income* since definition of NI involves the cost of capital which anyway is a usual ALM component of funding costs. In other words, the net income is largely related to the interest income (loans and securities) and cost (deposits and other debt) but takes into account also costly equity financing. Cost of capital (required by investors return on equity) is always considered by banks in their ALM process (Adam (2008)).

In order to reflect the investors' risk aversion in the goal function the way paved by the capital allocation techniques and economic value optimisation is followed (Adam, 2008).

Given strategy $(\pi_0^L, \pi_0^S, \pi_0^I)$ and denoting by $D^2(X)$ the variance of a given random variable X, the reward function is

$$J^{\pi_0^L, \pi_0^S, \pi_0^I}(L_0, S_0, I_0, D_0, F_0, DB_0) = \frac{\mathrm{NI}_1}{E_0} - \kappa D^2\left(\frac{\mathrm{NI}_1}{E_0}\right) \tag{8}$$

and the banks tends to maximize it choosing π_0^L, π_0^S and π_0^I at the beginning of $\mathbb{T}_0$.[20] The sensitivity parameter κ can be set either according to "EVA maximisation" type of reasoning

[18] The standard test was performed ($\alpha = 5\%$) assuming the asymptotic multivariate Gaussian distribution in the sample.

[19] The most compact formulation of banks target of profitability and risk refers to their desired rating. As Adam (2008) stipulates, the optimal strategy would be the one that guarantees the rating with highest probability. In practice, it implies that a bank is to hold economic capital at the level covering potential losses with probability of default associated with the desired rating. Eg., bank aiming for AA rating should keep its probability of default at the level of 0.05%-0.03%.

[20] The goal function can be reformulated to a standard mean-variance setting: for a vector of strategies π the bank would optimise a functional $A\pi - \pi^t Q\pi$ for appropriately defined vector of return parameters A and covariance matrix Q.

(Grant, 1998; Stoughton and Zechner, 2007) or "risk tolerance" related to option-implied (Bliss and Panigirtzoglou, 2004).[21] These two approaches should be consistent. EVA arbitrarily assumes that net income should be adjusted by the expected loss which in practice means the VaR of the income. In the world with risk being associated with the normally distributed random variables this adjustment translate to about 3 times the standard deviation of the net income. The EVA approach has the advantage over the risk tolerance in terms of market practice – it is widely used in banks to measure performance of capital allocation. Thus, it is easier in applications than the risk tolerance coefficient which nevertheless has more solid economic foundation (see Plantinga and Groot (2001); Kocherlakota (1996) for general discussion about risk tolerance, its measurement, range of feasible values it can take and implications for portfolio performance measurement).

3.3 Set of admissible strategies

Banks are allowed for reinvesting assets in such a way that certain rules are abided by. These are: balance sheet sum identity and solvency and liquidity constraints.

The first, and technical, condition that the strategies have to satisfy is the budget constraint reading as the balance sheet sum condition valid at the start ($t = 0$) and at the end ($t = 1$) of the period, i.e.

$$TA_t \colon = \sum_{i=1}^{K} D_t^i + \sum_{i=1}^{M} F_t^i + \sum_{i=1}^{M_2} DB_t^i + E_t$$

and

$$L_t + \sum_{i=1}^{N} S_t^i + I_t = TA_t.$$

A change of the balance sheet sum between t_0 and t_1 is assumed to be solely driven by the liability side, i.e.

$$\pi_0^L + \sum_{i=1}^{N} \pi_0^{S,i} + \pi_0^I = \sum_{i=1}^{N} S_0^i + aL_0 + I_0 + TA_1 - TA_0. \tag{9}$$

Loans are assumed to be perfectly illiquid assets which cannot be sold before maturity. Therefore, only the maturing part aL_0 can be reinvested. As opposed to loans, securities are liquid and are sold at the market price. It means that the cash available for reinvestment/restructuring is limited by $aL_0 + \sum_{i=1}^{K} S_0^i$. Not the whole cash amount needs to be allocated into loans, securities or interbank. A part of it can just be left as cash having zero risk weight (bearing no risk) and – as a consequence – being uncorrelated with other balance sheet categories.

Apart from maximising risk-adjusted profit the bank operates under the legally binding risk constraints. The most fundamental one is the capital adequacy ratio that requires that part of the risk-weighted assets (RWA) should be covered by the high quality capital. Assuming the standardised approach of the bank to fulfill its capital adequacy obligations, the weights in RWA need to be calibrated in such a way that:

1. total RWA equals the RWA reported by banks in EBA stress-testing

[21] The estimates of the risk aversion parameter range from 1 to 5. $\kappa = 2$ is the baseline case.

2. product breakdown of assets and liabilities (if available) gives the minimal bounds of the weights. For instance, a bank granting only mortgage loans would have lower weights (probably on average around 75%) than other bank with loan portfolio consisting of solely uncollateralised consumer loans (100%).

Each of the three groups of the asset categories (i.e. loans, securities and interbank lending) is assumed to have one, common risk weight. They can be calibrated based on the initial RWA and a known structure of assets at t_0. Consequently, the solvency (or capital adequacy) constraint has the following form:

$$E_1 > 8\% \cdot \left(w_L^{RW}((1-a)L_0 + \pi_0^L) + w_S^{RW} \sum_n \pi_1^{n,S} + w_I^{RW} \sum_n \pi_1^{n,I} \right).$$

It can be understood as a requirement imposed on the strategies such that banks are allowed for reinvesting their assets by assuring that they have enough capital to cover the related risks. The assumed 8% solvency threshold is in line with the Basel II definition of capital adequacy based on Tier I and Tier II capital. Basel III changes this rule referring only to Tier 1 capital ratio requiring it to remain above 6% but also defining the additional buffers: conservation buffer (additional 2.5%) and countercyclical buffer (up to 2.5% at discretion of the national supervisors). Our model can be used with any of this thresholds if needed. We concentrate on the total capital ratio which can be calculated based on banks' 2011 annual reports.

The equally important constraint is related to the short term liquidity. Banks want and are obliged[22] to hold liquid assets to cover short term liabilities. The constraint has the form:

$$w_L(L_1 + \pi_0^L) + w_S \sum_n \pi_0^{n,S} + w_I \sum_n \pi_0^{n,I} \geq w_D \sum_k D_0^k + w_F \sum_k F_0^k + w_{DB} \sum_k DB_0^n, \quad (10)$$

where w_L, w_S, w_I and w_D, w_F, w_{DB} are the liquidity weights. The liquidity constraint does not depend on the maturity structure but rather on the product breakdown.

Let us assume that the short term horizon is delimited by $l^* < 1Y$. Then, a very rough proxy for the liquidity weights of loans, interbank placements, interbank deposits and own debt issued would be the ratio of volume of a given balance sheet category maturing up to l^* and the total volume of this category. In this way, also the maturity structure factors in the liquidity constraint.

Summing up, the set of admissible strategies $\mathcal{A}$ is the following:

$$\begin{aligned} \mathcal{A} \colon = \Big\{ (\pi_0^L, \pi_0^S, \pi_0^I) \in \mathbb{R} \times \mathbb{R}^N \Big| \pi_0^L \geq 0,\ \pi_0^I \geq 0, \\ \pi_0^L + \sum_{i=1}^N \pi_0^{S,i} + \pi_0^I = \sum_{i=1}^N S_0^i + aL_0 + I_0 + TA_1 - TA_0 \Big\} \end{aligned} \quad (11)$$

and the bank solves for the following constraint optimal program:

$$\begin{aligned} &\max_{(\pi_0^L, \pi_0^S, \pi_0^I) \in \mathcal{A}} J^{\pi^L, \pi^S, \pi^I}(l, s, i, d, f) \qquad \text{s.t.} \qquad (12) \\ &E_0 > 8\% \cdot \mathrm{RWA}_0, \\ &w_L(L_1 + \pi_0^L) + w_S \sum_n \pi_0^{n,S} + w_I \sum_n \pi_0^{n,I} \geq w_D \sum_k D_0^k + w_F \sum_k F_0^k + w_{DB} \sum_k DB_0^n. \end{aligned}$$

[22] Very strictly under Basel 3 which specifies details of the classification of assets and liabilities to liquid categories.

What happens with the solution to the model if the capital constraint is violated already at the initial time t_0? The shortest answer is that there is no solution in this case since there is no admissible strategy satisfying the risk constraints. In order to allow for analysing undercapitalised banks, the model is slightly relaxed for these banks by changing the definition of the admissible set form A to A':

$$\mathcal{A}' :\, = \Big\{ (\pi_0^L, \pi_0^S, \pi_0^I) \in \mathbb{R} \times \mathbb{R}^N \Big| \pi_0^L \geq 0,\, \pi_0^I \geq 0, \\ \pi_0^L + \sum_{i=1}^{N} \pi_0^{S,i} + \pi_0^I \leq \sum_{i=1}^{N} S_0^i + aL_0 + I_0 + TA_1 - TA_0 \Big\} . \tag{13}$$

The difference $(\sum_{i=1}^N S_0^i + aL_0 + I_0 + TA_1 - TA_0) - (\pi_0^L + \sum_{i=1}^N \pi_0^{S,i} + \pi_0^I)$ in the optimum can be interpreted as cash C_1 used to pay back part of a bank's debt to lower the leverage. In this way we implicitly exclude the recapitalisation in form of new equity from issuance or debt conversion unless we simply assume an increase of the capital with appropriate adjustment of its cost and risk.

4 Fostering robustness of results

It turns out that the optimal balance sheet structures may substantially deviate from the observed ones. This is a natural consequence of the simplifications introduced to banks' optimisation of asset structures. More specifically, it may be attributable to the following issues:

1. A complex institution as a bank may not be capable of quickly reacting to unexpected changes in market conditions. Decision processes are usually complicated (going through the Asset-Liability Committee, risk committee accounting for different views of business areas, then management board). Moreover, it may be costly (in purely pecuniary terms but timing also matters) to change marketing tools or introduce new marketing campaign rebalancing the loan or deposit portfolio structure, etc. This inertia may result in some deviations of the balance sheet structure from (theoretically optimal) targeted composition.

2. The observed structure may be in a disequilibrium; due to the unforeseen market developments the structure may not be the one targeted by managers and shareholders.

3. The goal is multi-criterial instead of just mean-variance optimisation. This is driven by banks' management and shareholders' expectations and beliefs about the macroeconomic and financial markets developments. Notably, they have also different planning horizon. These beliefs are hidden to other parties, in particular to researchers.

4. Not all the important decision variables are taken into account. For instance, managers of a bank may target a given leverage ratio and a market share in some important products. This could be partly discovered by studying banks' annual reports where they may want to highlight particular changes in the asset and liability structures revealing their intentions.

4.1 Transaction costs

In order to be able to account for the first two points listed above, they are expressed in terms of the so-called *transaction costs*. Namely, a bank willing to restructure its balance sheet needs to incur additional costs. It may not be possible to reflect them fully in accounting terms (esp. time and lost opportunity costs). Nevertheless, they are assumed to impact the goal functional in the following way:

$$J^{\pi_0^L,\pi_0^S,\pi_0^I}(L_0,S_0,I_0,D_0,F_0,DB_0) = \frac{\mathrm{NI}_1}{E_0} - \kappa D^2\left(\frac{\mathrm{NI}_1}{E_0}\right) - K_\tau(\pi_0^L,\pi_0^S,\pi_0^I,L_0,S_0,I_0), \quad (14)$$

where

$$K_\tau(\pi_0^L,\pi_0^S,\pi_0^I,L_0,S_0,I_0) \quad = \quad \tau^L|\pi_0^L - aL_0| + \tau^S\sum_{i>i^*}\left(\pi_0^{S,i} - S_0^i\right)^- + \tau^I\sum_i|\pi_0^{I,i} - I_0^i|$$

The scalars $\tau^L > 0$, $\tau^S > 0$ and $\tau^I > 0$ are the cost parameters. The cost penalises primarily for upward deviations in volumes of loan portfolios but in the interbank portfolio as well. Secondly, a "fire sale" of securities carries transaction costs, here applied only to the longer-term securities ($i > i^*$). Liquidation of the portfolio may only be possible at an additional haircut. What is more, given the total volume of assets unchanged, a shrinkage in securities portfolio is inherently related to a growth in at least one of the other asset categories. In practice, as suggested in the literature, banks may experience even slightly increasing marginal costs of launching big campaigns to make a substantial change to the existing portfolio; in other words – there may be diseconomies of scale in this respect. Thirdly, the reduction of both loan and interbank asset portfolio is assumed to be costly since it may require changes in agreed goals of individual business units within the bank (time for planning, negotiations).

4.2 Correlation structure

The differences between optimal and observed balance sheet structures may also result from the estimated correlation matrix. We There are at least 3 reasons for that.

1. (inaccurate estimation) The matrix CM is tested only for values essentially different from 0. The confidence intervals are specified neither for positive nor for negative values in the matrix.

2. (backwards looking perspective) Most likely, the correlation used by banks in their strategic planning of the balance sheet structure takes into account managers expert judgment (beliefs mentioned in subsection 4) about future co-movements of important macroeconomic and financial variables.

3. (customer relationship management) Correlation inferred from macro variables and aggregate variables (eg interest on loans and deposits) do not take into account the strategic linking (bundling) of products (e.g. current accounts and mortgage loans, credit lines and investment products). In that case, pricing is not the only important factor driving income of the bank but building a long-term relationship as well.

Let us solve the reverse optimisation problem with respect to the correlation matrix. The reverse optimisation addresses the following question: what should the correlation matrix look

Figure 1: Assets and liabilities breakdown

ASSETS *group*	*model category*	LIABILITIES + CAPITAL *group*	*model category*
Loans to customers	(**L**) Outstanding volume	Deposits from customers	(**D_1**) on demand
			(**D_2**) up to 3M
			(**D_3**) 3M to 1Y
	(**NewL**) New loans		(**D_4**) 1Y to 5Y
Debt securities	(**S_1**) up to 3M		(**D_5**) above 5Y
	(**S_2**) 3M to 1Y	Interbank funding	(**F_S**) Short term up to 3M
	(**S_3**) 1Y to 5Y		(**F_L**) Long term (above 3M)
	(**S_4**) over 5Y	Debt securities issued	(**DB_S**) up to 3M
Interbank assets	(**I_S**) Short term (up to 3M)		(**DB_L**) above 3M
	(**I_L**) Long term (above 3M)	Other liabilities	(OL)
Other assets	OA	Capital	(**E**)

Source: own calculations

like to insure that the resultant optimal asset structure is close to the observed one? A Monte Carlo algorithm is proposed for sampling of matrices that are "close" to the estimated one and to search for those that give the least divergent optimal structure.

A 10 percentage point threshold is set for the deviation of the correlation matrix being robustified. It means that the each of the components of the adjusted correlation matrix should not differ more than 10 pp from the estimated one. Moreover, as in the case of gradient method, the sign of the correlation components is kept unchanged. It means that if the estimated value of correlation between item i and item j of the balance sheet is equal to q_{ij} and is greater then 0 then in the robust matrix

$$q_{ij}^r \in (\min(\max(0, q_{ij} - 0.1), 1), \min(\max(0, q_{ij} + 0.1), 1)) .$$

Analogously, if $q_{ij} < 0$ then

$$q_{ij}^r \in (\min(\max(-1, q_{ij} - 0.1), 0), \min(\max(-1, q_{ij} + 0.1), 0)) .$$

5 Data

The simulations of the optimal balance sheet structure encompass large European banks which were encompassed by the European Banking Authority (EBA) disclosures for the 2011 EU capital exercise.[23] The motivation for the particular selection of the sample was twofold. First, the group of banks from EBA exercise is a benchmark sample in the process of EU banking sector analysis. Moreover, banks in this group guarantee an availability of detailed data on banks' balance sheets. Second, the sample is reduced to those banks for which additional

[23] Due to limitations in data availability, not the whole sample was used but a majority of banks.

Table 1: Estimates of return and cost rates at the outset of the horizon (average for banks in a given country, standard deviation of the rates in %)

Country	L	L_New	S	I	D	F	DB	E
AA	3.1	4.5	2.3	1.4	1.3	1.6	4.5	7.2
BB	3.4	4.4	3.4	1.4	1.4	1.7	9.6	10.6
CC	6.6	5.9	5.1	1.3	1.9	1.3	20.1	14.0
DD	3.9	4.2	2.0	1.5	1.4	1.6	3.4	8.0
EE	3.8	5.6	3.1	1.7	1.3	1.8	6.9	6.2
FF	3.2	4.5	2.1	1.5	1.7	1.5	3.9	7.1
GG	4.1	4.6	1.9	1.6	1.6	1.4	3.3	8.2
HH	6.0	6.5	62.4	1.3	1.7	1.3	137.4	35.0
II	9.3	16.4	2.6	7.2	3.9	7.5	11.3	9.1
JJ	3.6	4.3	3.2	1.3	1.5	1.4	21.3	8.5
KK	4.0	5.0	3.2	1.4	0.9	1.5	10.0	6.8
LL	3.5	4.6	2.2	1.5	1.2	1.4	2.8	12.1
MM	4.6	7.8	4.0	1.3	3.3	1.5	24.0	13.3
NN	4.1	4.4	1.9	2.6	2.2	2.6	2.8	5.0
OO	5.1	5.4	2.7	1.5	1.2	1.8	8.6	6.7

Source: own calculations based on EBA, ECB, Bloomberg and SNL

information about maturity structure of their balance sheets can be found. This piece of information is only available from some public sources (annual and Pillar 3 reports) and is the main limitation further reducing the size of the analyse sample. The sum of total assets of the banks amounts to EUR 22,640 bn. The starting ($t_0 = 0$) composition of assets and liabilities is a direct aggregation of volumes disclosed by banks in their publicly available annual reports. The aggregation is performed according to the balance sheet categories specified in figure 1.

The proxies for return rates, cost rates and their volatility is derived using databases of Bloomberg, Bureau van Dijk's Bankscope and the ECB. The weighted average interest rate paid by loans takes into account maturity structure available from the financial reports, product structure (the same maturity composition was assumed for all product categories) and rates from the ECB statistics (similarly for the customer deposits broken down into retail and corporate portfolios and some maturity buckets). Yields of securities are approximated by the government and corporate yield curves (collected from different sources as Bloomberg and Reuters) and weighted by the share of corporate and government bonds in banks' securities portfolios. In order to estimate the risk of return and cost of all the categories a time series of 3 years (for the cost of capital – 5 years) of monthly data is applied, which is 36 points of time (60 points for the cost of capital).

The cost of capital is calculated with an arbitrary choice of US equity premium at the level of 5%.[24] Average country-specific bank P/E ratios are taken from Bloomberg. The β of the banks' stock prices is estimated from equation 5, with 1Y IRS rates as risk-free rates $r^{risk-free}$ (five years of monthly data), and with monthly log-returns from investing into the main stock exchange index in a given country r^{index}. The summary of the return and cost rates is presented in Table 1. For confidentiality reasons country-specific results are anonymised.

The probability of default (ρ) of loans in retail and corporate portfolios of banks is approximated by the bank-specific loss rates reported in the EBA's 2011 EU-wide stress test. The

[24] Damodaran (2009) estimates the premium based on the TBills or TBonds data in a range of roughly 4-7% depending on the data and method used.

Table 2: Estimates of the risk of return and cost rates at the outset of the horizon (average for banks in a given country, in %)

Country	L	L_New	S	I	D	F	DB	E
AA	10.2	9.9	23.4	12.8	10.1	13.2	45.8	13.2
BB	5.3	8.0	45.4	12.9	16.3	13.5	62.6	32.3
CC	9.6	14.6	32.3	12.5	8.0	12.5	155.4	80.6
DD	4.3	7.5	26.0	13.1	8.6	13.3	23.6	11.1
EE	6.9	15.0	41.4	15.5	9.7	15.9	58.8	6.9
FF	7.0	5.6	30.3	12.9	12.9	13.0	30.6	6.9
GG	5.8	10.0	25.9	9.6	9.7	9.4	27.3	30.5
HH	14.1	16.8	114.6	12.5	11.2	12.5	398.9	119.0
II	23.4	51.5	40.8	32.7	20.8	38.1	83.3	58.8
JJ	6.4	19.8	55.2	12.6	7.7	12.8	175.2	95.9
KK	9.7	11.8	46.2	12.8	6.5	13.1	66.2	21.9
LL	6.8	8.4	30.7	13.0	8.1	12.8	23.3	28.9
MM	12.5	36.9	41.5	12.5	30.9	13.1	150.2	46.4
NN	11.8	23.0	37.4	33.0	28.5	33.2	23.3	4.7
OO	10.6	14.8	22.8	13.1	6.7	13.6	67.5	14.0

Source: own calculations based on EBA, ECB, Bloomberg and SNL

new loans are assumed to have default probability which is equal to 0.2ρ.[25]

The risk weights (RW) in the solvency constraint are assigned in the following way. For securities portfolio there is an average RW (w_S^{RW}) weighted by the share of government securities (0% RW) and other securities (20% RW). The proportion of the two categories is based on the information about the securities portfolio structure obtained from Bankscope database. The interbank assets are assumed to have 20% RW (($w_I^{RW} = 20\%$)). The total RWAs of banks available in the dataset of the EBA disclosers combined with RW of securities and interbank placements imply the average RW for loans ((w_L^{RW})).

The liquidity weights (see constraint 10) are approximated in a way presented in table 3. Notably, the assumed coefficient for deposits (7.5%) is the average of the Basel 3 proposal of 5% and 10% Liquidity Coverage Ratio weights assigned to stable and less stable customer deposits. Coefficients that are equal to 33% reflect proportional monthly outflow (inflow) of liabilities (assets) maturing within 3 month.

6 Measures of model performance

6.1 Robust results with respect to the correlation matrix

In order to make the presentation of results more readable asset categories are aggregated at the country level. The most difficult part of the modeling exercise is to obtain the reliable optimal asset structure given the parameters derived directly from market data (without any stresses imposed on risk parameters, funding availability, etc.). Therefore, in this subsection the focus is on the outcomes of the model with unchanged parameters.

At the model-based optimum, banks in countries perceived as afflicted by the crises to a

[25]The particular choice is justified by findings in Engelmann (2011) – the average PD of a loan in its first year after origination was estimated to be approximately 5 times lower than its average PD in the next 5 years. The sensitivity test of the results of the model is presented in section 6.4.

Table 3: Liquidity weight of asset and liability categories

Category	*Liquidity ratio*
L	3M loans / Total Loans
S	Gvt securities / Total securities
I (short-term)	33.3%
I (long-term)	0.0%
D	7.5%
F (short-term)	33.3%
F (long-term)	0.0%
DB (short-term)	33.3%
DB (long-term)	0.0%

lesser extent – like countries DD, LL but also FF[26] – are expected to continue lending, being only limited by the imposed credit growth in the economy. On the opposite side are banks in countries BB, CC, HH and II but also KK and MM which are projected to severely cut on lending. These findings correspond to the estimates of risk, return and cost parameters in Tables 1 and 2, especially with respect to the funding risk and credit risk in loan portfolios. In this way the results meet the expectations about general loan supply constraints in EU following the crisis.

The impact of the liquidity constraints on the optimal asset structure of banks in the sample is also analysed. Notably, the liquidity constraints proved not to be binding and therefore are not included in further simulations and discussion. However, they are an important component of the model which only adds to flexibility of the framework.

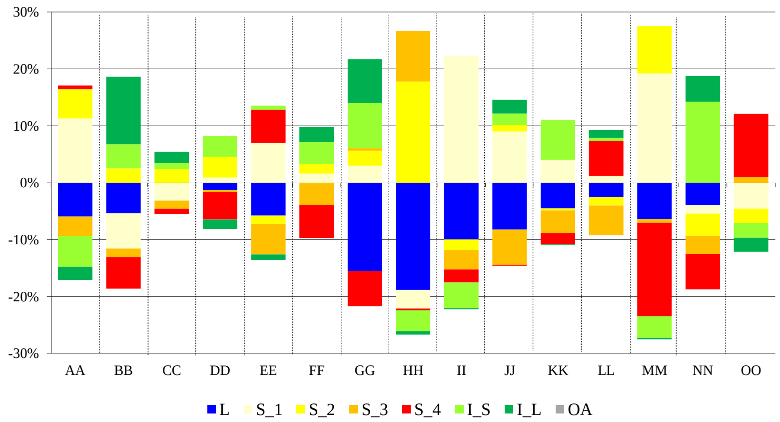
There are some puzzling results in terms of the theoretically optimal loan supply on country level. Banks GG and NN in the sample are seen as dramatically cutting on lending which is against the intuition given a low level estimates of risk accumulated in their balance sheets (see Table 2). Moreover, banks from country NN are in general projected to hold the asset structures far from the theoretically optimal one. A substantial increase of their interbank exposures may be attributable to a higher return on this categories than from securities whereas the risk of the two asset classes is very similar (see table 1). As already mentioned in section 4, part of the divergence may be explained by the estimation risk of the correlation structure in balance sheets.

6.2 Isolated shocks – does the model perform?

The model can be used to analyse consequences of various scenarios of market distresses to banks' activity. In order to check whether the model gives consistent results two simple, "isolated" scenarios are applied. Namely, it is done by changing of one of the market parameters and leaving all other conditions unchanged (i.e. at the end of 2011 level). It is an easier approach in terms of prediction and interpretations of results than the fully-fledged implementation of consistent stress-testing adverse scenarios (see section 7). Only robust correlation

[26]Due to confidentiality issues, the countries are anonymised by randomly assigning to them pairs of letters AA, BB,... ,OO.

Figure 2: Changes in country aggregate of the asset structures (observed end of 2011 composition and the optimal end of 2011 with robust correlation matrix CM, % of total assets)

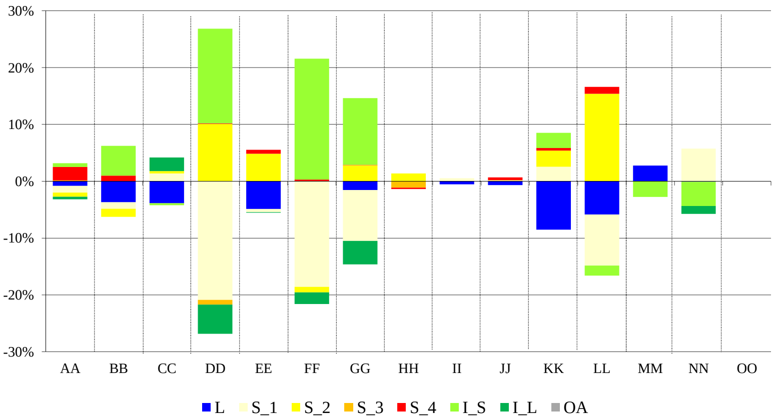


Source: own calculations

structure is applied (see figure 2). First, the PD of bank's outstanding loan portfolio increases by 100 bps and, accordingly with the assumptions, the new loan portfolio quality deteriorates by 0.2*100=20 bps. It implies a lower return but also a higher standard deviation of the return. The expected outcome of such a scenario is presented in Figure 3 where the theoretically optimal balance sheet structures in "no credit shock" and "credit shock" cases are compared. In the new optimum, all the banks lend less. However, the reactions of banks to this shock are heterogenous. Banks in BB, CC, EE, KK and LL decrease lending to a higher extent than their counterparts in other countries. Country MM banks are projected to accelerate lending, which may be explained by the correlation estimates. Namely, there is on average no correlation between income from new lending and from the outstanding loan portfolio and, therefore banks in MM benefit from diversification when they expand retail lending.

Second, banks' reactions to changes of funding availability are tested. The most interesting insight that the model can bring given current concerns about the credit supply in the European economy is the deleveraging depth triggered by the funding constraints of the banks. Obviously, funding constraints may not only impact the lending but potentially change the whole (theoretically optimal) asset structure of banks. The developed framework can help to analyse the deleveraging process concentrating on the consequences of one of the possible scenarios of funding disruptions. Namely, banks' responses to the partial inability to rollover their short-term interbank funding and their short-term debt issued are scrutinised (see figures 4). According to the subsection 3.3, the OUTFLOW bars in figure 4 reflect the magnitude of the interbank funding reduction of banks, heterogenous across countries depending on how the domestic systems rely on the interbank funding. In general, banks reduce their securities portfolios in the first instance, except for countries KK and MM where banks compensate funding constraints with a reduction of their lending and interbank exposures, respectively (see figure 4). However, there is some visible heterogeneity in loan portfolio dynamics. For

Figure 3: The shock to the credit quality of the loan portfolio (changes of the optimal balance sheet structure in the constant environment (all parameters at the end of 2011 level) and the optimal structure after a 100 bp shock to the PD of the outstanding volume of loans and 10 bp shock to the new one)



Source: own calculations

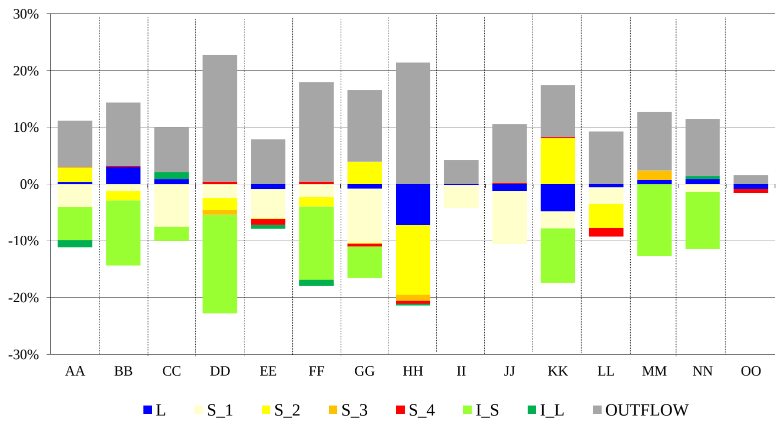
example, banks with substantial share of short-term wholesale funding in DD, FF and GG react differently. Banks from the first two mentioned countries deleverage primarily on the interbank portfolio and their theoretically optimal retail lending activities remain unaffected contrary to country GG where banks lend less to retail and corporate customers but retain the interbank lending. The differences in reactions confirm that the deleveraging speed may be strongly related to a particular asset-liability structure and dependence structure within the balance sheet which a relationship can be traced by the developed model.

6.3 Optimal structures and financial ratios

Some basic financial ratios of banks can be helpful in measuring of the model performance. They are borrowed from corporate finance studies and capture aggregate risk and return potential of banks (Damodaran, 2007). It can be expected that optimal design of banks' balance sheet structures should be related to these ratios. For instance, the deleveraging depth in loan portfolios should be explained by riskiness of loans, correlation of returns from loans with funding sources or relative level of returns from loans comparing with other asset classes.

The potentially relevant ratios are calculated based on the initial (t_0) balance sheet structures of banks and their risk and return parameters. Therefore, they can be called *micro-drivers*. We checked whether there is a relationship between the optimal volumes of loan portfolios and banks' financial ratios in the sample. The so-called deleveraging depth is measured by the ratio of the volume of the theoretically optimal loan portfolio to the observed volume at the end of 2010. There are 9 measures of individual banks' risk and interest income

Figure 4: Comparison of the country average optimal structure of assets with no funding constraints with the average optimal structure under funding constraint



Note: Funding constraint = 50% rollover rate for short-term interbank funding (F) and short-term debt issued (DB).
Source: own calculations

potential:

- SharpeLvsA – difference of the Sharpe ratio for loans (interest rate divided by the risk measure) and of the average Sharpe ratio of other asset categories;
- WACC – weighted average cost of capital as average interest paid on liabilities and capital weighted by the volumes of deposits, interbank funding, own debt issued and capital;
- SharpeWACC – WACC divided by weighted risk of liabilities and capital, analogous to the famous Sharpe ratio of return and risk measures;
- CDS – soveraign CDS spread;
- PD – average default probability of the loan portfolio (bank specific);
- ECost – cost of equity;
- Duration – weighted average maturity of liabilities;
- CorrLvsA – correlation of interest paid by loan portfolio with interest income earned on other asset categories (securities and interbank placements);
- NormalisedRisk – risk of balance sheet at the end of 2011 measured by the variance of income and cost related to its components and taking into account the estimated correlation, all divided by the total assets. It reflects the volatility of the income and the interest rate risk mismatch between assets and liabilities (i.e. the higher the measure the bigger the risk of the bank).

Table 4: Correlation of income, cost and risk parameters of the balance sheets with Deleveraging Depth

var	description	correlation (%)	p-value (%)
SharpeLvsA	Loan Sharpe ratio - avg other assets Sharpe ratio	22.3	11.8
WACC	Weighted Avg Cost of Capital	**-24.0**	**4.0**
SharpeWACC	WACC devided weighted risk of the cost	11.4	42.6
CDS	Sovereign risk spread	**-34.8**	**1.2**
PD	Avg default probability (loans)	**-31.2**	**1.3**
ECost	Cost of equity	**-27.8**	**4.8**
Duration	Duration of liabilities	9.1	51.9
CorrLvsA	Correlation of loans with other asset categories	25.4	7.4
NormalisedRisk	Aggregate BS risk := $[A \; - L]' * Q * [A \; - L]$; Q :=covariance	**-22.6**	**4.1**

Source: own calculations

As Table 4 shows, the most significant (at 5% signif. level) factors explaining the changes in the optimal volume of the loan portfolio are the weighted average cost of capital (WACC), sovereign risk of country to which a given bank is originated (CDS), loan portfolio PDs and the total (normalised) risk of balance sheet. The measures indicate that higher costs of funding and high risk of interest income exert adverse pressure on the loan portfolio making a bank more likely to deleverage on the loan portfolio.

The correlation between lending and various balance sheet measures explain at least part of the divergence between the observed and the optimal structures

6.4 Sensitivity analysis

There is a couple of arbitrary assumptions about parameters of our model. Therefore, it is interesting to analyse sensitivity of the simulated optimal asset structures to changes of these parameters in some reasonable ranges. Among the analysed parameters are: credit risk parameters (LGD and PD), measure of the risk aversion, risk premium estimate used in the calculations of the cost of capital, liquidity and capital constraints and, finally the estimates of the return and cost parameters.

The results of this exercise are presented in Table 5. They show variation of asset composition across banks given that one of the 6 chosen parameters is changed. At the center of this analysis are the assumptions about Loss Given Default of loans, the ratio defining probability of default of new loans in relationship to the overall estimated PD of loans (ρ), risk adjustment parameter κ, assumed US market risk premium, capital requirement threshold and the share of the HTM securities portfolio (i.e. a proportion of securities' portfolio not impacting the income measures through mark-to-market valuation). The LGD parameter was assessed as having the most significant impact on the optimal assets structure (e.g. decrease of the assumed LGD by 10pp to 40% leads on average to 3.5% change of the theoretically optimal lending volume; the impact is twice as much in the securities portfolio). Results of the simulations are also quite sensitive to the default probability of new loans. Notably, assumed changes of risk aversion measure κ, US risk premium and estimated share of HTM securities do not materially distort the optimal structures (much less than 1% changes across all banks and all portfolios).

The outcomes of the model are relatively stable with respect to parameters of solvency and liquidity constraints. Capital ratio increased to 9% or risk weights of loans or securities

Table 5: Sensitivity of the theoretically optimal banks' asset structures

Parameter	Tested value	Average across banks of absolute deviation of the optimal volume of a given category (%) divided by the total optimal volume		
		L	S	I
LGD of Loans (50%)				
	40%	3.51	7.68	6.57
	60%	3.03	6.19	5.39
PDs of New Loans (0.20 of $x\%$ data driven)				
	$0.15 * x\%$	2.12	1.33	1.15
	$0.25 * x\%$	1.39	1.22	0.76
κ (2)				
	1	0.44	0.46	0.22
	5	0.69	0.73	0.33
US premium (5%)				
	6%	0.06	0.09	0.02
Share of HTM portfolio (data driven $x\%$)				
	$\max(0, x - 10)$	0.52	1.20	0.70
Capital ratio (8%)				
	9%	0.20	0.20	0.00
Liquidity weight for S				
	-250 bps	0.33	1.57	0.52
Liquidity weight for D				
	+250 bps	0.98	2.45	2.91
Risk weight (RW) of S				
	+250 bps	0.00	0.02	0.02
Risk weight (RW) of L				
	+250 bps	0.08	0.12	0.06

Source: own calculations

Table 6: Sensitivity of the theoretically optimal banks' asset structures

	Parameter	Tested change	Mean of absolute deviation of the optimal volume of a given category (%) divided by the total optimal volume		
			L	S	I
Level	Interest rate - loans	25 bps up	0.59	2.59	2.41
	Interest rate - deposits	25 bps up	0.00	0.00	0.00
	Yield curves	25 bps up	0.09	0.36	0.35
	CDS spreads	25 bps up	0.00	0.00	0.00
Volatility	Interest rate - loans	5% up	0.19	0.44	0.40
	Interest rate - deposits	5% up	0.08	0.59	0.57
	Yield curves	5% up	0.54	3.16	2.74
	CDS spreads	5% up	0.14	1.40	1.31

Source: own calculations

increased by 250 bps have a very limited impact on the theoretically optimal asset structures. Average shifts between categories are lower than 0.2% of total assets. More sensitivity, still quite low, can be observed in case of weights applied in liquidity constraint. An increase of the outflow of customer deposits by 250 bps (to 10%) leads to changes of asset categories amounting up to 3% of total assets.

Results of the simulations can potentially be sensitive to the parameters estimated based on historical time series, in particular interest rats on loans and deposits, yield curves or CDS spreads. We verified that the optimal asset structures are most sensitive to changes in levels of interest rates in customer loan portfolio and to the volatility of yield curves (the later naturally related to the fact that yield curves determine interests paid by many balance sheet items both on asset and liability side). However, a general variability of the optimal structures is rather low and does not exceed 3.2% of total assets.

All in all, that results are quite robust to some degree of misspecification of model parameters.

7 Stress testing application

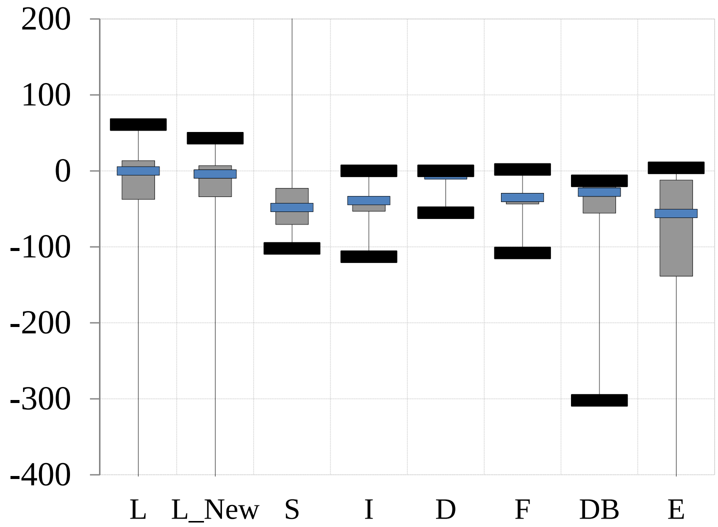
7.1 Design of the exercise

In the section, in order to illustrate the usefulness of the dynamic balance sheet tool in stress testing exercises some macro financial adverse scenarios are applied in the developed optimal asset structure framework. They impact all the parameters of the model, including volumes of deposits and interbank funding. The applied scenarios are taken from the Financial Stability Review (FSR) of the ECB (June 2012 edition). Two particular scenarios are taken into account: a mild baseline scenario and a severe adverse scenario. Both are designed as consistent macroeconomic and financial shocks to the financial system. The following assumptions are the most important components of the scenarios:[27]

- (*Yield curve twist*) A country specific interest rate shock is a result of the ECB in-house interest rate impulse response models that link various macroeconomic variables. The paths of macroeconomic variables used in the projections of rates are the EU Commission forecast of May 2012. The baseline GDP growth in the Euro Area is -0.3 in 2012 and +1.0 in 2013 and in the adverse case -1.5 and -1.2 respectively. An adverse development of the macroeconomic parameters is driven by the sovereign debt crises increasing interest rates and implying stock price declines and a domestic demand shock.
- (*Funding constraints*) In the adverse scenario, it is assumed that only 50% of the wholesale funding maturing in 2012-2013 can be renewed. The rate of the retail deposit outflow is related to the variability of the country aggregate stock of deposits in 2010 and 2011.
- (*Retail and corporate PD shock*) The evolution of default probabilities that impacts the return from the loan portfolios and its riskiness is estimated based on an the autoregressive distributed lag (ADL) model that allows for passing through it the assumed macroeconomic scenarios. The projected changes in the credit quality estimated on country level are applied to bank specific loss rates reported by EBA in 2011 EU wide stress test.

[27] A more extensive description can be found in ECB (2012).

Figure 5: Developments of income and cost rates between 2011 and 2013 in the baseline case scenario (minimum, first quartile, median, third quartile and maximum across countries, in bps)



Note: minimum and maximum presented only if it does not distort a clear picture (i.e. if it is not too far from the median).
Source: own calculations based on EBA, ECB, Bloomberg and SNL

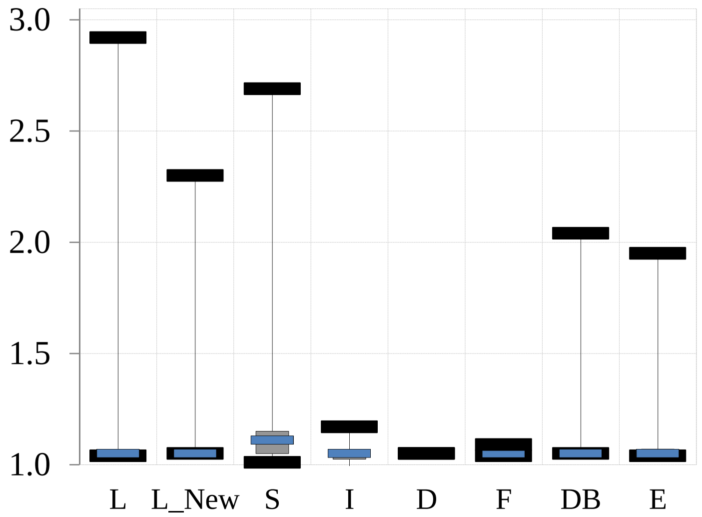
- (*Sovereign CDS shock*) The shock to the sovereign debt quality was calibrated to the changes in the ten-year government bonds. It was based on the estimated elasticities of the CDS changes to the variation of ten-year government bonds for each country.

Both in FSR and in the discussed approach the assumed paths of evolution of the macroeconomic and financial shocks are translated into changes in return, cost and risk parameters. Notably, in the case of risk parameters, the projected path of income and cost rates is included into calculation of the risk measure (i.e. standard deviation of the effective interest rates including the projected paths).

The two year stress testing scenarios are applied to the sequence of the one period models described in section 3. The resulting balance sheet structure in a given period serves as the starting point for optimization in the next adjacent period. This setup of myopic sequential decisions allows for the implementation of feed-back effects since the model is autonomously solved in each period.

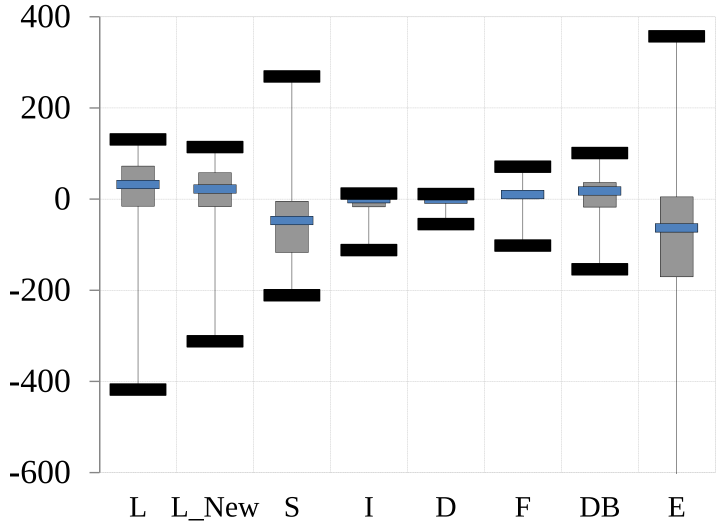
There is at least one important issue concerning the resultant optimal structure used over the stress test forecast horizon that we discuss here extensively before proceeding to the implementation. It is related to the divergence of the optimal and the observed asset structures even in the absence of shocks (see section 6.1). "Robustification" (e.g. by means of the adjusted correlation structure, see 4.2) does not completely remove the divergence between optimal and observed structures. It may be particularly acute when performing a sequence of simulations as it is done in the proposed approach. The starting balance sheets should anyway be the observed structures. However, there is an open issue how to iterate the balance sheet

Figure 6: Developments of riskiness of income and cost rates between 2011 and 2013 in the baseline case scenario (minimum, first quartile, median, third quartile and maximum across countries, factor)



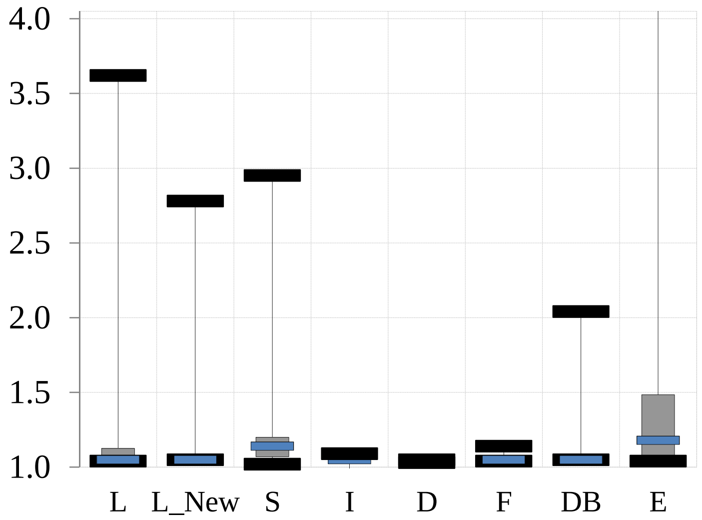
Note: the factor indicates how much time the risked changed (e.g. 1.0 factor for country X's interbank loans (I) means that the risk of the interest income of the interbank placement in X does not increase between 2011 and 2013 in the assumed adverse scenario).
Source: own calculations based on EBA, ECB, Bloomberg and SNL

Figure 7: Developments of income and cost rates between 2011 and 2013 in the adverse case scenario (minimum, first quartile, median, third quartile and maximum across countries, in bps)



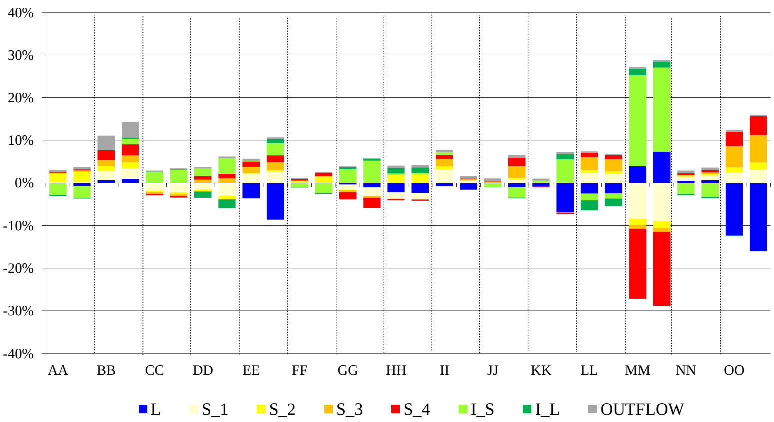
Source: own calculations based on EBA, ECB, Bloomberg and SNL

Figure 8: Developments of riskiness of income and cost rates between 2011 and 2013 in the adverse case scenario (minimum, first quartile, median, third quartile and maximum across countries, factor)



Note: the factor indicates how much time the risked changed (e.g. 2.1 factor for country X's interbank loans (I) means that the risk of the interest income of the interbank placement in X increases 2.1 times between 2011 and 2013 in the assumed adverse scenario).
Source: own calculations based on EBA, ECB, Bloomberg and SNL

Figure 9: Evolution of the country aggregate asset structures under *baseline* macro-financial scenario



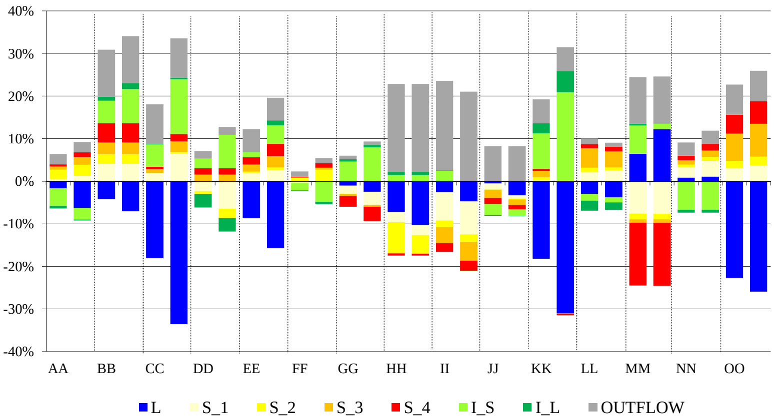
Note: For each country there are two stacked bars representing changes in asset structures between (starting) 2011 and two projections for year 2012 and 2013.
Source: own calculations

composition given the projected trajectory of the adverse scenarios and the related theoretically optimal asset structure. In order to get the optimal asset structure at a given horizon T, only the resulting optimal accrued differences of asset composition between time $t = 0$ vs. $t = 1$, $t = 1$ vs. $t = 2$ up to and $t = T - 1$ vs. $t = T$ are applied. Notably, the resulting asset structure is unlikely to be optimal. However, this disadvantage must be weighed against the benefit of obtaining a more stable asset evolution. This “incremental” approach is the preferred option. Its purpose is not necessarily to follow the theoretically optimal asset-liability structures, but rather to incorporate a dynamic optimising behavior of banks into the process, consistent with a given scenario while being based on the observed balance sheet structure used in the stress test. This notwithstanding, there is an unavoidable trade-off between theoretical consistency and the first period structural jump in the asset composition.

7.2 Discussion

(**Balance sheet parameters**) The assumed scenarios for the macroeconomic variables change the relative riskiness and profitability of all the asset and liability categories. Notably, in the baseline scenario the (interest related) return on assets is in general lower than in the adverse case due to stable interest rate curve in the baseline setting (compare figures 5 and 7. However, the cost of funding is also lower as well as the risk of return from loan portfolios. The average outcomes of the applied shocks comply with an intuition about the direction and magnitude of changes in risk / return parameters; the intuition based on the general assessment of countries’ macroeconomic and financial strength. For countries having most serious problems related to current crisis, a return from new lending is expected to drop significantly which may trigger

Figure 10: Evolution of the country aggregate asset structures under *adverse* macro-financial scenario



Note: For each country there are two stacked bars representing changes in asset structures between (starting) 2011 and two projections for year 2012 and 2013.
Source: own calculations

Table 7: Correlation of income, cost and risk balance sheet ratios with Deleveraging Depth

var	description	correlation (%)	p-value (%)
SharpeLvsA	Loan Sharpe ratio - avg other assets Sharpe ratio	21.6	12.6
Diff_WACC	Weighted Avg Cost of Capital (change Y2 vs Y0)	**-32.3**	**2.1**
SharpeWACC	WACC devided weighted risk of the cost	**-30.5**	**2.9**
DepoOutflow	Deposit and wholesale funding	-22.7	10.0
CDS	Sovereign risk related spread	**-26.8**	**4.6**
Diff_CDS	Sovereign CDS spread (change Y2 vs Y0)	**-41.2**	**0.3**
Diff_LoanPD	Avg loan portfolio PD (change Y2 vs Y0)	**-36.6**	**0.9**
ECost	Cost of equity	**-42.1**	**0.2**
Duration	Duration of liabilities	**33.2**	**1.7**
Diff_Duration	Duration of liabilities (change Y2 vs Y0)	-23.2	10.0
CorrLvsA	Correlation of loans with other asset categories	25.0	7.7
NormalisedRisk	Aggregate BS risk := $[A - L]^\top * Q * [A - L]$; Q :=covariance	-24.2	8.6

Note: prefix *Diff_* indicates the difference between end of 2013 and 2011.
Source: own calculations

deleveraging in the loan portfolios. The decline is particularly acute for countries HH and CC. Due to the increase of the sovereign risk (measured by the CDS spreads) the securities portfolios are expected to devalue substantially and consequently to decrease the expected return from investment between 2011 and 2013. It is again evident for programme countries but also for some other countries. The cost of the wholesale funding is going to rise in most of the countries in the sample or at least, even if falling in absolute terms, increase in relative terms comparing to the return from other asset categories (HH, JJ and MM). Only NN country banks in the sample are expected to have less costly funding sources.

In line with the cost / return parameters also the risk of cost of funding and the risk of return from all asset classes change materially within the analysed horizon. Notably, the risk in loan portfolios of country HH and CC banks increases the most which outcome is consistent with the general deterioration of households and corporate financial strength. On the funding side, the one category that was strongly affected was the the estimated expected return demanded by the equity holders (compare figures 6 and 8). This conclusion applies to all the countries severely hit by the crisis except for CC and JJ.

(**Deleveraging in loan portfolios**) One of the very important implications of the model refers to the reduction of lending following either baseline or adverse evolution of the macro-financial variables. The deleveraging in the loan portfolios is mostly seen in banks from countries BB, CC, EE, JJ, KK, OO but also for LL banks. Obviously, the magnitude of deleveraging is different in the two scenarios (compare figures 9 and 10). A significant deposit outflow combined with higher increase of loan portfolio PDs implies much deeper cut on lending for banks in countries CC, HH and OO.[28] Notably, in the adverse scenario also country AA banks are forecasted to lend less (contrary to the baseline scenario). Given the estimated parameters of the model, it is quite counterintuitive that MM banks sizably increase lending in the unfavorable macroeconomic conditions implied by the scenario. However, the return parameters decrease relatively mildly within the analysed horizon whereby the risk parameters remain unchanged. This translates into higher landing since in general lending margins are higher comparing to the other portfolios (i.e. lending is a profitable business is the credit risk is moderate and stable). Moreover, banks in general reduce asset categories which are highly correlated with partly rolled over funding sources.

A very simplistic statistical analysis of the relationship between the deleveraging depth and the changes of various of meaningful balance sheet ratios is very supportive in understanding the drivers of the results. Table 7 shows the indicators that explain the dynamics of the loan supply. First, the increase in the weighted average cost of funding adversely affects lending. However, if the risk of funding cost increases (relative to the cost) then the optimal response on the credit side is to extend lending. Second, the increase of sovereign credit spread reflecting the general weakening of the economy implies less credit. Quite obviously, the same conclusion applies to the increase in the general probability of default of the borrowers. Third, the risk of funding measured either by the cost of capital or duration of liabilities decreases lending incentives.

(**Caveats**) The advantages of the proposed approach must be weighted against a few important caveats. First, the model is based on the very simplified concept of banks' decision-making processes. In reality, banks face multi-criteria objectives (see section 4.2). Second, the model

[28] Since banks try to adequately price credit risk into customers' interest rates, increasing PDs could also be compensated by rising interest rate on loans without reduction of the volumes. This however could be a risky strategy if the increase of PDs is substantial, as implied by the scenarios.

has only one period. In fact, banks combine short term objectives to allow for showing "good results" annually with the long-term strategic planning taking care of business continuity and favorable market share development. The long-term horizon may bring some important leeway into the restructuring of the portfolio. Additionally, some liquidity effects may play an important role in preventing banks from excessive reshaping of the portfolio since it can be subject to bad signalling effects. Third, banks may react to stress by trying to raise capital or optimising risk weighted assets rather than risk adjusted return. Nevertheless, the empirical results are consistent with the applied adverse macroeconomic and financial scenarios. The approach is a substantial step towards less mechanistic rules in the stress testing domain. Fourth, there is some arbitrariness in estimation of the parameters of the model, in particular in the chosen length of the time series and in the model for the cost of capital. Fifth, the forecasted direction and depth of banks' responses to the changing environment may be distorted by new regulatory regime of banks (e.g. new capital or liquidity regulation, recapitalisation of banks) or of the financial markets (e.g. central clearing payments), or by some non-standard monetary policy measures.

8 Conclusions

A framework for analysing banks' reactions through adjustments made to their asset structures following various economic and financial shocks is proposed. In a resultant 1-period model of risk-reward optimal choice of asset composition, simulations of the theoretically optimal balance sheet restructuring are performed in response to credit risk deterioration (in loan portfolios) and severe funding constraints for a sample of large European banks. In this way, the sensitivity and heterogeneity of banks' reactions – esp. regarding deleveraging depth – to stressful market conditions is studied. The most prominent example of the application of our framework is the analysis of the theoretically optimal responses of banks to some real macroeconomic scenarios (baseline and adverse) that were originally developed by the ECB experts for stress testing of the EU banking system and were presented in June 2012 Financial Stability Review (FSR). For that purpose, the 1-period models were applied sequentially, i.e. the optimal asset structure at the end of a given period was a starting structure for the next period.

Banks are modeled as investment portfolios with risky funding and solvency and liquidity constraints. Their optimal, one period investment plan is assumed to maximise the risk-adjusted return on equity. It is a very computationally tractable and at the same time flexible setup. It allows for considering many different shocks (as comprehensive as the FSR one) to the balance sheets and quantify the banks' responses to them. It was verified that the changes of the theoretically optimal asset portfolios correlate with some financial ratios (e.g. Weighted Average Cost of Capital, cost of equity or a measure of the average risk accumulated in a balance sheet). Moreover, in the adverse scenario implying increasing funding risk and lower expected return from loans suggests increasing deleveraging pressure in countries deeply effected by the crises. In general, correlation of assets and funding sources explains the trends in the optimal asset evolution: banks reduce assets that are correlated with decreasing deposits. This notwithstanding, the outcomes should be interpreted with caution since the model may not take into account some important features of banking business like many competing objectives at the same time, longer term perspective, banks' own beliefs about financial and macroeconomic developments or dynamic regulatory framework of banking market.

The proposed tool and its applications have some appealing policy implication. This flexible tool provides insight into the way banks may optimally respond to the changing market conditions. It gives consistent results both with the general understanding of the developments in the banking sector (assessment of various analysts) and investment portfolio theory relating asset allocation with the dynamics of the risk / reward relationship. Static balance sheet assumption in the stress testing exercises can be relaxed with this tool. Usually, in context of the top-down stress tests, analysts apply some pre-defined changes to the balance sheets of the analysed banks. The presented approach allows for an application of the well established optimal portfolio theory in modeling banks' reactions to stressful market conditions.

The outcomes of the model can serve as a cross-checking of other models' predictions about credit supply in economies affected by the adverse macroeconomic or financial shocks. Consequently, it may positively contribute to understanding of

- lending activities of banks in the crisis situation; The model gives the micro-foundation perspective of the credit expansion which is usually absent in the credit supply models. Therefore, the proposed modelling framework potentially improves the policy responses to the crises situation on the market for credit;

- the strength of the credit channel of the monetary policy transmission mechanism.

One natural extension of the model could be the fully dynamic model where[29]:

- decisions are made sequentially in the given investment horizon (possibly infinite) and already take into account their potential consequences in all the following periods;

- banks face the risk of bankruptcy meaning that the shareholders are deprived of a part of the future divided flow;

- (most difficult to implement) the level of the loan and deposit margins are at the banks' discretion, i.e. banks can actively manage margins to attract funding and boost income earned on loan portfolio. Any, even the simplest but solvable implementation of the feedback effects would be a great value added to such a modeling framework since by making the parameters of the model dependent on the bank strategies one would obtain extremely nonlinear problem, with potential amplifying feedback loops.

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[29]It could be formulated in a dynamic mean-variance portfolio setting studied by eg. Leippold et al. (2011)

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