



EUROPEAN CENTRAL BANK

EUROSYSTEM

# Discussion of “Expecting the unexpected”

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# Summary, focusing on methodology

**Estimate G@R conditional on macro and financial factors**

$$q_{\alpha}(y_{t+1}|\Omega_t) = \mu(\tau) + \phi(\tau)y_t + \beta_1(\tau)F_{1t} + \beta_2(\tau)F_{2t}$$

**Question:** How does G@R change when factors are under stress?

**Answer:** Use Growth in Stress (GiS)

$$\begin{aligned} \min q_{\alpha}(y_{t+1}|\Omega_t) \\ g(F_{1t}, F_{2t}; \alpha) = 0 \end{aligned}$$

# Methodological issues

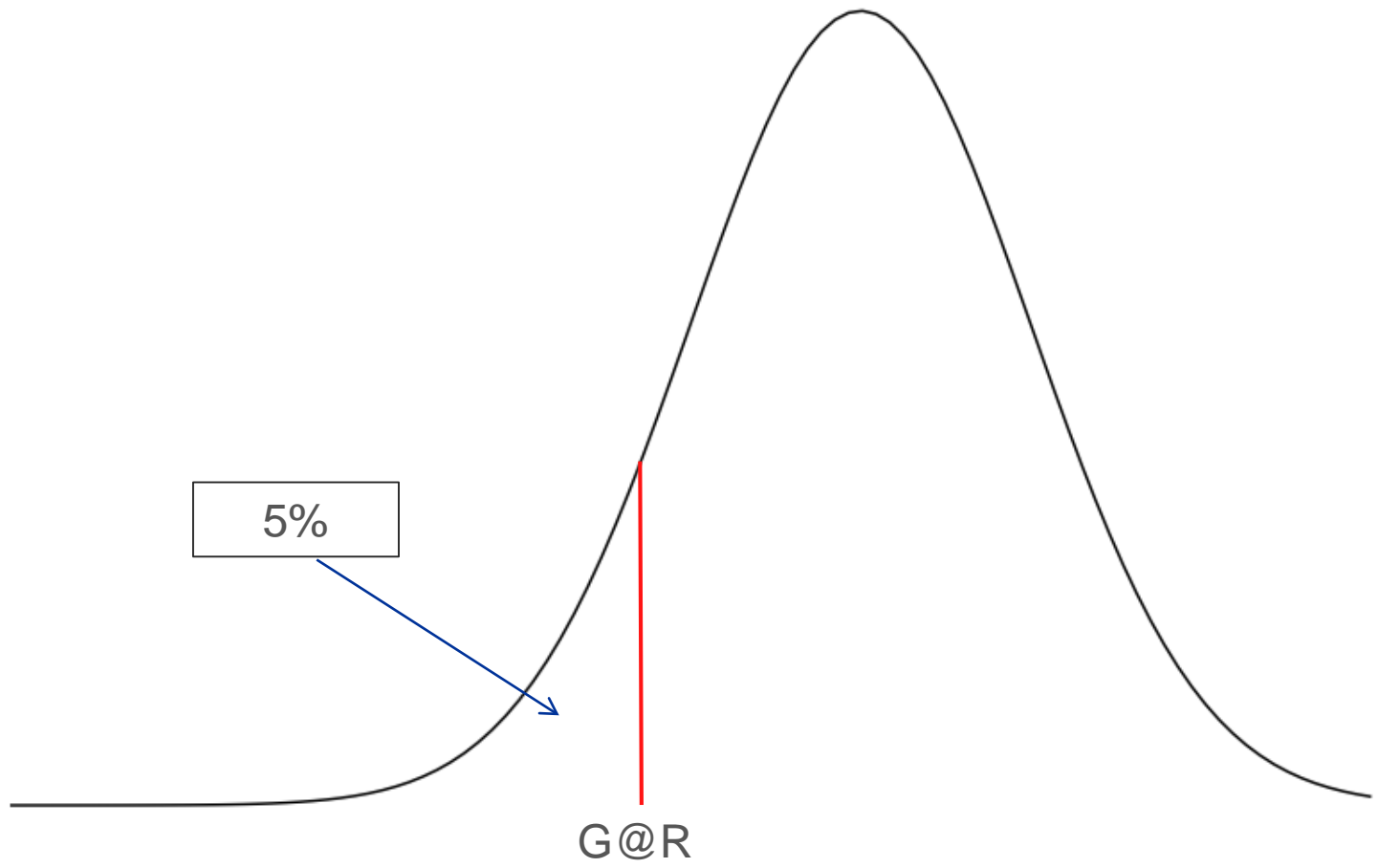
1. Definition of multivariate quantile
2. Quantile factors
3. Identification

# 1. Definition of multivariate quantiles

## Univariate definition

$$q_\tau(F_t) = \inf\{F_t: G(F_t) \geq \tau\} \quad \tau \in (0,1)$$

The definition exploits the canonical ordering of the real line.



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## Multivariate definition

Problem: canonical ordering no longer exists for  $R^2$  when  $d \geq 2$

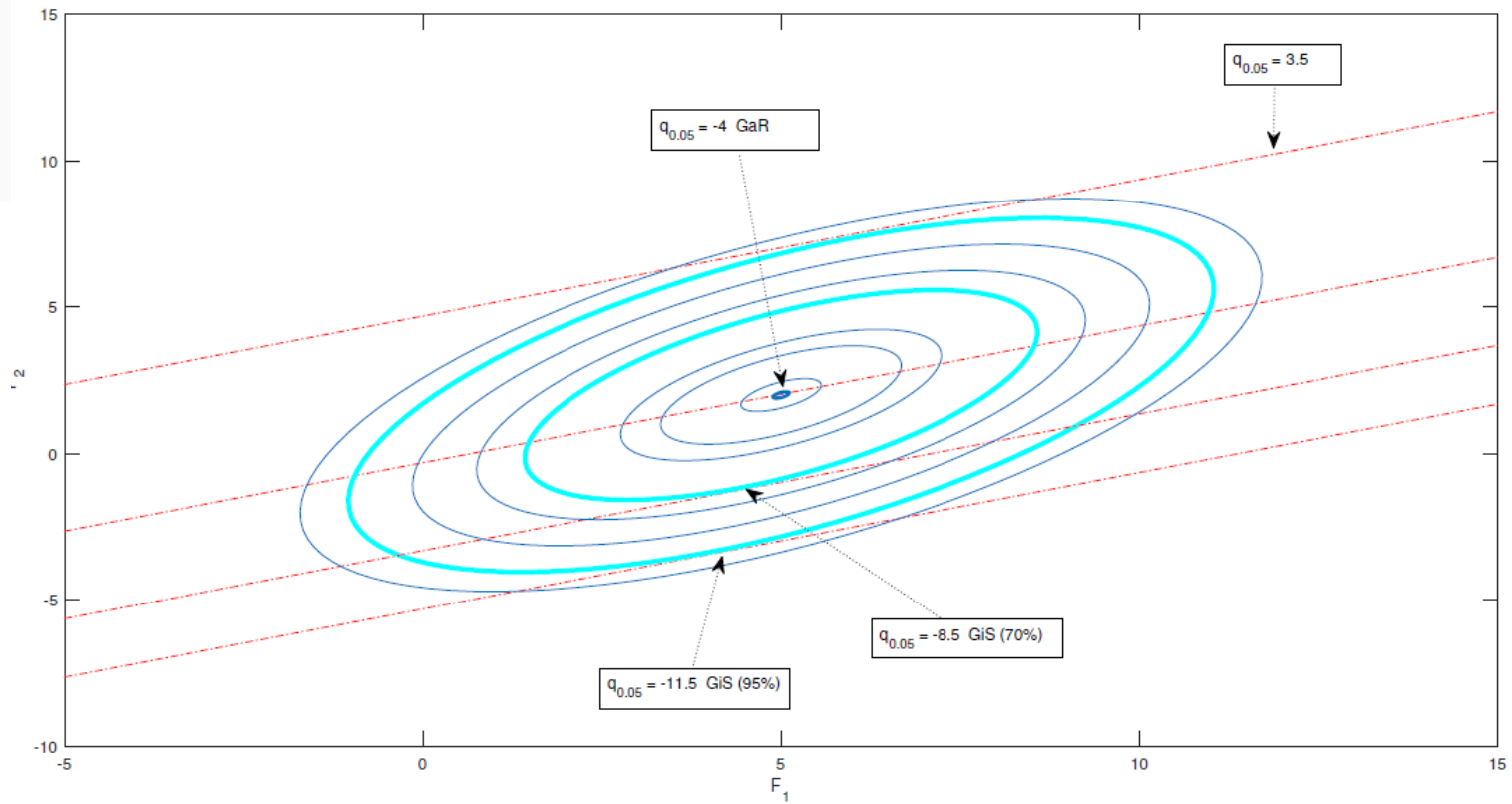


Figure 1: GaR and GiS.

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- Stratified quantile regression (Wei 2008)
- Directional approaches (Hallin, Paindaveine and Siman 2010)
- Optimal transport approach (Carlier, Chernozhukov, Galichon 2016)

See recent survey by Hallin and Siman in Handbook of Quantile Regression



## 2. Quantiles factor models

**Chen, Dolado, Gonzalo (2021)**

Suppose DGP is:

$$X_{it} = \beta_i F_{1t} + \eta_i F_{2t} \varepsilon_{it}$$

The  $\alpha$ -quantile is:

$$q_\tau(X_{it}) = \beta_i F_{1t} + \eta_i F_{2t} q_\tau(\varepsilon_{it})$$

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Principal component analysis would extract only location-shifting factor  $F_{1t}$ .

It cannot identify the scale factor  $F_{2t}$ .

**Financial factors are likely to affect both 1<sup>st</sup> and 2<sup>nd</sup> moments**

# 3. Quantile VAR

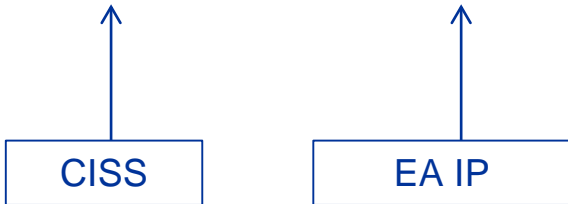
## Chavleishvili and Manganeli (2019)

All variables are endogenous

To be able to shock the factors, you need identification assumptions

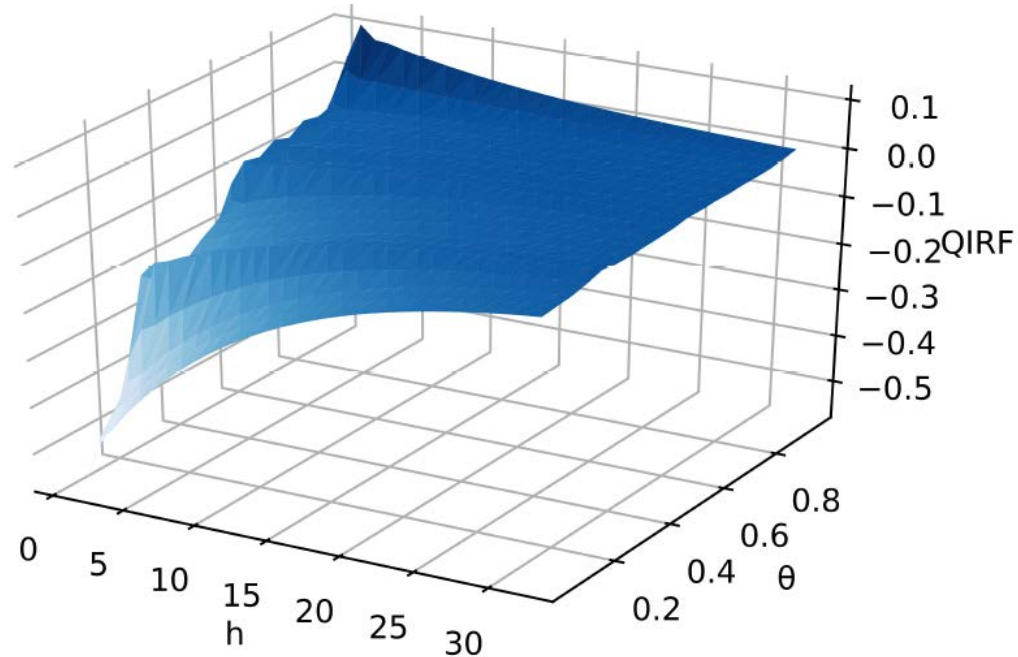
$$Y_{1,t+1} = \omega_1 + a_{11}Y_{1,t} + a_{12}Y_{2,t} + \varepsilon_{1,t+1}$$

$$Y_{2,t+1} = \omega_2 + a_0Y_{1,t+1} + a_{21}Y_{1,t} + a_{22}Y_{2,t} + \varepsilon_{2,t+1}$$



# Quantile impulse response function for IP

Shock to CISS



Source: Chavleishvili and Manganelli (2019)

# Conclusion

Punchline: G@R should be computed under situation of stress of the endogenous variables.

This represents a welcome contribution to the debate.

But:

- Mixing quantile and OLS techniques is not ideal.
- Combine recent quantile factor models with quantile VAR models.

# References

G. Carlier, V. Chernozhukov, and A. Galichon. Vector quantile regression. *Annals of Statistics*, 44:1165-1192, 2016.

Chavleishvili and Manganelli (2019), “Forecasting and stress testing with quantile vector autoregression”, Working Paper Series No 2330, European Central Bank.

Chavleishvili, Engle, Fahr, Kremer, Manganelli and Schwaab (2021), The risk management approach to macro-prudential policy, Working Paper Series No. 2565, European Central Bank.

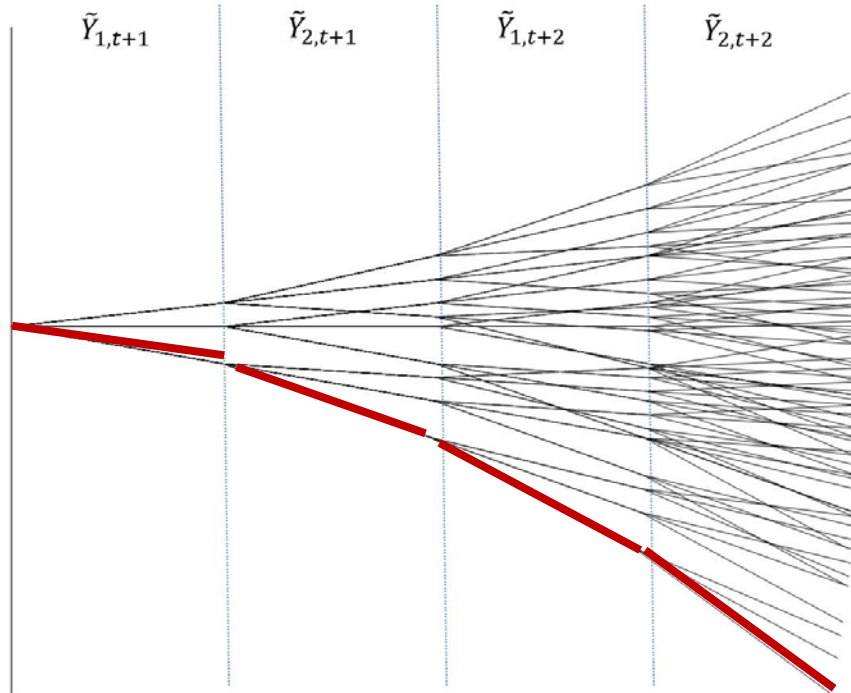
Chen, L., J. Dolado, and J. Gonzalo (2021), Quantile factor models, *Econometrica*.

M. Hallin, D. Paindaveine, and M. Siman. Multivariate quantiles and multiple-output regression quantiles: From L1 optimization to halfspace depth (with discussion). *Annals of Statistics*, 38:635-669, 2010.

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# Annex

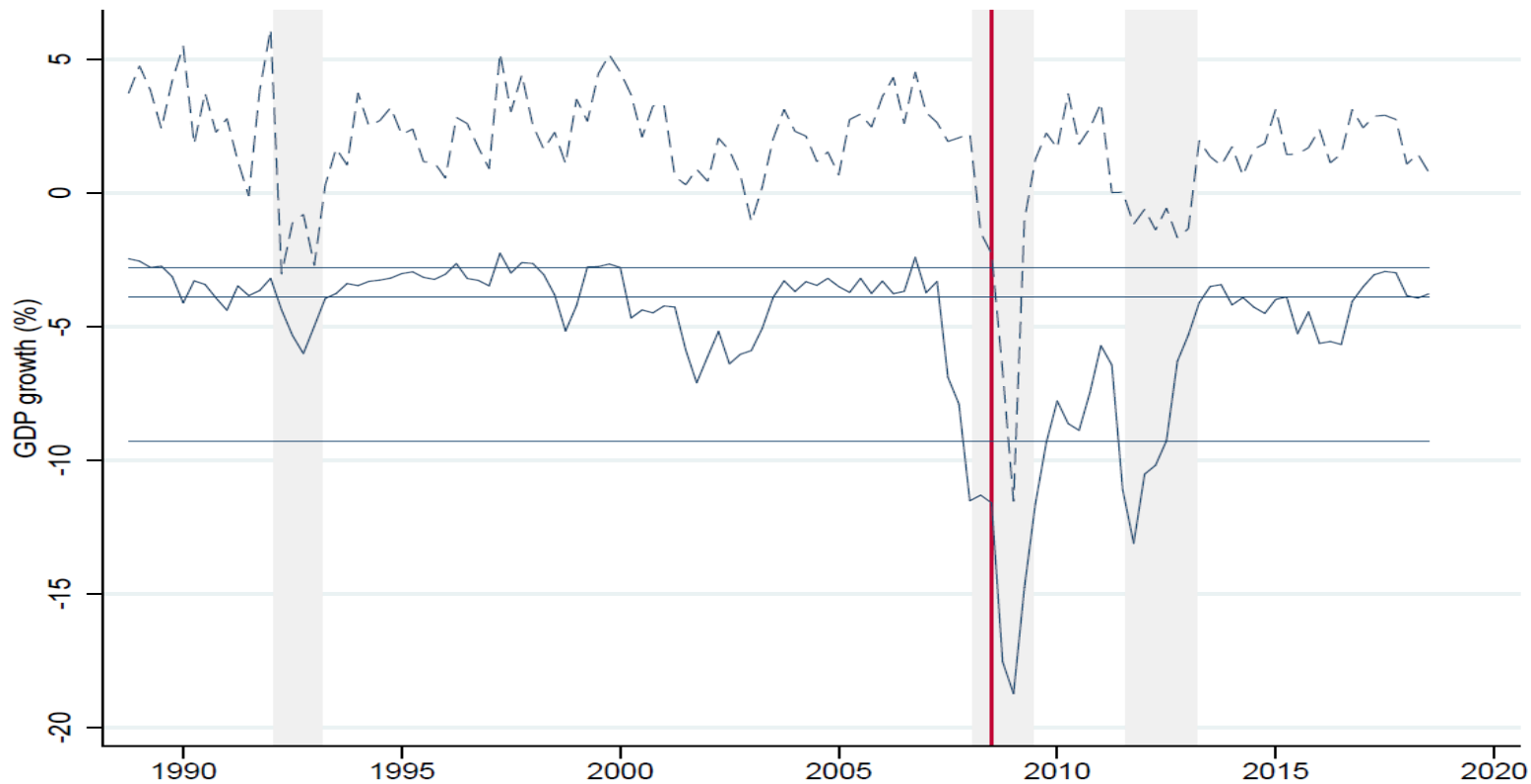
# Stress testing with QVAR



- Choose the quantile probabilities to match 2009 Q2 GDP contraction four quarters ahead
- Apply these quantile probabilities at each point in time



# Vulnerability to GFC-sized shock



Source: Chavleishvili, Engle, Fahr, Kremer, Manganelli and Schwaab (2021)