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A MODEL OF BORROWER REPUTATION AS INTANGIBLE COLLATERAL

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Abstract

In this paper, we build a Kiyotaki-Moore style collateral amplification framework which generates large endogenous fluctuations in the leverage available to investing firms. We assume that defaulting borrowers lose not only their tangible collateral but also their future debt market access. The possibility of such market exclusion can lead to the emergence of intangible collateral in equilibrium alongside the tangible collateral which is usually studied in the literature. Fluctuations in the value of intangible collateral are isomorphic to fluctuations in the downpayments they need to make in their purchases of productive assets. This modification of the Kiyotaki-Moore model substantially increases its amplification of exogenous shocks.

JEL Classification: E44.

Key Words: Collateral constraints, Aggregate fluctuations.

Non-technical summary

This paper builds a model of the leverage cycle in which endogenous fluctuations in down-payment requirements on capital/housing purchases generate substantial amplification of small exogenous shocks. The financial boom and bust cycle of 2005-2009 was characterised by a big increase and subsequent fall in the permissible leverage for many sectors of the economy, most notably households and financial institutions. This led to substantial asset price and output volatility, raising questions about the linkages between financial conditions, asset prices and real quantities during the financial crisis.

The standard collateral amplification mechanism (Kiyotaki and Moore (1997)) relies on the interaction of asset prices and credit constraints in order to generate volatility in investment and real activity. When debt access depends on collateral values, high asset prices relax credit constraints and allow constrained firms or households to increase their expenditure.

The main drawback of the standard approach is that it assumes that down-payment requirements on leveraged durable or financial asset purchases are exogenous. In contrast, as Geannakoplos (2009), Gorton (2010) and many others have argued, the recent crisis has been at least in part driven by higher 'haircuts' on securities in repo contracts and on increasing down-payment requirements on mortgage loans. Motivated by the empirical evidence on the leverage cycle, this research introduces endogenous variation in down-payments into an otherwise standard business cycle model with credit constraints and examines how this additional source of leverage fluctuations affects output volatility.

I introduce variations in down-payment requirements by appealing to the value of a borrower's clean repayment record. When defaulting borrowers can be excluded from debt markets, this makes a clean repayment record a valuable intangible asset which can be pledged to creditors alongside real or financial assets.

In booms, the value of debt market access increases and this makes it more costly for borrowers to default because this would result in debt market exclusion. The higher cost of default then allows borrowers to secure more debt relative to the value of their tangible assets (e.g. houses or securities). In other words, down-payment requirements decline and leverage rises. In recessions, this process goes into reverse, down-payment requirements increase and leverage falls.

The leverage cycle mechanism identified in our paper substantially increases the volatility of the economy in response to exogenous shocks. This amplification is due to the interaction of the intangible collateral mechanism identified in this paper with the traditional tangible collateral mechanism of Kiyotaki and Moore (1997). When down-payment requirements decrease in booms, this boosts asset prices and increases the net worth of leveraged borrowers. This interaction helps to generate more volatility in output and financial prices.

1 Introduction

The financial boom and bust cycle of 2005-2009 was characterised by a substantial increase and subsequent fall in the permissible leverage for all sectors of the economy. Downpayment requirements on housing, capital and financial asset purchases fell during the boom and then increased sharply as the financial crisis unfolded during 2008. At the same time, asset prices and output fell sharply across the world, raising questions about the linkages between financial conditions, asset prices and real quantities during the financial crisis. And while we have a good theoretical understanding of how credit constraints affect the interaction between output and asset prices, there has been comparatively less work on downpayment requirements and other aspects of the financial conditions facing private borrowers.

In this paper, we build a framework which generates fluctuations in downpayment requirements by appealing to changes in the value of borrowers ' reputation for repayment. We build a heterogeneous entrepreneur economy similar to Kiyotaki and Moore (1997) and Kiyotaki (1998). Due to limited commitment, collateral values play an important role in the allocation of productive resources to the best uses. In our environment, however, borrowers cannot keep their anonymity; only savers can. The absence of anonymity in debt markets allows lenders to punish defaulting borrowers by excluding them from future borrowing. We show how the possibility of such market exclusion can lead to the emergence of reputational (intangible) collateral in equilibrium alongside the tangible collateral which is usually studied in the literature.

One of the key contributions of this paper is to show how the financial contract in a model with tangible and intangible collateral can still be represented as a borrowing constraint which is linear in the market value of tangible collateral. A decline in the value of intangible collateral manifests itself in a higher 'haircut' (or downpayment) while a rise in the value of intangible collateral can manifest itself as a lower haircut. This result is useful because it substantially reduces the computational compexity of the model[1].

[1] The borrowing constraint is exactly linear in the steady state or during perfect foresight dynamics. Under uncertainty and risk aversion, the linearity of the borrowing constraint is only true up to an approximation, which is very good unless the degree of risk aversion is very high.

The intangible collateral is the value of a borrower's reputation for debt repayment. We find that this collateral form can back a very significant part of the liabilities of the private sector. In our baseline calibration, the value of intangible collateral fluctuates over the business cycle and generates volatility in debt relative to tangible assets. We find that the values of tangible and intangible collateral interact in ways that amplify business cycle fluctuations. Shocks that increase the value of future debt access permit borrower to obtain higher leverage in their capital purchases. This boosts capital demand and increases capital prices, thus boosting further the net worth of existing capital holders. Hence intangible collateral can substantially increase the power of the traditional tangible collateral amplification mechanism.

The structure of the paper is as follows. Section 2 reviews the related literature. Section 3 presents some data on US LTV ratios on new mortgages as a way of motivating our focus on downpayment requirements on capital good purchases. Section 4 outlines the model used in the paper. Section 5 discusses the baseline calibration of our model economy. Section 6 uses numerical simulations to display the properties of the model. Section 7 concludes.

2 Related Literature

This paper studies the nature of debt contracts in an environment with permanent exclusion from credit markets. There is a large literature on dynamic optimal contracts (DOC) starting with the seminal contributions of Kehoe and Levine (1993) who developed the first general equilibrium model with endogenous borrowing constraints. Subsequently, work by Alvarez and and Jermann (2000) showed how the allocation of Kehoe and Levine (1993) can be decentralised by a set of state contingent borrowing limits in an endowment economy with permanent exclusion from risk sharing arrangements. Subsequent work in this literature (Kehoe and Perri (2002), Krueger and Perri (2006) as well as others) have used the DOC framework to study a number of issues such as international business cycles or consumption inequality. The model in this paper differs from the above papers in two important respects. First of all, we assume incomplete markets (borrowing can only be done using uncontingent debt) whereas most of the DOC literature assumes that there exist a full set of Arrow Debreu securities but agents are (endogenously) quantity constrained in issuing some of them due to limited enforcement. Secondly, the DOC literature assumes multiperiod financial contracts whereas in this paper debt contracts last only one period.

Our paper is also related to the collateral amplification literature started by Kiyotaki and Moore (1997), Kiyotaki (1998) and Bernanke, Gertler and Gilchrist (1999). These papers have shown that when debts are collateralised, leverage magnifies the impact of small shocks on the net worth of producers, thus amplifying and propagating impulses over time. However, work by Cordoba and Ripoll (2004) has shown that a calibrated version of the standard Kiyotaki-Moore model does not generate much amplification of exogenous shocks. In the Cordoba and Ripoll (2004) paper, borrowers can only commit to repay a certain exogenous fraction of collateral values. In contrast, our paper explicitly models the fluctuations in such 'haircuts' (or downpayments) as a function of the value to a borrower of being able to access credit markets in future. This improves the model's ability to generate large output fluctuations due to small exogenous technology shocks.

Hellwig and Lorenzoni (2007) is another interesting paper which studies the possibility that exclusion from debt markets can lead to self-enforcing debt. They study an endowment economy with limited commitment in which there is no collateral to secure borrowing. Because the autrarkic equilibrium is dynamically inefficient, stationary bubbles on intrinsically worthless assets can exist. Hellwing and Lorenzoni show that when private borrowers can be permanently excluded from future credit market access, an equilibrium with bubbles on inside liquidity (private debt) can achieve an idential allocation to the equilibrium with rational bubbles (outside liquidity).

Gertler and Karadi (2010) is closer to this paper in the sense that they model banks' ability to borrow by appealing to the value of excess returns in an equilibrium with no bubbles. Their mechanism is similar to the intangible collateral studied in this paper. In Gertler and Karadi (2010) the bank is threatened with bankruptcy and the loss of the opportunity to operate as a banker. In our model, a defaulting entrepreneur can immediately set up a new firm and continue producing. However, she loses her access to future credit, which is costly because she can no longer lever up to maximise the returns from good business ideas (high productivity spells in the model).[2] Our contribution is also in demonstrating the fact that fluctuations in the value of intangible collateral can be observationally equivalent to fluctuations in downpayment requirements on tangible assets.

We find that counter-cyclical variation in idiosyncratic production risk is a key mechanism behind the counter-cyclical movements in haircuts which amplify the business cycle. Angeletos and Calvet (2006), Perez (2006) and, more recently, Bloom (2009) and Bloom et al (2011) are several papers that examine the importance of idiosyncratic production risk for the business cycle. They share the conclusion that the presence of uninsurable idiosyncratic production risk can have a profound impact on risk-taking and capital accumulation. And if the degree of idiosyncratic production risk varies in a counter-cyclical fashion (i.e. it is higher in recessions), these papers show that this can amplify the business cycle by affecting entrepreneurs' investment into risky projects. In our paper, the focus is mainly on the impact of idiosyncratic production uncertainty on capital good downpayments. High idiosyncratic productivity variability causes the expected return from production (in utility terms) to decline and this reduces the value of borrowing. Consequently borrowers must invest with less leverage reflecting their reduced incentives to protect their clearn repayment record. Because production uncertainty increases in recessions, this channel is capable of producing counter-cyclical downpayment requirements, which work as an additional amplification factor.

3 Motivating Observations

There is a lot of evidence that permissible leverage fluctuates very substantially for many private borrowers over the business cycle. Figure 1 below shows the movement of the monthly LTV ratio for new home buyers in the US. This ratio has a slight upward trend over time and displays considerable cyclical movements.

[2] In addition, our paper makes the technical contribution of generalising the dynamic contracting framework to an environment of risk-averse consumer-producers while still retaining the tractability of the linear borrowing constraints of the Kiyotaki-Moore (1997) model.

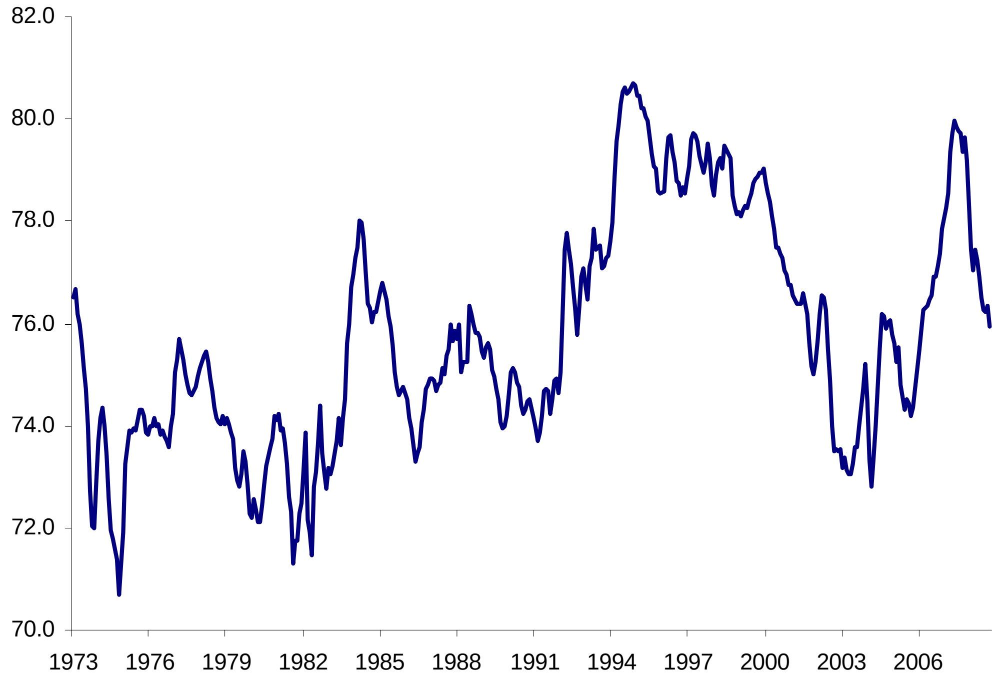

Figure 1: Loan to Value Ratios in the US: 1973-2008 (Source: FHFA)

Figure 2 below examines the same data but at the annual frequency and after applying the HP-filter. For comparison we also plot the cyclical component of GDP. The cyclical component of LTVs has an annual standard deviation of 1.35; it is also correlated with the cyclical component of GDP (the correlation coefficient is 0.55). In this paper we build a model in which tangible and intangible collateral interact to generate time varying and pro-cyclical LTV ratios similar to those found in the data.

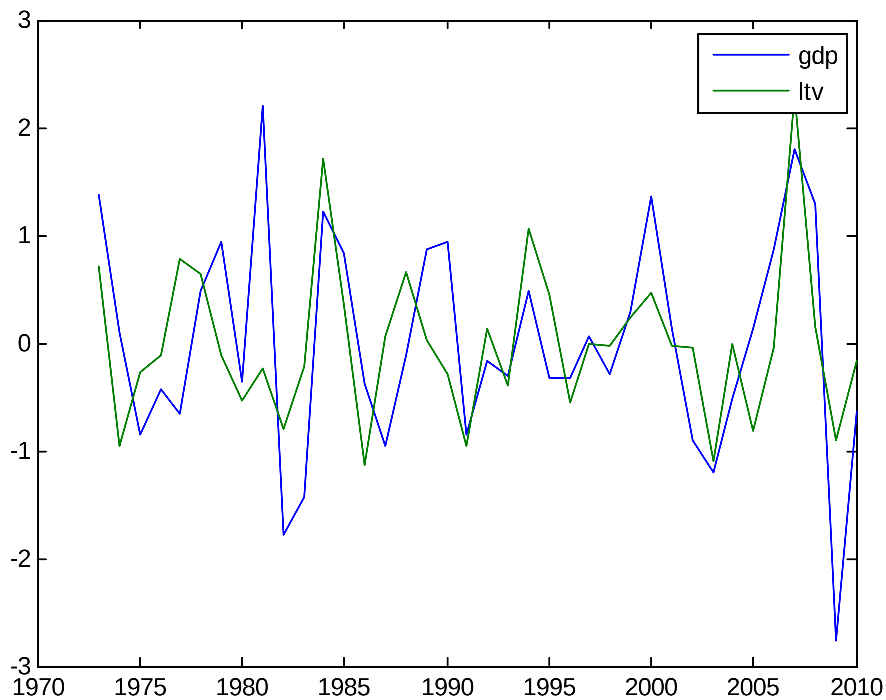

Figure 2: HP-filtered LTV and GDP

4 The Model

4.1 The Economic Environment

The economy is populated with a continuum of infinitely lived entrepreneurs of measure 1 and a continuum of infinitely lived workers also of measure 1. Each entrepreneur is endowed with a constant returns to scale production function which uses land, labour and capital to produce output y.

$$y_{t+1} = a_{t+1}A_{t+1}\left(\frac{l_t}{\alpha}\right)^{\alpha}\left(\frac{k_t}{\eta}\right)^{\eta}\left(\frac{h_t}{1-\alpha-\eta}\right)^{1-\alpha-\eta}$$

l is land (which does not depreciate and is fixed in aggregate supply), k is capital which depreciates at the rate of $1-\gamma$ and h is labour.

a_t is the idiyosyncratic component of productivity which differs between firms both in terms of its **ex ante** expected value at the time of investment as well as in its **ex post** realised value at the time of actual production. A fraction of firms who we will refer to as 'high productivity' firms have a high idiosyncratic expected productivity:

$$E_t a_{t+1} = a^H > 1$$

The other firms have a low expected idiosyncratic productivity level ($a^L \equiv 1$).

Both types of firms face 'ex post' idiosyncratic risk too. If they are lucky, ex post idiosyncratic productivity is high

$$a_{t+1} = a^i \varepsilon^H (A_{t+1}), i = H, L$$

which happens with probability 0.5 and, if they are unlucky, ex post idiosyncratic productivity is low

$$a_{t+1} = a^i \varepsilon^L (A_{t+1}), i = H, L$$

which happens with probability 0.5. In order to match the empirical evidence in Bloom et al (2011) we will allow the variance of the ex post idiosyncratic productivity shock to co-move negatively with the aggregate state of the economy as measured by aggregate TFP A_t. This means that $\varepsilon^H \left(A^L\right) > \varepsilon^H \left(A^H\right)$ and $\varepsilon^L \left(A^L\right) < \varepsilon^L \left(A^H\right)$. In other words, the ex post TFP shocks become more volatile in recessions.

The ex ante component of idiosyncratic productivity evolves according to a Markov process. Following Kiyotaki (1998) let $n\delta$ be the probability that a currently unproductive firm becomes productive and let δ be the probability that a currently productive firm becomes unproductive. This implies that the steady state ratio of productive to unproductive firms is n. A is the aggregate component of productivity (which also can be high A^H or low A^L). The aggregate state also evolves according to a persistent Markov process.

Workers do not have productive opportunities and instead only supply labour in the model.

4.2 Entrepreneurs

4.2.1 Preferences

Entrepreneurs are ex-ante identical and have logarithmic utility over consumption streams

$$U^e = E_0 \left[\Sigma_{t=0}^{\infty} \beta^t \ln c_t \right]$$

4.2.2 Flow of Funds

Agents purchase consumption (c_t), capital goods (k_t), land (l_t) at price q_t pay wages to labour ($w_t h_t$), and borrow using debt securities b_t at risk free interest rate R_t[3].

$$c_t + q_t l_t + k_t + w_t h_t - \frac{b_t}{R_t} \leqslant y_t + q_t l_{t-1} + \gamma k_{t-1} - b_{t-1} \equiv z_t$$

We assume incomplete markets for idiosyncratic risk, meaning that Arrow securities contingent on the idiosyncratic state will not trade in equilibrium.

4.2.3 Collateral constraints

Due to moral hazard in the credit market, agents will only honour their promises if it is in their interests to do so. We assume that an entrepreneur who borrows funds at time t has the ability to default at $t+1$. Following Kiyotaki and Moore (1997) we assume that lenders can seize the entrepreneur's land and capital holding which has value $q_{t+1} l_t + \gamma k_t$ as well as a potentially time-varying fraction ϕ_t of the firm's revenues y_{t+1}. The entrepreneur keeps the rest of the firm's output ($1 - \phi_t$ fraction). Furthermore, we assume that, upon default, entrepreneurs are permanently excluded from future borrowing. However, they can anonymously lend to other entrepreneurs or produce without any leverage.

Individuals will repay their debts whenever the value of repaying exceeds the value of defaulting. Let $V(s_t, X_t)$ denote the value of an entrepreneur who has never defaulted and let $V^d(s_t, X_t)$ denote the value of an entpreneur who has defaulted in the past. $s_t \equiv (z_t, a^i)$ is the idiosyncratic state where z_t is individual wealth and a^i is the expected idiosyncratic level of TFP. X_t is a vector of aggregate state variables which will be described later. We

[3] We focus on a no-default allocation so that debts are risk-free.

focus on a no-default allocation. Following Kiyotaki and Moore (1997) and Kiyotaki (1998) we assume that default (or debt renegotiation) can only occur before the realisation of the aggregate or the idiosyncratic shock at time $t+1$. So the borrowing constraint is cast in terms of expected values:

$$E_t V\left(s_{t+1}, X_{t+1} | s_t, X_t\right) \geqslant E_t V^d\left(s_{t+1}, X_{t+1} | s_t, X_t\right)$$

At this stage we conjecture that this value function comparison can be reduced to a linear collateral constraint of the following form.

$$b_{t+1} \leqslant E_t\left[\theta_t y_{t+1} + q_{t+1} l_t + \gamma k_t\right]$$

The value of intangible collateral is equal to the amount of borrowing unbacked by tangible assets which can be seized by the lender

$$\left(\theta_t - \phi_t\right) E_t y_{t+1}$$

We verify subsequently that this is indeed the case.

4.3 Entrepreneurial behaviour

The entrepreneurs in our economy have to make two types of decisions. They have to choose consumption over time optimally (the consumption problem) and they have to choose the (real and financial) assets they invest in (the portfolio problem). Fortunately, the budget constraint is linear in all the assets at the entrepreneur's disposal and as a result we can utilise the result due to Samuelson (1968), which states that we can solve separately the consumption and portfolio decisions.

4.3.1 The consumption problem

Due to logarithmic utility, consumption is a fixed fraction of wealth at each point in time for all entrepreneurs regardless of their level of idiosyncratic productivity. This result is proved in Appendix A and it greatly simplifies the aggregation of consumption decisions.

$$c_t = (1-\beta) z_t$$

4.3.2 The production/portfolio problem

Entrepreneurs choose their holdings of three assets (land, capital and debt) as well as their labour input into production under the presence of a collateral constraint. The first order conditions for the three assets and labour input are given below.

The first order condition for land is:

$$-\lambda_t q_t + \beta E_t \left[\frac{\alpha y_{t+1}}{l_t} + q_{t+1} \right] \lambda_{t+1} + \mu_t E_t \left[\theta_t \frac{\alpha y_{t+1}}{l_t} + q_{t+1} \right] = 0 \tag{1}$$

where $\lambda_t = 1/c_t$ is the lagrange multiplier on the flow of funds constraint while μ_t is the lagrange multiplier on the collateral constraint. The first order condition for capital investment is:

$$-\lambda_t + \beta E_t \left[\frac{\eta y_{t+1}}{k_t} + \gamma \right] \lambda_{t+1} + \mu_t E_t \left[\theta_t \frac{\eta y_{t+1}}{k_t} + \gamma \right] = 0 \tag{2}$$

The first order condition for labour demand is:

$$-\lambda_t w_t + \beta E_t \left(\frac{(1 - \alpha - \eta)\, y_{t+1}}{h_t} \lambda_{t+1} \right) + \mu_t E_t \left(\theta_t \frac{(1 - \alpha - \eta)\, y_{t+1}}{h_t} \right) = 0 \tag{3}$$

Finally the first order condition for debt holdings is:

$$-\frac{\lambda_t}{R_t} + \beta E_t \lambda_{t+1} + \mu_t = 0 \tag{4}$$

Combining (1), (2) and (4) we get an expression for the optimal mix between land and reproducible capital:

$$\frac{l_t}{k_t} = \frac{\alpha}{\eta} \frac{u_t^{k,i}}{u_t^{l,i}}, i = L, H \tag{5}$$

where $u_t^{k,i}$ and $u_t^{l,i}$ are, respectively, the user cost of capital and land for the two types of entrepreneurs.

The user cost of capital is the same for the two groups of entrepreneurs because the price of capital is non-stochastic

$$u_t^{k,H} = u_t^{l,L} = 1 - \frac{\gamma}{R_t} \tag{6}$$

In the case of the user cost of land, fluctuations in the value of land creates some differences in the user cost for constrained and unconstrained entrepreneurs.

$$u_t^{l,H} = q_t - \frac{E_t q_{t+1}}{R_t} - E_t \left((q_{t+1} - E_t q_{t+1}) \frac{\lambda_{t+1}^H}{\lambda_t^H} \right) \tag{7}$$

is the user cost of land for high productivity entrepreneurs for whom borrowing constraints bind and $\mu_t > 0$.

$$u_t^{l,L} = q_t - E_t\left(q_{t+1}\frac{\lambda_{t+1}^L}{\lambda_t^L}\right) \tag{8}$$

is the user cost of land for low productivity entrepreneurs who are unconstrained. λ_t^H and λ_t^L are, respectively, the shadow values of funds for high and low productivity entrepreneurs. Finally, the optimal mix between land and labour is given by:

$$\frac{l_t}{h_t} = \frac{\alpha}{1-\alpha-\eta}\frac{w_t}{u_t^{l,i}}, i = L, H \tag{9}$$

Let

$$\nu_t^i = \left(u_t^{l,i}\right)^\alpha \left(u_t^k\right)^\eta w_t^{1-\alpha-\eta}, i = L, H$$

be the unit cost of investment for entrepreneur of type i (which can be high or low productivity). This cost depends on the user cost of land, the user cost of capital and the cost of labour. Then the ex post rate of return on production for the two types of entrepreneurs is given by:

$$\frac{\varepsilon_{t+1}a^i A_{t+1}}{\nu_t^i}$$

When the value of a unit of productive investment for high productivity entrepreneur exceeds the value of a unit of safe debt, borrowing constraints bind and productive agents borrow up to the limit.

$$E_t\left(\frac{\varepsilon_{t+1}a^H A_{t+1}}{\nu_t^H}\lambda_{t+1}^H\right) > R_t E_t\left(\lambda_{t+1}^H\right)$$

Low productivity entrepreneurs are active in production when credit constraints are sufficiently tight and the following condition is satisfied:

$$E_t\left(\frac{\varepsilon_{t+1}a^L A_{t+1}}{\nu_t^L}\lambda_{t+1}^L\right) = R_t E_t\left(\lambda_{t+1}^L\right)$$

4.4 Borrowing limit determination

Our economy is a limited commitment one. Borrowers repay their debts only if it is in their interests to do so. Upon default, a borrower loses his tangible assets as well as his reputation for repayment. This results in permanent exclusion from debt markets in future. As we now show, entrepreneurs will be allowed to borrow up to the value of the tangible and intangible assets they can lose when they default.

4.4.1 The value of a non-defaulting entrepreneur

Let $V(s_t, X_t)$ be the value of a non-defaulting entrepreneur with idiosyncratic state s_t when the aggregate state is X_t.

$$V(s_t, X_t) = \max_{c_t, k_t, h_t, l_t, b_t} \{\ln c_t + \beta E_t V(s_{t+1}, X_{t+1})\}$$

In Appendix B we show that the value function takes the following form

$$V(s_t, X_t) = \varphi(s_t, X_t) + \frac{\ln z_t}{1-\beta}$$

where the intercept $\varphi(s_t, X_t)$ satisfies a functional equation:

$$\varphi(s_t, X_t) = \ln(1-\beta) + \max_{k_t, h_t, l_t, b_t} \beta E_t \left[\frac{\ln \beta}{1-\beta} + \frac{\ln r^i_{t+1}}{1-\beta} + \varphi(s_{t+1}, X_{t+1}) \right] \tag{10}$$

Intuitively, the value of an entrepreneur depends on his current wealth (this is the term in $\ln z_t$) as well as the rates of return the entrepreneur can earn on his wealth in future (this is the $\varphi(s_t, X_t)$ term). Looking at (10) we can see that, if the rate of return on wealth is equal to the inverse of the rate of time preference at all times ($r^i = 1/\beta$), the intercept $\varphi(s_t, X_t)$ will be equal to $\ln(1-\beta)/(1-\beta)$ and the value of an entrepreneur will be solely determined by his current wealth. In contrast, values of r^i above $1/\beta$ would generate a positive value of φ reflecting the net present value of 'excess returns' to the entrepreneur.

When the borrowing constraint binds, high productivity entrepreneurs borrow up to the limit in order to invest in productive projects. The leveraged rate of return on wealth for a high productivity entrepreneur is given by the following expression:

$$r^H_{t+1} = \frac{(\varepsilon_{t+1} A_{t+1} - \theta_t E_t A_{t+1}) a^H + \left(\alpha \nu_t / u_t^{l,H}\right)(q_{t+1} - E_t q_{t+1})}{\nu_t - \theta_t E_t A_{t+1} a^H / R_t} \tag{11}$$

Equation (11) allows us to trace the effects of leverage on the rate of return on wealth. The denominator of (11) gives us the downpayment that entrepreneurs need to make on their investments. The unit cost of investment is ν_t but the entrepreneur can finance part of this by borrowing $\theta_t E_t A_{t+1} a^H / R_t$ against future revenues. When θ_t increases, this downpayment declines at the expense of a lower numerator - the value of post debt repayment revenues in the following period. When the expected rate of return on production exceeds the interest rate on debt, more leverage increases the rate of return on wealth.

Low productivity entrepreneurs invest in production (without leverage) as well as in loans to other entrepreneurs. In equilibrium, they earn a rate of return on wealth which is given by:

$$r_{t+1}^L = \frac{Y_{t+1}^L + q_{t+1}(1 - L_t) + \gamma K_t^L + B_t}{w_t H_t^L + q_t(1 - L_t) + K_t^L + B_t/R_t} \tag{12}$$

where Z_t^L and Y_t^L, and $1 - L_t$ are, respectively, the aggregate wealth, output, and land investments of low productivity workers. K_t^L and H_t^L are the capital and employment choices of low productivity firms[4].

4.4.2 The value of a defaulting entrepreneur

Let $V^d(s_t, X_t)$ denote the value of an entrepreneur who has defaulted in the past. This is given as follows:

$$V^d(s_t, X_t) = \varphi^d(s_t, X_t) + \frac{\ln z_t}{1 - \beta}$$

The intercept of the value function satisfies the now familiar functional equation:

$$\varphi^d(s_t, X_t) = \ln(1 - \beta) + \max_{k_t, h_t, l_t, b_t} \beta E_t \left[\frac{\ln \beta + \ln r_{t+1}^{di}}{1 - \beta} + \varphi^d(s_{t+1}, X_{t+1}) \right]$$

where $r_{t+1}^{di}$is the return on wealth for an entrepreneur who is in productivity state $i = H, L$ (high or low productivity) and who is excluded from debt markets because he has defaulted in the past. When such an entrepreneur has low productivity, he has the same portfolio as other low productivity entrepreneurs with a clean repayment record. This is because low productivity agents do not use leverage.

When the defaulting entrepreneur is in a high productivity state, he cannot use leverage

[4]Since low productivity agents are indifferent between producing and lending to others, the structure of their portfolios is determined by market clearing in the market for land, labour and goods.

and must self-finance. This implies that he faces a higher user cost of land equal to:

$$u_t^{l,d} = q_t - E_t \left\{ q_{t+1} \frac{\lambda_{t+1}^{H,d}}{\lambda_t} \right\} > u_t^{l,H}$$

and a higher user cost of capital equal to:

$$u_t^{k,d} = 1 - \gamma E_t \left(\frac{\lambda_{t+1}^{H,d}}{\lambda_t} \right) > u_t^{k,H}$$

In the absence of borrowing opportunities, the defaulting entrepreneur faces a shadow cost of funds equal to his own valuation of future wealth. This will tend to be higher compared to those who have some access to debt markets because under a binding collateral constraint, high productivity agents value future wealth less than the market price. The high user cost of land and capital investments implies that defaulted high productivity entrepreneurs will economise on land and capital investments and their input mix will be heavily skewed towards labour. Such a distortion of the input mix reduces the rate of return on productive investments

$$r_{t+1}^{d,H} = \frac{\varepsilon_{t+1} A_{t+1} a^H}{\nu_t^{d,H}} \tag{13}$$

below the rate of return enjoyed by those with full debt access who can finance much of their land and capital purchases with borrowed funds. This is mainly because the lack of borrowing capacity increases production costs as captured by

$$\nu_t^{d,H} = \left(u_t^{l,d} \right)^{\alpha} \left(u_t^{k,d} \right)^{\eta} w_t^{1-\alpha-\eta} > \nu_t^H$$

But debt exclusion also carries the additional cost because $\theta_t = 0$ for defaulting borrowers. To see why this is costly, recall the discussion of the effect of θ_t on equation (11). As long as productive projects yield more than the interest rate on debt, being able to borrow against future revenues in order to minimise the downpayment in the current period boosts the rate of return on invested wealth. At $\theta_t = 0$, $E_t r_{t+1}^{d,H}$ is still higher than the risk free rate but much less so compared to $E_t r_{t+1}^H$.

The reduction in the expected rate of return on wealth as a result of bankruptcy and debt exclusion is reflected in the intercept term of the value function $\varphi^d (s_t, X_t)$. Lower future rates of return lead to a large fall in this term. This is the cost (in terms of the entrepreneur's value function) of debt exclusion.

The benefit from default can be seen from the value function. If state (s_{t+1}, X_{t+1}) realises following her decision to default, the entrepreneur's value will be given by:

$$\begin{aligned} V^d(s_{t+1}, X_{t+1}) &= \varphi^d(s_{t+1}, X_{t+1}) + \frac{\ln z^d_{t+1}}{1-\beta} \\ &= \varphi^d(s_{t+1}, X_{t+1}) + \frac{\ln[(1-\phi_t) y_{t+1}]}{1-\beta} \end{aligned}$$

The wealth of a defaulting entrepreneur is the $1-\phi_t$ fraction of output she gets to keep post default. This is higher than the wealth she would have had under repayment, because the defaulting entrepreneur gains wealth equal to $(\theta_t - \phi_t) y_{t+1}$ by avoiding repayments on the debt secured by intangible collateral. So default carries immediate benefits in terms of higher current wealth but costs in terms of lower future returns on wealth.

4.4.3 Solving for the borrowing limits

Alvarez and Jermann (2000) solve for borrowing limits which are 'not too tight' as the highest possible borrowing limit consistent with repayment. In our setting this is given by the incentive compatibility constraint which equates the expected value of repayment with the expected value of defaulting.

$$E_t V(s_{t+1}, X_{t+1}) = E_t V^d\left(s^d_{t+1}, X_{t+1}\right)$$

The expectation operator is taken with respect to the distribution of the aggregate as well as the idiosyncratic productivity shocks hitting the firm. This implies that the expected loss of reputation due to default (LHS of the expression below) exactly offsets the one-off gain from having one's unsecured debt written off (the RHS of the expression below).

$$\begin{aligned} & (1-\beta) E_t\left[\left(\varphi(s_{t+1}, X_{t+1}) - \varphi^d(s_{t+1}, X_{t+1})\right)\right] \\ \geqslant\ & E_t \ln[(1-\phi_t) y_{t+1}] - E_t \ln[y_{t+1} + q_{t+1}k_{t+1} - b_{t+1}] \end{aligned}$$

Using the approximation:

$$E \ln x \approx \ln Ex - \frac{1}{2} var(\ln x)$$

we get:

$$\begin{aligned} & (1-\beta) E_t\left[\left(\varphi(s_{t+1}, X_{t+1}) - \varphi^d(s_{t+1}, X_{t+1})\right)\right] - \Omega_t \\ \geqslant\ & \ln E_t[(1-\phi) y_{t+1}] - \ln E_t[y_{t+1} + q_{t+1}k_{t+1} - b_{t+1}] \end{aligned}$$

where

$$\Omega_t = \frac{1}{2}\left\{var_t(\ln\left[y_{t+1} + q_{t+1}k_{t+1} - b_{t+1}\right]) - var_t(\ln\left[\left(1-\phi_t\right)y_{t+1}\right])\right\}$$

is an approximate risk premium term which reflects the greater ex post wealth variability for repaying entrepreneurs. Re-arranging we have:

$$b_{t+1} \leqslant \left\{\frac{\Delta\left(s_{t+1}, X_{t+1}|\phi_t\right) + \phi_t - 1}{\Delta\left(s_{t+1}, X_{t+1}|\phi_t\right)}\right\} y_{t+1} + q_{t+1}k_{t+1} \tag{14}$$

where

$$\Delta\left(s_{t+1}, X_{t+1}|\phi_t\right) \equiv \exp\left\{\left(1-\beta\right)E_t\left(\varphi\left(s_{t+1}, X_{t+1}|\theta\right) - \varphi^d\left(s_{t+1}, X_{t+1}\right)\right) - \Omega_t\right\}$$

Solving for the borrowing constraints requires us to solve for the value function and for the borrowing constraints until both have converged. See Appendix B for further details on the computational procedure.

4.4.4 Discussion

The entrepreneur's borrowing limit is determined by the trade off between the benefits of gaining some current wealth by defaulting against the costs of permanently losing the ability to borrow. The benefit from defaulting is determined by the size of unsecured borrowing - $\left(\theta_t - \phi_t\right)y_{t+1}$. The costs are dominated by the gap between the expected value of being a non-defaulting entrepreneur $\left(E_t\varphi\left(s_{t+1}, X_{t+1}|\theta_t\right)\right)$ and the value of defaulting $\left(E_t\varphi^d\left(s_{t+1}, X_{t+1}\right)\right)$. This gap is driven by the utility value of the entrepreneur's stream of excess returns relative to current financial wealth.

Because most of these excess returns are in the future, the discount factor is one of the main determinants of the value of repayment. A discount factor of 0.95 implies that the entrepreneur is indifferent between a 1pp increase in his rate of return on wealth in perpetuity and a 19% increase in his current financial wealth. With a discount factor of 0.9, the consumer is only willing to accept a 9.5% increase in current wealth in exchange for a 1pp increase in returns.

Another crucial determinant of the size of intangible collateral is the probability of remaining highly productive. If this probability is high, then debt access is valuable because a borrower is likely to remain productive for some time and would like therefore to keep borrowing in order to boost his return on wealth. In an environment with persistent investment opportunities, intangible collateral is high and entrepreneurs have a much higher borrowing capacity than the value of their tangible assets alone.

Finally, the value of intangible collateral depends crucially on the expected excess return of productive projects over the risk free rate.

$$\frac{E_t A_{t+1} a^H}{\nu_t^H} / R_t$$

The higher this excess return, the costlier it is for agents to be excluded from future debt market access. The level of the excess return in turn depends on the tightness of borrowing constraints. In economies in which borrowing constraints do not bind, this excess return will be small and the value of intangible collateral will be small. We return to this issue is section 6 below.

4.5 Workers

Workers are identical and maximise the following preferences

$$U^W = E_t \sum_{t=0}^{\infty} \beta^t \left(c_t - \chi \frac{h_t^{1+1/\omega}}{1+1/\omega} \right)$$

which are linear in consumption and convex in hours worked. The workers' budget constraint is given by:

$$c_t + \frac{b_t}{R_t} = b_{t-1} + w_t h_t$$

In addition, workers are also subject to limited commitment and cannot borrow unless the debt is backed by collateral. Because workers have no productive opportunities this implies they cannot borrow:

$$b_t \geqslant 0$$

Labour supply is given by

$$h_t = (1/\chi)\, w_t^{\omega}$$

Due to linear preferences in consumption, consumers choose to set savings equal to zero whenever $R_t < \beta^{-1}$. We will verify that this is indeed the case throughout any numerical simulation of our model. Since workers do not save, they consume their entire wage income in every period.

4.6 Market clearing

There are four market clearing conditions in our model economy - the debt market, the land market, the labour market and the goods market.

In the debt market, debts sum up to zero in the aggregate

$$\int b_t^i di = 0 \tag{15}$$

In the land market, land purchases sum up to the aggregate land stock (normalised to unity).

$$\int l_t^i di = \overline{L} \equiv 1 \tag{16}$$

In the labour market, labour demand equals labour supply:

$$\int h_t^i di = (1/\chi)\, w_t^{\omega}$$

In the goods market, consumption and investment equal total output.

$$\int c_t^i di + \int \left(k_t^i - \gamma k_{t-1}^i\right) di = \int y_t^i di \tag{17}$$

4.7 Behaviour of the aggregate economy

Due to the presence of binding borrowing constraints, high and low productivity entrepreneurs have different demands for assets at a given level of wealth. High productivity agents prefer to invest in production in order to take advantage of high productivity. Low productivity agents have a more balanced portfolio - they invest in production too but also lend funds to the high productivity entrepreneurs through the debt market. This implies that the wealth distribution does matter for equilibrium. But even though the individual decision rules differ according to idiosyncratic productivity, these decision rules remain linear in wealth which means that a within-groups aggregation result obtains. The economy behaves as if it is populated by two agents (a high productivity and a low productivity one). Following Kiyotaki and Moore (1997), we can concentrate on just two moments of the wealth distribution - the mean of the wealth distribution Z_t and the share of wealth owned by high-productivity agents d_t.

At any given date, the state of the aggregate economy can be summarised by the state vector

$$X_t = \{A_t, Z_t, d_t\}$$

consisting of the level of aggregate productivity, the level of aggregate wealth and the share of aggregate wealth held by productive agents. A_t evolves according to an exogenous two state Markov process while the evolution of the two state variables Z_t and d_t is governed by the following relations.

$$Z_{t+1} = \beta \left[d_t r^H_{t+1} + (1 - d_t) r^L_{t+1} \right] Z_t \tag{18}$$

$$d_{t+1} = \frac{(1-\delta) d_t r^H_{t+1} + n\delta (1 - d_t) r^L_{t+1}}{d_t r^H_{t+1} + (1 - d_t) r^L_{t+1}} \tag{19}$$

where r^H_{t+1} and r^L_{t+1} are the rates of return on wealth of, respectively, high productivity and low productivity agents.

In equilibrium, productive agents' wealth grows at rate

$$r^H_{t+1} = \frac{(\varepsilon_{t+1} A_{t+1} - \theta_t E_t A_{t+1}) a^H + \left(\alpha \nu_t / u^{l,H}_t\right) (q_{t+1} - E_t q_{t+1})}{\nu_t - \theta_t E_t (A_{t+1}) a^H / R_t} \tag{20}$$

while for unproductive agents ' wealth grows at rate

$$r^L_{t+1} = \frac{Y^L_{t+1} + q_{t+1} (1 - L_t) + \gamma K^L_t + B_t}{w_t H^L_t + q_t (1 - L_t) + K^L_t + B_t / R_t} \tag{21}$$

where L_t is the aggregate land holding of high productivity agents. The stock of debt B_t is given by aggregating individual borrowing constraints:

$$B_t = E_t \left(q_{t+1} L_t + \gamma K^H_t + \theta_t Y^H_{t+1} \right) \tag{22}$$

K^i_t and H^i_t $(i = H, L)$ are the aggregate demands for capital and labour by high and low productivity firms. L_t, K^H_t and H^H_t are determined by (5) and (9) and by the aggregate budget constraints of productive agents, together with the borrowing constraint (22) and the aggregate consumption function:

$$\left(q_t - \frac{E_t q_{t+1}}{R_t} \right) L_t + \left(1 - \frac{\gamma}{R_t} \right) K^H_t + w_t H^H_t = \beta d_t Z_t + \theta_t E_t Y^H_{t+1}$$

Due to log utility, individual and aggregate consumption are linear in individual and aggregate wealth. Hence goods market clearing implies:

$$(1-\beta) Z_t + w_t \left(H_t^H + H_t^L\right) + K_t^H + K_t^L - \gamma \left(K_{t-1}^H + K_{t-1}^L\right) = Y_t^H + Y_t^L$$

Labour market clearing implies that:

$$H_t^H + H_t^L = (1/\chi)\, w_t^\omega$$

4.8 Competitive equilibrium

Recursive competitive equilibrium of our model economy is a price system $u_t^{l,H}$, $u_t^{l,L}$, u_t^k, w_t, q_t, R_t, $u_t^{l,d}$, $u_t^{k,d}$ household decision rules l_t^i, k_t^i, h_t^i, b_t^i and c_t^i, $i = H, L$, value functions φ_t^H, φ_t^L, $\varphi_t^{d,H}$ and $\varphi_t^{d,L}$, borrowing limits θ_t and equilibrium laws of motion for the endogenous state variables (18) and (18) such that

(i) The value functions φ_t^H, φ_t^L, $\varphi_t^{d,H}$ and $\varphi_t^{d,L}$ describe the maximum life time utility of agents conditional upon the aggregate and idiosyncratic state as well as the default histories.

(ii) The decision rules l_t^i, k_t^i, h_t^i, b_t^i and c_t^i, $i = H, L$ solve the household decision problem conditional upon the price system $u_t^{l,H}$, $u_t^{l,L}$, u_t^k, w_t, q_t, R_t, $u_t^{l,d}$, $u_t^{k,d}$ and the borrowing limits θ_t.

(iii) The process governing the transition of the aggregate productivity and the household decision rules l_t^i, k_t^i, h_t^i, b_t^i and c_t^i $i = H, L$ induce a transition process for the aggregate state s given by (18) and (19).

(iv.) The borrowing limits θ_t are consistent with no default on the equilibrium path conditional upon the value functions φ_t^H, φ_t^L, $\varphi_t^{d,H}$ and $\varphi_t^{d,L}$.

(v) All markets clear

5 Calibration

We calibrate our model economy as follows. We set α, the share of land in national income, equal to 0.1 in line with Davis and Heathcote (2004). The total share of tangible assets (capital and land) in national income is set to 0.36 and hence η, the share of reproducible capital in output, is equal to 0.26. For the baseline calibration, I set ϕ, the percentage of output that can be seized in the event of default, to zero. So any collateralisability of output in the steady state is due to the value of intangible collateral.

Calibrating the cross-sectional dispersion of TFP is important because the quantitative importance of the intangible collateral studied in this paper depends crucially on the differences in entrepreneurs' expected productivity levels as well as the risks that arise due to idiosyncratic ex post production risk.

Bernard et al. (2003) report an enormous cross-sectional variance of plant level value added per worker using data from the 1992 US Census of Manufactures. The standard deviation of the log of value added per worker is 0.75 in the data while their model is able to account for only around half this number. The authors argue that imperfect competition and data measurement issues can account for much of this discrepancy between model and data. In addition, the study assumes fixed labour share across plants so any departures from this assumption would lead to more variations in the measured dispersion of labour productivity.

In a comprehensive review article on the literature on cross-sectional productivity differences, Syverson (2009) documents that the top decile of firms has a level of TFP which is almost twice as high as the bottom decile. Finally, Bloom (2009) and Bloom et al (2011) document that the dispersion of plant level TFP displays a clear counter-cyclical pattern - dispersion is almost twice as high in recessions as it is in booms.

In our model, dispersion in firm level TFP arises due to the persistent ex ante component (which can take high or low values) as well as the ex post production uncertainty which is purely transitory. We calibrate both of these productivity components in order to match the empirical evidence reviewed above.

We choose the dispersion of ex post production risk shocks (ε) to match the evidence in Bloom et al (2011). In booms, $\varepsilon^H(A^H) = 1.1$ and $\varepsilon^L(A^H) = 0.9$ while in recession $\varepsilon^H(A^L) = 1.2$ and $\varepsilon^L(A^L) = 0.8$. The ex ante component of production is set in line with Aoki et al. (2009) who argue that $a^H/a^L = 1.15$ is broadly consistent with the empirical evidence. This productivity process implies that in recessions the highest value of productivity is 1.35 while the lowest is 0.8; in booms, the highest productivity is 1.25 while the lowest is 0.9. This implies a high (and counter-cyclical) level of TFP dispersion in line with the empirical evidence.

The discount factor β, the probability that a highly productive entrepreneur switches to low productivity δ, and the ratio of high to low productivity entrepreneurs n are parameters I pick in order to match three calibration targets - the ratio of tangible assets to GDP, aggregate leverage and the leverage of the most indebted decile of firms.

I use data on tangible assets and GDP and expenditure components from the BEA National Accounts. The asset data is for the 1952-2011 period. The concept of tangible assets includes Business and Household Equipment and Software, Inventories, Business and Household Structures and Consumer Durables. The GDP, Investment and Consumption annual data is for the 1929-2011 period. GDP excludes government value added so it is a private sector output measure.

Aggregate leverage is defined as the average ratio of the value of the debt liabilities of the non-financial corporate sector to the total value of assets. Leverage measures can be obtained from a number of sources. In the US Flow of Funds, aggregate leverage is approximately equal to 0.5 for the 1948-2008 period. This is broadly consistent with the findings of Covas and den Haan (2011) who calculate an average leverage ratio of 0.587 in Compustat data from 1971 to 2004. Covas and Den Haan (2011) also examine the leverage of large firms and find that it is slightly higher than the average in the Compustat data set. Firms in the top 5% in terms of size have leverage of around 0.6. Covas and Den Haan (2007) have similar findings in a panel of Canadian firms. There the top 5% of firms have leverage of 0.7-0.75 compared to an average of 0.66 for the whole sample. High productivity entrepreneurs in our economy run larger firms so differences in productivity and therefore leverage could be one reason for the findings of Covas and Den Haan (2007 and 2011). But the perfect correlation of firm size and leverage that holds in our model will not hold in the data. So if we are interested in the distribution of firm leverage, the numbers in Covas and Den Haan will be an underestimate. This is why we pick a target for the average leverage of the top 10% most indebted firms to be equal to 0.75. This number is broadly consistent with the findings in Covas and Den Haan.

We set $\omega = 1/3$ giving a Frisch elasticity of labour supply of 3. χ is set to ensure that labour supply is equal to 1/3 in line with time use evidence.

Finally, the high (low) realisations of the aggregate TFP shock (A^H and A^L) are picked to ensure that the standard deviation of annual GDP in the model matches that of HP-filtered annual US GDP. (2.80% in our data sample). The probability that the economy remains in the same aggregate state it is today is equal to 0.86[5]. Table 1 below displays the match between data and model moments at the baseline calibration.

Table 1: Model and data moments

Moment (Model concept)	Data	Model
Ratio of tangible assets to GDP	3.300	3.300
Average corproate leverage	0.500	0.535
Leverage of most indebted firms	2.000	2.000
Average fraction of time worked	0.333	0.333
Annual st. dev. of GDP (%)	2.799	2.800

Table 2 below displays a summary of the baseline calibration.

[5] This corresponds to an autocorrelation of TFP shocks at the quarterly frequency of 0.95.

Table 2: Baseline calibration

Parameter Name	Parameter Value
β	0.939
δ	0.287
n	0.040
α	0.100
η	0.260
ω	3.000
χ	3.650
ϕ	0.000
p_{HH}	0.860
p_{LL}	0.860
$\triangle^A$	0.010
$\varepsilon^H\left(A^H\right),\varepsilon^L\left(A^H\right)$	1.1,0.9
$\varepsilon^H\left(A^L\right),\varepsilon^L\left(A^L\right)$	1.2,0.8
a^H/a^L	1.150

6 Numerical Results for the Baseline Economy

6.1 Steady state comparative statics

In this section we consider how the steady state value of intangible collateral varies with different features of the economy's production technology and nature of contract enforcement. Figure 3 below shows the value of intangible collateral as a percentage of output. We compute the value of intangible collateral as the size of firms' debts which are not secured by tangible collateral, expressed as a percentage of steady state output. The three lines on the chart correspond to three different values of a^H/a^L - the ex ante productivity differential between high and low productivity entrepreneurs. In the absence of any long term punishments for defaulters, all three lines on the figure should be zero - the downpayment should be exactly pinned down by the collateralisability of the firm's capital and output. But in our framework borrowing capacity is determined by the values of a borrower's reputation for repayment as well as by the value of tangible assets.

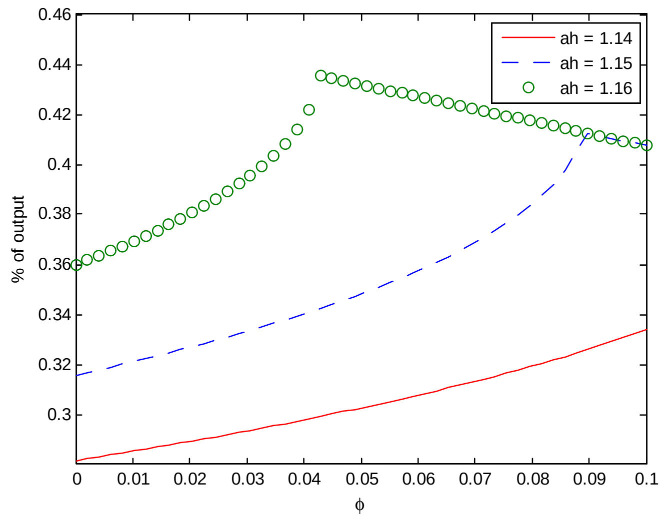

Figure 3: Intangible collateral as a % of output

We can see from the figure that intangible collateral first increases with ϕ before declining once a critical level of ϕ is reached. The figure also shows that when the amount of tangible collateral (ϕ) is low, a higher ex ante productivity differential a^H/a^L is associated with more intangible collateral in equilibrium.

Figure 4 below shows the excess return on wealth for high productivity entrepreneurs - a crucial determinant of the value of a borrower's reputation. The evolution of reputational collateral in response to changes in ϕ is governed by the interplay of the impact of rising land prices and falling real interest rates on the leveraged rate of retun on wealth for high productivity entrepreneurs. While the economy is productively inefficient ($Y^L > 0$), rising ϕ increases the price of land and this depresses the rate of return on production for low productivity entrepreneurs. Because low productivity savers need to be indifferent on the margin between making loans and producing using their own technology, the lower rate of return on low productivity projects also pushes down on the risk-free real interest rate.

Rising capital prices and falling real interest rates increase the leverage available to high productivity entrepreneurs, boosting the excess rate of return during high productivity spells. This in turn makes access to borrowing more attractive, driving up intangible collateral values higher and helping to increase leverage and capital prices even more. Here there is something of a multiplier effect. Higher leverage boosts excess returns and increases the value of intangible collateral, thereby securing further increases in excess returns and a further increase in the value of intangible collateral. What caps the increase in corporate leverage is the growing immediate benefit from default which comes with a high quantity of borrowing which is not secured by tangible assets (land or pledgable future production).

The reason for the non-linearity in the relationship between tangible and intangible collateral arises due to the fact that once ϕ becomes high enough, high productivity entrepreneurs have enough financing capacity to purchase the entire stocks of land and capital and low productivity firms stop producing. At this point, the economy achieves productive efficiency even though borrowing constraints still bind. Once the economy becomes productively efficient, further increases in ϕ boost demand for credit by more than they increase the supply of savings. This starts to bid up the real interest rate and reduces high productivity firms' excess return on wealth in the process. Lower excess returns, in turn, errode the value of reputational collateral. The value of the reputation for repayment reaches zero at the point at which borrowing constraints stop binding and the excess return disappears.

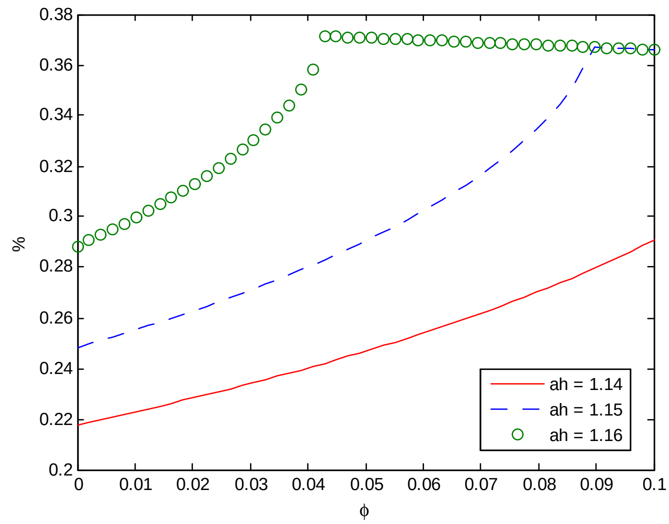

Figure 4: Excess return for high productivity entrepreneurs

Figures 3 and 4 also show that the value of repayment increases as the productivity differential a^H/a^L rises. The bigger the productivity advantage the greater the benefit of leverage and therefore the greater the leverage a borrower can obtain by mortgaging his tangible assets and reputation for repayment.

6.2 Numerical results for the stochastic economy

In this subsection we examine the cyclical properties of our model economy. Our focus here is on whether the intangible collateral model can generate more amplification compared to the model which has only tangible collateral. In other words, does the intangible collateral mechanism of this paper add to the amplification of the basic Kiyotaki-Moore framework?

Table 3 below presents the cyclical volatility in the US data to that generated by the model with intangible collateral (the column named IC Model) and by the model with only tangible collateral (the column named KM Model). The volatility of exogenous technology shocks has been chosen so that the baseline intangible collateral model matches the volatility of output in the data. Investment in the model is more volatile than output though less volatile than in the data. Consumption in the model is slightly more volatile than in the data. As the fifth row of Table 3 shows, the intangible collateral mechanism generates leverage volatility which is around a quarter of that in the data on US home purchase LTVs. The co-movements with GDP are captured accurately by the model.

Table 3: Business Cycle second moments - Model vs US Data

	US Data	IC Model	TC Model
STD (output)	2.80	2.80	2.20
STD (investment)	8.27	3.87	3.02
STD (consumption)	1.91	2.58	2.06
STD (leverage)	1.35	0.37	0.09
COR(leverage,output)	0.55	0.52	-0.18

Note: IC model - Intangible Collateral Model, TC model - model with only tangible collateral assets

The third column of Table 3 shows the cyclical behaviour of the economy in a version of our model in which no exclusion from future borrowing is possible (the KM Model column). In such a model, borrower anonymity ensures that only tangible assets can serve as collateral and there is no role for reputation to back debt in equilibrium. This model is parameterised in exactly the same way as the baseline IC model. The only difference is that ϕ (the fraction of output that can be seized by creditors) has been adjusted so as to match average leverage in the baseline model. We can see that the volatility of output declines substantially and leverage ratios show almost no variability. The only source of leverage variability in the model without intangible collateral arises due to fluctuations in the real interest rate and expected land price changes. As the final column of the table shows, such fluctuations are small and generate counter-cyclical leverage. The counter-cyclical behaviour of leverage in the Kiyotaki-Moore model occurs for the reasons discussed in Brunnermeier and Pedersen (2009). If default is determined by comparing the value of debt to the expected collateral value in the following period, then expected collateral price appreciation boosts leverage today. This is an example of the 'stabilising margins' discussed in Brunnermeier and Pedersen (2009). When a negative temporary shock hits, collateral prices decline but are expected to appreciate in the future when the economy eventually switches to the good state. This leads to an increase in the amount of debt borrowers can obtain relative to the current value of tangible assets.

To generate some more intuition on the behaviour of the model with and without intangible collateral, Figure 5 below displays a deterministic simulation in which the economy switches between the high and low state every 100 periods. The figure focuses on the key variables for the intangible collateral amplification mechanism. Leverage in the figure is given by the inverse of the downpayment requirement on agents ' productive expenditures:

$$\frac{1}{\nu_t - \theta_t E_t A_{t+1} a^H / R_t}$$

The borrowing limit is the sequence for θ_t. We also plot the evolution of the land price and of aggregate output in order to demonstrate the additional amplification of the mechanism in this paper.

Throughout the simulation, agents expect the aggregate state switches to occur according to the switching probabilities in Table 2. Each panel of the figure contains two lines - the solid red line is a simulation path of the baseline intangible collateral while the dashed blue line is a simulation of the model with only tangible collateral.

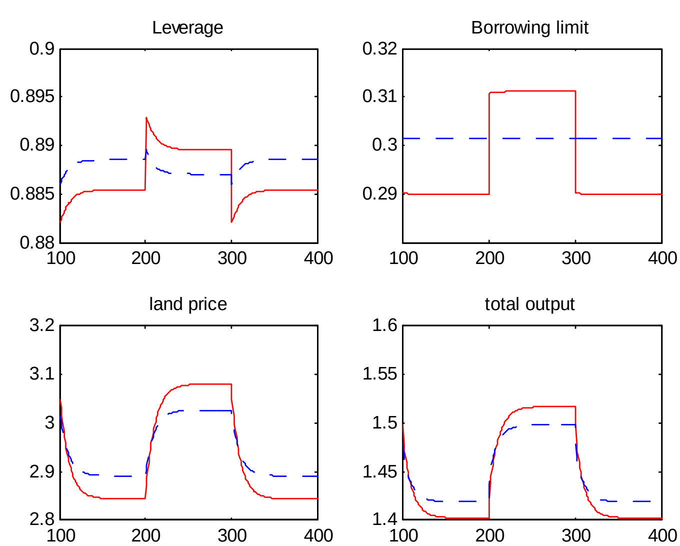

Figure 5: IRF to a technology shock (baseline)

The figure clearly shows the different qualitative and quantitative behaviour of leverage in the two versions of the model. The figure also underlines the way in which pro-cyclical leverage helps to boost the power of the model to amplify the underlying structural shocks hitting the economy. Both versions of the model are hit with the same size exogenous technology shocks but the intangible collateral model displays larger fluctuations in land prices and output.

Figure 6 below tries to look deeper into the mechanism that generates pro-cyclical leverage movements by examining the behaviour of the economy when we switch off idiosyncratic production risk. In the baseline version of the model this risk varies in a counter-cyclical fashion in line with the evidence presented in Bloom et al (2011). The figure shows that counter-cyclical idiosyncratic production risk is a key channel which helps the model to match the evolution of leverage over the business cycle. Without such cyclical variability in firm-specific productivity, the model behaves in a very similar manner to the standard tangible collateral model. The intuition for this result is that aggregate TFP shocks affect the rates of return on wealth of all entrepreneurs equally. Consequently, the attractiveness of being a leveraged producer relative to being an unleveraged producer changes very little, leading to roughly constant leverage over the business cycle.

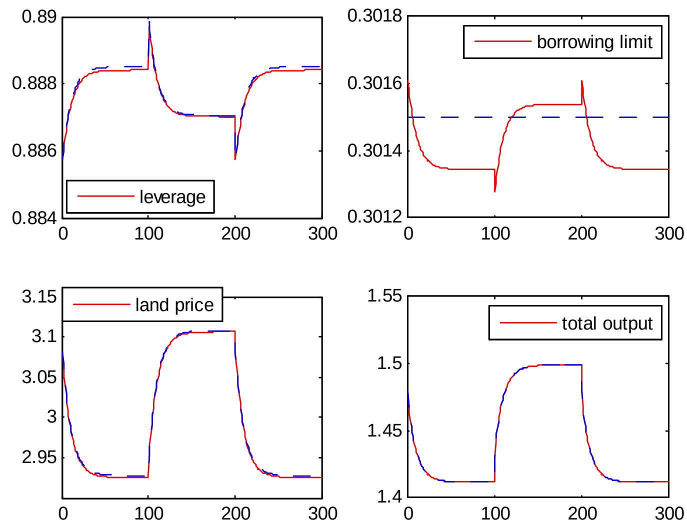

Figure 6: IRF to a technology shock (no idiosync. TFP risk)

7 Conclusions

This paper extends the collateral amplification framework of Kiyotaki and Moore (1997) and Kiyotaki (1998) by considering the possibility that defaulting borrowers can be permanently excluded from future borrowing. Because debt access is valuable, a borrower's clean repayment record becomes another collateral asset, which can guarrantee debt repayment in the same way as the more traditional tangible collateral. When credit constraints bind, leveraged high productivity entrepreneurs have a rate of return on investments which exceeds the market interest rate. Leveraging this excess productive return can substantially boost high productivity agents' rate of return on wealth. Consequently exclusion from debt markets is costly to these entrepreneurs. This is what generates intangible collateral in our model: it is the value of a borrower's reputation for repayment.

We study the way such intangible collateral varies with the nature of technology and contract enforcement in the economy both in steady state and over the business cycle. Steady state intangible collateral is higher the larger the excess return of leveraged production relative to saving. This is the case when the productivity differential between the high and low efficiency technology is large and when the collateralisability of tangible assets is high.

When we introduce aggregate uncertainty we find that the baseline model predicts that the value of intangible collateral is procyclical and it greatly adds to the model's ability to amplify business cycle fluctuations. This is because of the way tangible and intangible collateral interact over the business cycle. Shocks that affect the value of intangible collateral increase downpayments on capital goods and reduce capital demand. This in turn depresses the value of tangible collateral adding to the model's ability to amplify the underlying shocks hitting the economy.

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9 Appendix A: Optimal consumption

Suppose the entrepreneur has optimally chosen her investments in land, goods investment and debt securities. This means that she can earn a state contingent rate of return on invested wealth of $R\left(a_t^i, X_{t+1}\right)$ where a_t^i is the ex ante idiosyncratic TFP component of the agent. The first order condition for optimal consumption becomes:

$$\frac{1}{c_t} = \beta \sum_{X_{t+1}} \sum_{a_{t+1}} \pi\left(X_{t+1}|X_t\right) \pi\left(a_{t+1}|a_t\right) R\left(a_t^i, X_{t+1}\right) \frac{1}{c\left(a_{t+1,} X_{t+1}\right)}$$

We guess that the entrepreneur consumes a fixed fraction of her available resources:

$$c_t = (1-\beta) z_t$$

This means that

$$z_{t+1} = \beta R\left(a_t^i, X_{t+1}\right) z_t$$

Substituting into the consumption Euler equation we have:

$$\begin{aligned}
\frac{1}{(1-\beta) z_t} &= \beta \sum_{X_{t+1}} \sum_{a_{t+1}} \pi\left(X_{t+1}|X_t\right) \pi\left(a_{t+1}|a_t^i\right) R\left(a_t^i, X_{t+1}\right) \frac{1}{(1-\beta) z_{t+1}} \\
&= \beta \sum_{X_{t+1}} \sum_{a_{t+1}} \pi\left(X_{t+1}|X_t\right) \pi\left(a_{t+1}|a_t^i\right) R\left(s_t, X_{t+1}\right) \frac{1}{(1-\beta) \beta R\left(a_t^i, X_{t+1}\right) z_t} \\
&= \frac{1}{(1-\beta) z_t}
\end{aligned}$$

This confirms our initial guessed consumption function.

10 Appendix B: Computing value functions

10.1 The value function of a non-defaulting entrepreneur

We now combine the optimal consumption and portfolio choices of entrepreneurs to derive the value function that characterises their maximum lifetime utility. Let $V\left(a_t^i, X_t\right)$ be the value of a non-defaulting entrepreneur with idiosyncratic state s_t when the aggregate state is X_t.

$$V\left(a_t^i, X_t\right) = \max_{c_t, k_t, l_t, h_t, b_t} \left\{ \ln c_t + \beta \sum_{X_{t+1}} \sum_{a_{t+1}} \pi\left(X_{t+1}|X_t\right) \pi\left(a_{t+1}|a_t^i\right) V\left(a_{t+1}, X_{t+1}\right) \right\}$$

We guess a solution of the form:

$$V\left(a_t^i, X_t\right) = \varphi\left(a_t^i, X_t\right) + \varsigma\left(a_t^i, X_t\right)\ln z_t$$

Hence the value function equals:

$$\varphi\left(a_t^i, X_t\right) + \varsigma\left(a_t^i, X_t\right)\ln z_t \tag{23}$$

$$= \max_{k_t, l_t, h_t, b_t}\left\{\begin{array}{c} \ln(1-\beta) + \ln z_t + \\ \beta \sum_{X_{t+1}} \sum_{a_{t+1}} \pi\left(X_{t+1}|X_t\right)\pi\left(a_{t+1}|a_t^i\right) \\ \left[\varphi\left(a_{t+1}, X_{t+1}\right) + \varsigma\left(a_{t+1}, X_{t+1}\right)\ln z_{t+1}\right] \end{array}\right\} \tag{24}$$

$$= \max_{k_t, l_t, h_t, b_t}\left\{\begin{array}{c} \ln(1-\beta) + \ln z_t + \\ \beta \sum_{X_{t+1}} \sum_{a_{t+1}} \pi\left(X_{t+1}|X_t\right)\pi\left(a_{t+1}|a_t^i\right) \\ \left[\varphi\left(a_{t+1}, X_{t+1}\right) + \varsigma\left(a_{t+1}, X_{t+1}\right)\left(\ln\beta + \ln R\left(a_t^i, X_{t+1}\right) + \ln z_t\right)\right] \end{array}\right\}$$

Equating coefficients we have:

$$\varsigma\left(a_t^i, X_t\right) = 1 + \beta \sum_{X_{t+1}} \sum_{a_{t+1}} \pi\left(X_{t+1}|X_t\right)\pi\left(a_{t+1}|a_t^i\right)\varsigma\left(a_{t+1}, X_{t+1}\right) \tag{25}$$

and

$$\begin{aligned} \varphi\left(a_t^i, X_t\right) &= \ln(1-\beta) \\ &+ \max_{k_t, l_t, h_t, b_t} \beta \sum_{X_{t+1}} \sum_{a_{t+1}} \pi\left(X_{t+1}|X_t\right)\pi\left(a_{t+1}|a_t\right) \\ &\left[\varsigma\left(a_{t+1}, X_{t+1}\right)\left(\ln\beta + \ln R\left(a_t^i, X_{t+1}\right)\right) + \varphi\left(a_{t+1}, X_{t+1}\right)\right] \end{aligned} \tag{26}$$

Equation (25) implies that

$$\varsigma\left(a_t^i, X_t\right) = \frac{1}{1-\beta}$$

Plugging this into (26) we have

$$\begin{aligned} \varphi\left(a_t^i, X_t\right) &= \ln(1-\beta) \\ &+ \max_{k_{t+1}, x_{t+1}, b_{t+1}} \frac{\beta}{1-\beta} \sum_{X_{t+1}} \sum_{a_{t+1}} \pi\left(X_{t+1}|X_t\right)\pi\left(a_{t+1}|a_t^i\right) \\ &\left[\ln\beta + \ln R\left(a_t^i, X_{t+1}\right) + (1-\beta)\varphi\left(a_{t+1}, X_{t+1}\right)\right] \end{aligned} \tag{27}$$

10.2 Value function iterations

Let $r(a^i, X_{t+1})$ and $r^d(a^i, X_{t+1})$ denote, respectively, the rates of return on wealth for non-defaulting and defaulting entrepreneurs. We are now ready to compute the value functions by iterating on the functional equation below.

$$\begin{aligned} & \varphi\left(a^H, X_t\right) \\ = \; & \ln(1-\beta) + \frac{\beta}{1-\beta}\sum \pi\left(X_{t+1}|X_t\right)\pi\left(a_{t+1}|a^H\right) \\ & \left[\ln\beta + \ln r\left(a^H, X_{t+1}\right) + (1-\beta)\varphi\left(a_{t+1}, X_{t+1}\right)\right] \end{aligned} \tag{28}$$

$$\begin{aligned} & \varphi\left(a^L, X_t\right) \\ = \; & \ln(1-\beta) + \frac{\beta}{1-\beta}\sum \pi\left(X_{t+1}|X_t\right)\pi\left(a_{t+1}|a^L\right) \\ & \left[\ln\beta + \ln r\left(a^L, X_{t+1}\right) + (1-\beta)\varphi\left(a_{t+1}, X_{t+1}\right)\right] \end{aligned} \tag{29}$$

$$\begin{aligned} & \varphi^d\left(a^H, X_t\right) \\ = \; & \ln(1-\beta) + \frac{\beta}{1-\beta}\sum \pi\left(X_{t+1}|X_t\right)\pi\left(a_{t+1}|a^H\right) \\ & \left[\ln\beta + \ln r^d\left(a^H, X_{t+1}\right) + (1-\beta)\varphi^d\left(a_{t+1}, X_{t+1}\right)\right] \end{aligned} \tag{30}$$

$$\begin{aligned} & \varphi^d\left(a^L, X_t\right) \\ = \; & \ln(1-\beta) + \frac{\beta}{1-\beta}\sum \pi\left(X_{t+1}|X_t\right)\pi\left(a_{t+1}|a^L\right) \\ & \left[\ln\beta + \ln r^d\left(a^L, X_{t+1}\right) + (1-\beta)\varphi^d\left(a_{t+1}, X_{t+1}\right)\right] \end{aligned} \tag{31}$$

where $r\left(a^H, X_{t+1}\right)$ is given by (11), $r\left(a^L, X_{t+1}\right)$ and $r^d\left(a^L, X_{t+1}\right)$ are given by (21) and $r^d\left(a^H, X_{t+1}\right)$ is given by (13). The value of intangible collateral θ_t can be computed from (14).

For given state contingent land price functions, we compute the value functions as well as the borrowing limit θ_t as follows:

(i) Pick a starting value of θ_t and solve (28) - (31) by value function iteration.

(ii) Update the value of θ_t from (14).

(iii) Return to the value function step (i) above.

(iv) Iterate until value functions and borrowing limits have converged up to a pre-specified tolerance level.

11 Appendix C: Computing aggregate equilibrium

1. In solving for aggregate equilibrium at time t we use the borrowing limits as a function of the aggregate state $\theta(X_t)$. We obtain these using the value function iteration method described in Appendix B above. We also need to compute the state contingent evolution of the land price (q_t). We do this using the parameterised expectations approach of den Haan and Marcet (1990). We parameterise the land price value at $t+1$ as a log linear function of the current continuous state variables (Z_t and d_t) and as a discrete function of the current and future aggregate technology state (A_t and A_{t+1}):

$$\ln q(A_{t+1}) = \varkappa_0(A_t, A_{t+1}) + \varkappa_1(A_t, A_{t+1}) \ln Z_t + \varkappa_2(A_t, A_{t+1}) \ln d_t \tag{32}$$

In other words, we parameterise the value of the land price at time $t+1$ as a log linear function of Z_t and d_t where the coefficients depend upon the realisation of aggregate technology at both time t and $t+1$. Since we have a two-point aggregate technology state distribution, this gives us four different land price functions.

2. Once we have borrowing limit functions and land price functions we can solve for time t equilibrium as a function of the aggregate state (A_t, Z_t and d_t) using a zero-finding routine. I use Matlab's own fsolve.m routine.

3. Next use the state evolution equations to compute next period's state vector:

$$W_{t+1} = \left[d_t r^H_{t+1} + (1-d_t)\, r^L_{t+1}\right] \beta W_t \tag{33}$$

$$d_{t+1} = \frac{(1-\delta) d_t r^H_{t+1} + n\delta\,(1-d_t)\, r^L_{t+1}}{d_t r^H_{t+1} + (1-d_t)\, r^L_{t+1}} \tag{34}$$

4. Simulate the economy for a large number of periods. I simulate for 11,000 periods and throw away the first 1,000 periods. Using the simulated data, update the land price function (32) using linear regression.

5. Repeat steps (1)-(4) above until the coefficients on the land price functions have converged up to an error tolerance level.

6. Check the approximation errors on the land price functions (32). If the maximum absolute error over the simulated sample is less than 1% of the land price value in that state, stop. Otherwise, add more moments to (32) and repeat steps (1)-(5) until accuracy improves.