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Accounting for Macro-Finance Trends: Market Power, Intangibles, and Risk Premia*

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Abstract

Real risk-free interest rates have trended down over the past 30 years. Puzzlingly in light of this decline, (1) the return on private capital has remained stable or even increased, creating an increasing wedge between public and private rates of return; (2) stock market valuation ratios have increased only moderately; (3) investment has been lackluster. We use a simple extension of the neoclassical growth model as an accounting framework to diagnose the nexus of forces that jointly accounts for these developments. We find that rising market power, rising unmeasured intangibles, and rising risk premia, play a crucial role, over and above the traditional culprits of increasing savings supply and technological growth slowdown.

JEL codes: E34 ,G12.

Keywords: equity premium, investment, profitability, price-earnings, valuation ratios, price-dividend, labor share, competition, markups, safe assets.

1 Introduction

Over the past thirty years, most developed economies have experienced large declines in risk-free interest rates and increases in asset prices such as housing or stock prices, with occasional sudden crashes. At the same time, and apart from a short period in the 1990s, economic growth, in particular productivity growth, has been rather disappointing, and investment has been lackluster. Earnings growth of corporations has been strong however, leading in most countries to an increase in the capital share and to stable or slightly rising profitability ratios. Making sense of these trends is a major endeavor for macroeconomists and for financial economists.

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Given the complexity of these phenomena, it is tempting to study them in isolation. For instance, a large literature has developed that tries to understand the decline in risk-free interest rates. But studying these trends independently may miss confounding factors or implausible implications. For instance, an aging population leads to higher savings supply which might well explain the decline in interest rates. However, higher savings supply should also reduce profitability, and increase both investment and stock prices. Hence, a potential driver that is compelling judged on its ability to explain a single trend, may be implausible overall, because it makes it harder to account for the other trends.

Another way to highlight these tensions is to note that the stable profitability of private capital and declining risk-free rate lead to a rising wedge, or spread, between these private and public rate of returns. What gives rise to this spread? A narrative that has been recently attracted significant interest is the possibility of rising market power. On the other hand, rising risk premia could also account for the wedge. The only way to disentangle these potential causes is to consider additional implications - for instance, everything else equal, rising market power should imply a lower labor share, and rising risk premia should be reflected in lower prices of risky assets such as stocks.

These simple observations motivate our approach. We believe that structural analysis of the past thirty years should account for these trends *jointly.* A novel feature of our analysis is that we aim to account both for macro trends and finance trends. The first step we take is to document a set of broad macro-finance trends which we believe are of particular interest. We focus on six broad indicators, that involve the evolution of economic growth, risk-free interest rates, profitability, the capital share, investment, and valuation ratios (such as the price-dividend or price-earnings ratio).

The second step in our paper is to develop an accounting framework to disentangle several potential drivers of these trends. We focus on five broad narratives that have been put forward to explain some or all of these trends. The first narrative is that the economy experienced a sustained growth decline, owing to lower population growth, investment-specific technical progress, or productivity growth. The second narrative is that savings supply has increased, perhaps owing to population aging (or to the demand of emerging markets for store of values). A third narrative involves rising market power of corporations. A fourth narrative focuses on technological change, coming from the introduction of information technology, which may have favored capital or skilled labor over unskilled labor, or the rise of hard-to-measure intangible forms of capital. A fifth narrative, which we will emphasize, involves changes in perceived macroeconomic risk.

Our approach is simple enough to allow for a relatively clean identification of the impact of these drivers on the facts that we target. Here our contribution is to propose a simple macroeconomic framework - a modest extension of the neoclassical growth model - that allows to account for the "big ratios" familiar to macroeconomists as well as for the "financial ratios" of financial economists. The familiar difficulty here is the disconnect between macro and finance, e.g. the equity premium puzzle: it is difficult to use macro models to fit asset price data. Our model does this in a way that allows for interesting feedbacks between macroeconomic and financial variables. For example, the investment-output ratio is affected by market power and macroeconomic risk as well as savings supply and technological parameters. At the same time, our framework preserves the standard intuition and results of macroeconomists

and financial economists, and hence serves a useful pedagogical purpose.

In our baseline estimation, we abstract from intangibles. Our main empirical result here is that the rising spread between private and public capital is driven mostly by a confluence of two factors: rising market power *and* rising macroeconomic risk. This rising macroeconomic risk in turns implies that the equity premium, which previous researchers have argued fell in the 1980s and 1990s, may have risen since 2000. Moreover, we show how previous researchers, who have used models without risk, have attributed too big a role to rising market power. We also find little role for technical change. Our estimates offer a better understanding of the drivers of investment, profitability, and valuation ratios. Finally, stepping outside of the model, we provide further independent corroborative evidence of the increase in the equity premium using simple reduced-form methods.

When we incorporate intangibles, we see that a significant increase in their unmeasured component can help explain the rising wedge between the measured marginal product of capital and the risk-free rate. Interestingly, we find that intangible capital reduces the estimated role of market power in our accounting framework, while preserving the role of risk.

The rest of the paper is organized as follows. The remainder of the introduction discusses the related literature. Section 2 presents the main trends of interest. Section 3 presents our model, which is a modest generalization of the neoclassical growth model. Section 4 explains our empirical methodology and identification. Section 5 presents the main empirical results. Section 6 discusses some extensions and robustness. Finally, section 7 discusses some outside evidence on the rise in the equity premium, markups, and intangibles. Section 8 concludes.

1.1 Literature review

Our paper, given its broad scope, makes contact with many other studies that have separately tried to understand one of the key trends we document. We discuss in more detail the relation of our results to the recent literature on market power, intangibles and risk premia in section 7.

First, there is a large literature that studies the decline of interest rates on government bonds. Hamilton et al. (2015) provide a long-run perspective, and discuss the connection between growth and the interest rates. Rachel and Smith (2017) is an exhaustive analysis of the role of many factors that affect interest rates. Carvalho et al. (2016) and Gagnon et al. (2016) study the role of demographics in detail. Del Negro et al. (2017) emphasize the role of the safety and liquidity premia. Our analysis will incorporate all these factors, though in a simple way.

Second, a large literature documents and tries to understand the decline of the labor share in developed economies. Elsby et al. (2013) document the facts and discuss various explanations using US data, while Karabarbounis and Neiman (2013) study international data and argue that the decline is driven by investment-biased technical change. Rognlie (2015) studies the role of housing.

Perhaps the most closely related papers are Marx, Mojon and Velde (2017) and the contemporaneous work by Eggertsson, Robins and Wold (2018). Marx, Mojon and Velde (2017) also find, using a different methodology, that an increase in risk may explain the observed pattern for the risk-free rate. They do not explicitly target the evolution of other variables such as investment or the price-dividend ratio. On

the other hand, Eggertsson, Robins and Wold (2018) target some of the same big ratios that we study, but there are differences in terms of methodology and in terms of results. Methodologically, our approach uses a more standard and simple model, which allows a closed-form solution and clear identification. Substantively, we find a more important role for macroeconomic risk whereas they contend that rising market power is the main driving force.

2 Some macro-finance trends

This section presents simple evidence on the trends affecting some key macro-finance moments. We focus on six groups of indicators: interest rates on safe and liquid assets such as government bonds, measures of the rate of return on private capital, valuation ratios (i.e., price-dividend or price-earnings ratio for publicly listed companies), private investment in new capital, the labor share, and growth trends. We first present simple graphical depictions, then add some statistical measures.

Our focus is on the United States, but we believe that these facts hold in other developed economies and hence likely reflect worldwide trends. Like many macroeconomic studies, we will mostly consider the post-1984 period, which is associated with low and stable inflation era together with relative macroeconomic stability (the "Great Moderation"). We present the changes in the simplest possible way by breaking our sample equally in the middle, i.e. at the millennium. However, we will also discuss briefly the longer trends and present continuous indicators using moving averages.

One important decision to make is whether to study the entire private sector, or to exclude housing and focus for instance on nonfinancial corporations. On one hand, the savings decision should apply to the entire economy; on the other hand, the housing sector may be better modeled differently, and hence we might want to explicitly recognize the heterogeneity of capital goods. We will in this section present indicators that cover both, but in the end we target the entire private sector. For the most part, the trends that we focus on are apparent both for nonfinancial corporations and in the aggregate.

2.1 Graphical evidence

We summarize the evolution of the six groups of indicators as six facts.

Fact #1: Real risk-free interest rates have fallen substantially

The top panels of figure 1 present proxies for the one-year and ten-year real interest rates by substracting inflation expectations from nominal Treasury yields.[1] As many authors and policymakers have noted before, there has been a strong downward trend in these measures since 1984. The short-rate exhibits some clear cyclical fluctuations, while the long rate has a smoother decline. Table 1 shows that the average one-year rate falls from around 2.8% in the first half of our sample (1984–2000) to -0.3% in

[1] We use median consumer price inflation expectations from the Philadelphia Fed survey of professional forecasters (SPF). Very similar results for the trend are obtained if one uses the mean expectation rather than the median; or the Michigan survey of consumers rather than the SPF. For the one-year rate, one can also replace expectations with ex-post inflation or lagged inflation. For the ten-year rate, one can also use the TIPS yield for the sample where it is available.

the second half of our sample (2001-2016). The long-term rate similarly falls from 3.9% in the first half to 1.1% in the second half.

Fact #2: The profitability of private capital has remained stable or increased slightly

In contrast, there is little evidence that the return on private capital has fallen; if anything, it appears to have increased slightly. Gomme, Ravikumar and Rupert (2011; thereafter GRR) construct from national income (NIPA) data a measure of aggregate net return on physical capital, roughly profits over capital. The bottom left panel depicts their series. The rising spread between their measure, which can be thought as a proxy for the marginal product of capital, and the interest rate on US Treasuries, is an important trend to be explained for macro- and financial economists.

GRR construct their series using detailed data from NIPA and other sources, but one can construct a simple approximation using the ratio of operating surplus to capital for the nonfinancial corporate sector; Table 1 show that this ratio is also fairly constant. In our estimation exercise, we will focus on gross profitability, and, to ensure consistency between our measures, will construct it simply as the ratio of the profit-output ratio (i.e., one minus the labor share) to the capital-output ratio. This measure is depicted in the bottom right panel of figure 1; the overall level is higher, in part because it is gross rather than net, but the broad trend is similar to the GRR measure.

Fact #3: Valuation ratios are stable or have increased moderately

The top two panels of figure 2 present measures of valuation ratios for the US stock market. The top left panel shows the ratio of price to dividends from CRSP, while the top right panel shows the price-operating earnings ratio for the SP500.[2] The later is essentially trendless, while the former exhibits a huge volatility around 2000 before settling down to a higher value. Another commonly used valuation ratio is the price-smoothed earnings ratio of Shiller (CAPE), which divides the SP500 price by a ten-year moving average of real earnings, and is reported in table 1. While all these ratios are quite volatile, overall they exhibit only a moderate increase from the first period to the second period. Our analysis will emphasize that this limited increase is puzzling given the large decline of the risk-free rate (Fact #1).

Fact #4: The share of investment in output or in capital has fallen slightly

The bottom two panels of figure 2 depict the behavior of investment. As several authors have noted recently (e.g. Eberly and Lewis (2016), Gutierrez and Philippon (2017)), investment has been relatively lackluster over the past decade or more; but the magnitude of this decline is quite different depending on how exactly one measures it. Because the price of investment goods falls relative to the price of consumption goods, it is simpler to focus on the expenditure share of GDP (left panel) or the ratio of nominal investment to capital (evaluated at current cost; right panel). Both ratios ought to be stationary in standard models, and they appear nearly trendless over long samples. Comparing the two periods of interest, investment spending exhibits a very cyclical pattern, increasing faster than

[2] We focus on operating earnings which exclude exceptional items such as write-offs and hence are less volatile. In particular, total earnings were negative in 2008Q4 because banks marked down the values of their assets substantially.

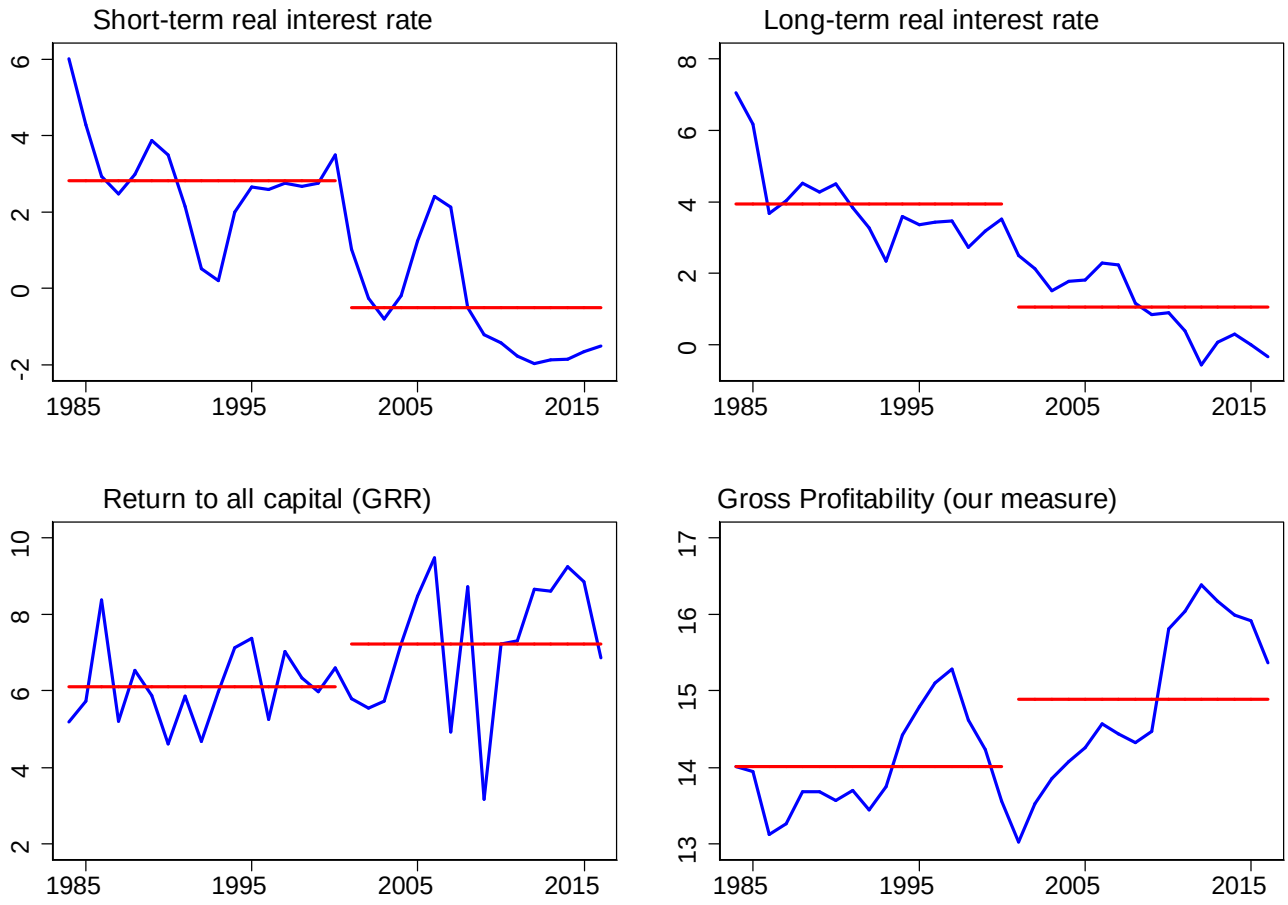

Figure 1: The top left panel displays the difference between the 1-year Treasury bill rate and the median 1-year ahead CPI inflation expectations from the Survey of Professional Forecasters (SPF). The top right panel displays the difference between the 10-year Treasury note rate and the median 10-year ahead CPI inflation expectations from the SPF. The bottom left panel presents the estimate of the pretax return on all capital from Gomme, Ravikumar and Rupert (2011). The bottom right panel presents our measure of gross profitability, the ratio of (1-labor share) to the capital-output ratio. The horizontal lines represent the mean in the first and second half of the samples (1984-2000 and 2001-2016 respectively).

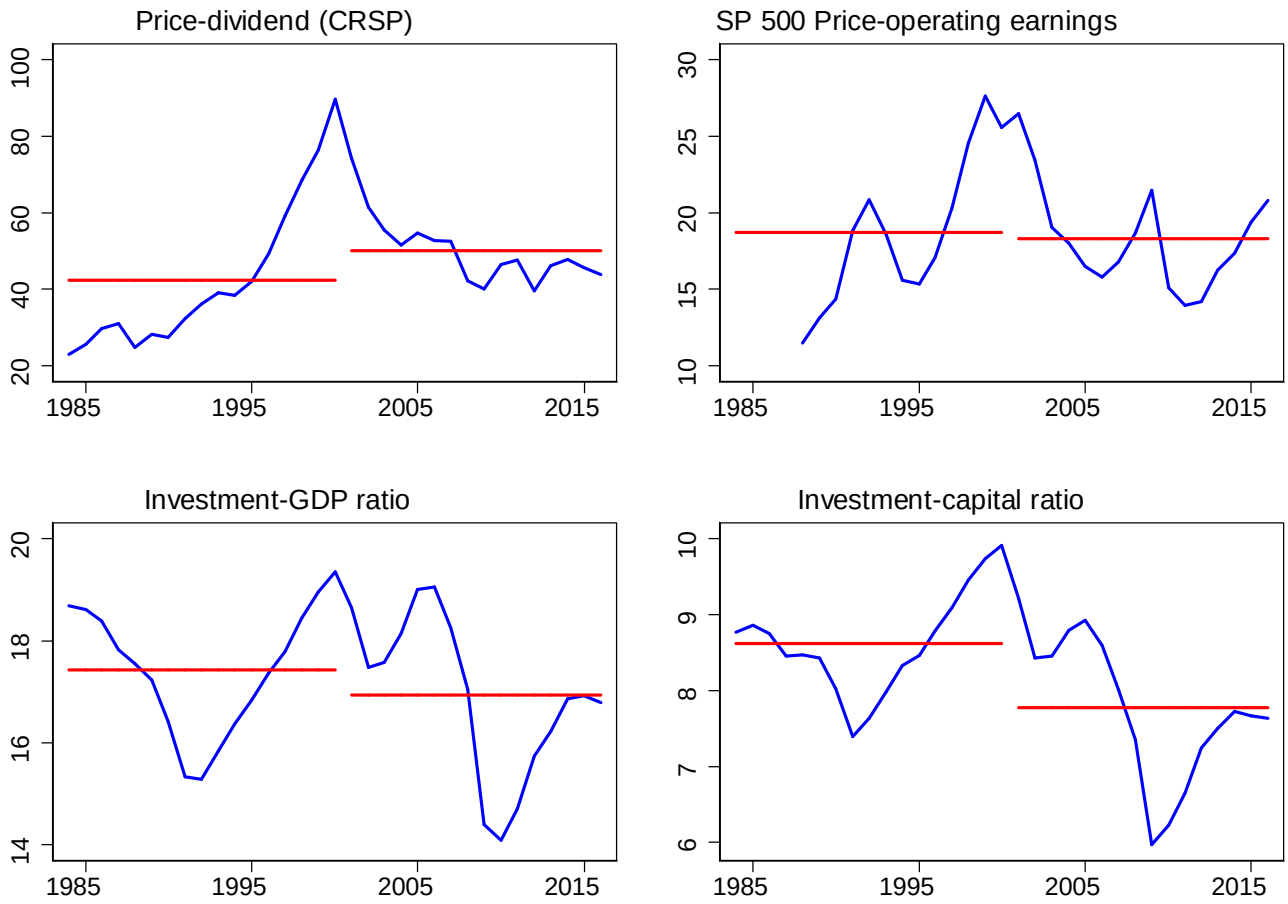

Figure 2: The top left panel displays the price-dividend ratio from CRSP. The top right panel shows the ratio of price to operating earnings for the SP500. The bottom left panel is the ratio of nominal investment spending to nominal GDP. The bottom right panel is the ratio of nominal investment to capital (at current cost). The horizontal lines represent the mean in the first and second half of the samples (1984-2000 and 2001-2016 respectively).

GDP during expansions and falling faster than GDP during recessions, but overall both ratios appear to exhibit small to moderate declines. Table 1 also report the ratios for the nonresidential sector (i.e., business fixed investment), which behaves very similarly, so our results are not driven by housing. The table also reports two measures of the evolution of the capital-output ratio; first, the ratio of capital at current cost to GDP; and second, the ratio of a real index of capital services[3] (from the BLS) to real output.(which we normalize to one in 1984). Both ratios exhibit an increase of about 0.15.[4]

Fact #5: Total factor productivity and investment-specific growth have slowed down, and the employment-population ratio has fallen

There has been much public discussion that overall GDP growth has declined over the past couple of decades. This decline is in part attributable to a decline in the employment/population ratio, largely due to demographic factors (Aaronson et al. (2015)), shown as the top right panel in Figure 3. Beyond that, the decline in output per worker growth is large between the two samples, from 1.8% per year to 1.2% per year according to table 1. This decline is largely driven by lower total factor productivity (TFP) growth and lower investment-specific technical progress; Table 1 shows that the growth rate of the Fernald TFP measure goes from 1.1% per year to less than 0.8% per year, while the growth rate of the relative price of investment goods to nondurable and service consumption goes from -1.8% per year to -1.1% per year; both are depicted in the bottom panels of Figure 3.

Fact #6: The labor share has fallen

Finally, the top left panel of figure 3 presents a measure of the gross labor share for the nonfinancial corporate sector; table 1 includes a measure that covers the entire US economy. As has been noted by many authors (e.g., Karabarbounis and Neiman (2013); Elsby et al. (2013); Rognlie (2015)), the labor share exhibits a decline that starts around 2000 in the United States.

Of course, all of these facts are somewhat difficult to ascertain graphically given the short samples and the noise in some series. This leads us to evaluate next the statistical significance of these changes.

2.2 Statistical evaluation

To summarize the trends in these series in a more formal way, Table 1 reports several statistics for the series presented in figures 1-3 above as well as for some alternative series that capture the same concepts. Columns 1-4 report the means in the first and second subsamples, which are depicted in figures 1-3 as horizontal lines, together with standard errors. Column 5 reports the difference between the means in the second and first sample, and column 6 is the associated standard error. Column 7 is the regression coefficient of the variable of interest on a linear time trend, and column 8 is the associated standard error. (The standard errors are calculated using the Newey-West method with five (annual) lags.)

[3] This index aggregates underlying capital goods using rental prices, which is the correct measure for an aggregate production function. In contrast, the capital at current cost is a nominal value which sums purchase prices.

[4] Over the long term, these ratios do behave differently, however. The BLS index exhibits an upward trend since the mid 1970s due to the decline of the price of investment goods, but this trend has slowed down recently. In contrast, the current cost capital/output ratio is nearly trendless.

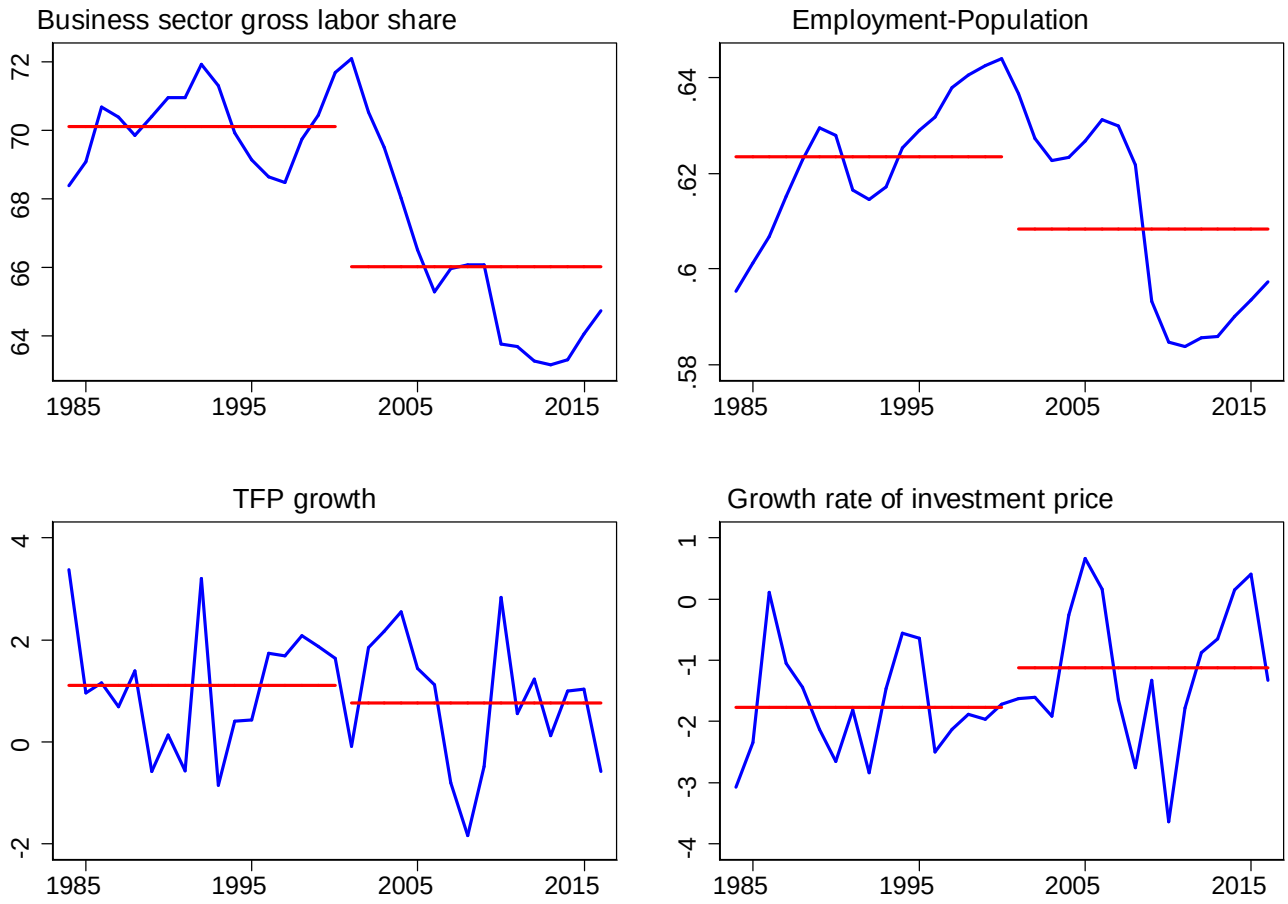

Figure 3: The top left panel shows the gross labor share for the nonfinancial corporate sector, measured as the ratio of nonfinancial business labor compensation to gross nonfinancial business value added. The top right panel is the employment-population ratio. The bottom left panel shows the growth rate of total factor productivity (TFP). The bottom right panel is the growth rate of the relative price of investment goods and consumption goods. The horizontal lines represent the mean in the first and second half of the samples (1984-2000 and 2001-2016 respectively).

For some indicators, there is little evidence of a break between the samples, while for others, there is overwhelming evidence of a break. Specifically, interest rates, the labor share, total factor productivity, and the investment-capital ratios are markedly lower in the second sample. On the other hand, valuation ratios and the return on capital appear fairly stable.

2.3 Longer historical trends

Figure 4 presents the evolution of nine of the moments we described above, but over a longer sample, since 1950. (These nine moments will be our estimation target below.) For clarity, we add a 11–year centered moving average to each series, so we depict the evolution from 1955 to 2011. One motivation for studying a longer sample is that real interest rates were also low in the 1970s and to some extent the 1960s, and hence one question is whether the abnormal period is the early 1980s when real interest rates were very high. The figure shows, however, that the analogy does not apply to all variables. It is true that profitability was high in the 1960s, but the price-dividend ratio was lower, and the labor share and the investment-capital ratio were relatively high, in contrast to the more recent period. Overall, neither the 1960s nor the 1970s are similar in all respects to the post 2000 period. Moreover, a serious consideration of the role of inflation is warranted to study the 1970s and early 1980s, as inflation likely affected many of the macroeconomic aggregates depicted here. This is why, for now, we focus on the post 1984 sample. But we will present some results below starting in 1950 to illustrate what our approach implies for these earlier periods.

3 Model

This section introduces a simple macroeconomic model to account for the macro-finance moments. The framework builds on Gourio (2012), adding macroeconomic risk and monopolistic competition to a neoclassical growth model. Given our focus on medium-run issues, we abstract from nominal rigidities and adjustment costs. This provides a very standard, well-understood framework to study the role played by each factor in driving the observed trends.

3.1 Model setup

We consider a standard dynastic model with inelastic labor supply. In order to highlight the role of risk, we use Epstein-Zin preferences:

$$V_t = L_t \left((1-\beta) c_{pc,t}^{1-\sigma} + \beta E_t \left(V_{t+1}^{1-\theta} \right)^{\frac{1-\sigma}{1-\theta}} \right)^{\frac{1}{1-\sigma}}, \tag{1}$$

where V_t is utility, L_t is population size (which is exogenous and deterministic), $c_{pc,t}$ is per-capita consumption at time t, σ is the inverse of the intertemporal elasticity of substitution of consumption, and θ the coefficient of relative risk aversion. We assume that labor supply is exogenous and equal to $N_t = \overline{N} L_t$ where $\overline{N}$ is a parameter that captures the employment-population ratio.

Group	Variable	Averages						Trend	
		1984-'00	SE	2001-'16	SE	Diff.	SE	Coeff.	SE
Interest rate	One year maturity*	2.79	.45	-.35	.59	-3.14	.75	-.17	.02
	Ten year maturity	3.94	.41	1.06	.46	-2.88	.69	-.18	.01
Return on capital	GRR: all, pretax	6.1	.2	7.24	.45	1.14	.45	.07	.02
	GRR: business, pretax	8.59	.32	10.46	.62	1.87	.62	.11	.03
	Nonfin. corps. GOS/NRK	7.59	.34	7.87	.36	.27	.51	.04	.01
	Gross profitability* (see text)	14.01	.26	14.89	.49	.88	.6	.07	.02
Valuation ratios	Price-dividend ratio* CRSP	42.34	8.56	50.11	3.4	7.78	8.39	.67	.36
	Price-operating earnings SP500	18.7	2	18.31	1.09	-.39	1.75	.03	.12
	Price-smoothed earnings Shiller	22.07	4.41	24.36	1.25	2.29	4.5	.33	.17
Investment	Investment share in GDP	17.43	.53	16.93	.65	-.5	.76	-.04	.04
	Nonres. invest. share in GDP	12.94	.40	12.79	.18	-.15	.43	0	.02
	Investment-capital: all*	8.1	.25	7.23	.35	-.88	.38	-.04	.02
	Investment-cap.: nonresidential	10.95	.39	10.2	.24	-.76	.4	-.03	.02
Capital-output	Fixed asset	2.13	.03	2.28	.03	.15	.04	.01	0
	Real index (BLS)	1.06	.02	1.18	.01	.13	.02	.01	0
Labor share	Nonfarm business (BLS) gross	62.07	.31	58.56	1.01	-3.51	1.11	-.21	.04
	Nonfinancial corps. gross*	70.11	.34	66.01	1.21	-4.1	1.29	-.24	.05
Growth	Output per worker	1.80	.22	1.22	.23	-.58	.29	-.03	.02
	Total factor productivity*	1.10	.31	.76	.32	-.34	.36	-.02	.02
	Population*	1.17	.08	1.1	.06	-.07	.08	0	0
	Price of investment: all*	-1.77	.15	-1.13	.34	.64	.26	.03	.02
	Price of investment: nonresid.	-2.38	.19	-1.75	.29	.63	.25	.04	.02
	Price of invt: equipment	-3.62	.60	-3.27	.53	.34	.72	.02	.04
	Price of invt: IPP	-1.71	.30	-2.15	.36	-.44	.52	0	.02
	Employment-pop. ratio*	62.34	.58	60.84	0.94	-1.51	1.06	-.07	.06

Table 1: The table reports, for each variable, the mean in the sample 1984-2000, in the sample 2001-2016, and the differences of means, as well as the coefficient on a linear time trend, all with standard errors. Stars indicate moments targeted in our estimation exercise. GRR stands for Gomme, Ravikumar and Rupert (2011), GOS for gross operating surplus, NRK for non-residential capital, and IPP for intellectual property products. Variables construction detailed in appendix.

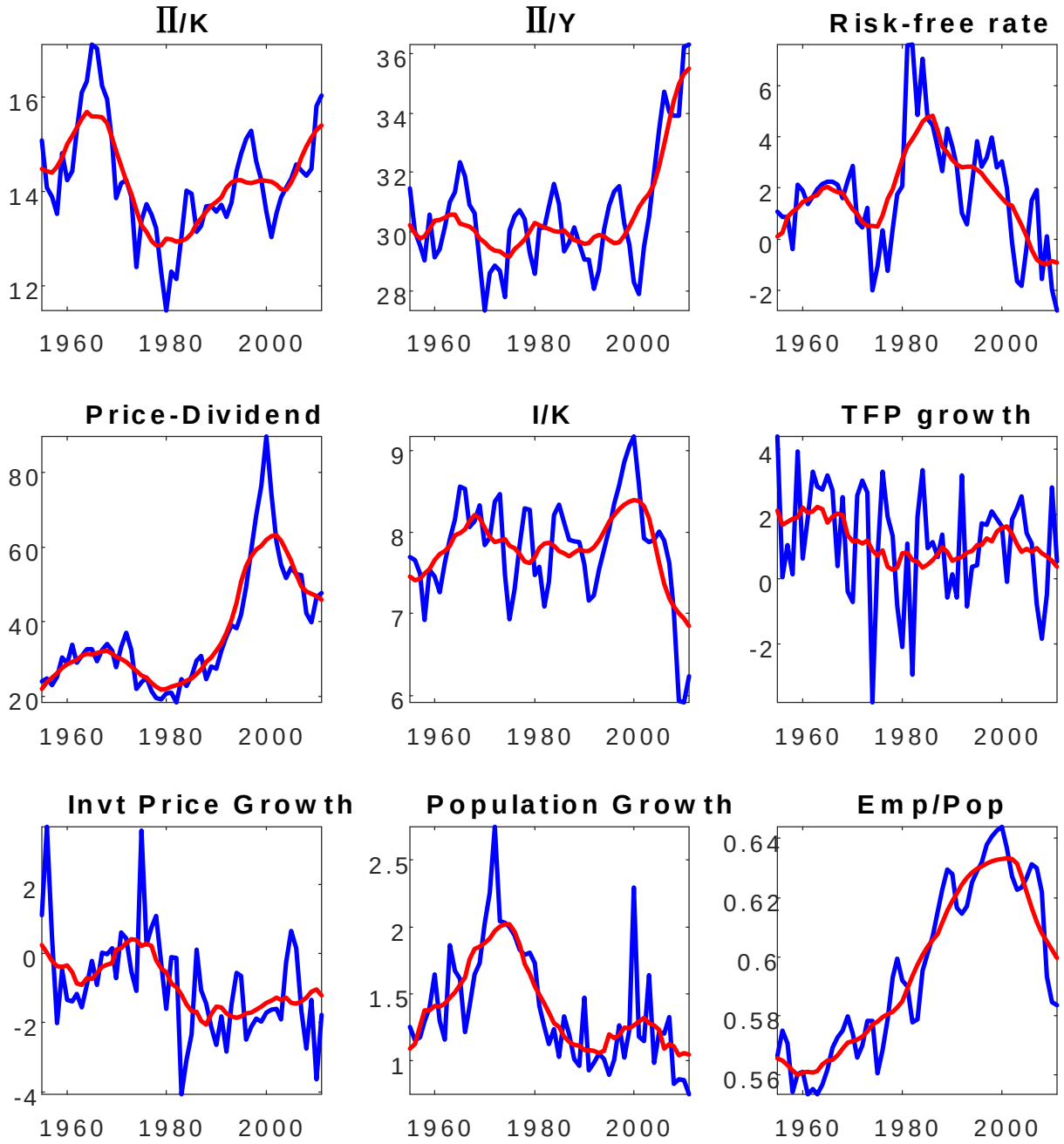

Figure 4: This figure presents the nine series used in our estimation exercise over the 1955-2011 sample, together with a 11–year centered moving average.

Final output is produced using constant return to scale from differentiated inputs,

$$Y_t = \left(\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $\varepsilon > 1$ is the elasticity of substitution. These intermediate goods are produced using a Cobb-Douglas production function,

$$y_{it} = Z_t k_{it}^{\alpha} (S_t n_{it})^{1-\alpha},$$

where k_{it} and n_{it} are capital and labor in firm i at time t, Z_t is an exogenous deterministic productivity trend, and S_t is a stochastic productivity process, which we assume to be a simple unit root:

$$S_{t+1} = S_t e^{\chi_{t+1}}, \tag{2}$$

where χ_{t+1} is *iid*.

Capital is accumulated using a standard investment technology, but is subject to an aggregate "capital quality" shock ψ_{t+1}:

$$k_{it+1} = \left((1-\delta) k_{it} + Q_t x_{it} \right) e^{\psi_{t+1}}.$$

Here Q_t is an exogenous deterministic process reflecting investment-specific technical progress as in Greenwood, Hercowitz, and Krusell (1997). The relative price of investment and consumption goods is $1/Q_t$.

Capital and labor can be reallocated frictionlessly across firms at the beginning of each period. Given the constant-return-to-scale technology, firms then face a constant (common) marginal cost mc_t. It is easy to see[5] that the economy aggregates to a production function

$$Y_t = Z_t K_t^{\alpha} (S_t N_t)^{1-\alpha}, \tag{3}$$

[5] This footnote details the mechanics. Each firm sets its price p_{it} and output y_{it} to maximize profits, subject to its demand curve:

$$\max_{y_{it},\, p_{it}} \left\{ (p_{it} - mc_t) y_{it} \right\},$$
$$s.t. \quad : \quad y_{it} = Y_t \left(\frac{p_{it}}{P_t} \right)^{-\varepsilon},$$

where P_t is the price index, which we can normalize to one as a numeraire. This program leads to the optimal markup equal to the inverse of the demand elasticity:

$$\frac{p_{it} - mc_t}{p_{it}} = \frac{1}{\varepsilon}.$$

Hence all firms set the same price, and in equilibrium we obtain that $n_{it} = N_t$, $k_{it} = K_t$, $y_{it} = Y_t$, $p_{it} = P_t = 1$ and marginal cost is

$$mc_t = \frac{\varepsilon - 1}{\varepsilon}.$$

Marginal cost can be calculated as the cost of expanding production using either labor or capital, or

$$mc_t = \frac{w_t}{MPN_t} = \frac{R_t}{MPK_t},$$

where w_t is the real wage, R_t the rental rate of capital, and MPN_t and MPK_t are the marginal products of labor and capital respectively. This leads to the first order conditions

$$\begin{aligned} (1-\alpha) \frac{Y_t}{N_t} &= \frac{\varepsilon}{\varepsilon - 1} w_t, \\ \alpha \frac{Y_t}{K_t} &= \frac{\varepsilon}{\varepsilon - 1} R_t. \end{aligned}$$

and that markups distort the firms' first order conditions, leading to

$$(1-\alpha)\frac{Y_t}{N_t} = \mu w_t, \tag{4}$$

$$\alpha\frac{Y_t}{K_t} = \mu R_t, \tag{5}$$

where $\mu = \frac{\varepsilon}{\varepsilon-1} > 1$ is the gross markup, w_t is the real wage and R_t the rental rate of capital.

Moreover, the law of motion for capital accumulation also aggregates,

$$K_{t+1} = \left((1-\delta)K_t + Q_t X_t\right) e^{\psi_{t+1}}. \tag{6}$$

The choice of investment is determined by the (common) marginal product of capital, leading to the Euler equation:

$$E_t\left[M_{t+1}R^K_{t+1}\right] = 1, \tag{7}$$

where M_{t+1} is the real stochastic discount factor and R^K_{t+1} is the return on capital, which is given by:

$$R^K_{t+1} = \left(\frac{\alpha Y_{t+1}}{\mu K_{t+1}} + (1-\delta)\frac{1}{Q_{t+1}}\right) Q_t e^{\chi_{t+1}}. \tag{8}$$

This expression is a standard user cost formula, which incorporates the rental rate of capital of equation (5) but also depreciation, the price of investment goods, and the capital quality shock. Given the preferences assumed in equation (1), the stochastic discount factor is

$$M_{t+1} = \beta\left(\frac{L_{t+1}}{L_t}\right)^{1-\sigma}\left(\frac{c_{pc,t+1}}{c_{pc,t}}\right)^{-\sigma}\left(\frac{V_{pc,t+1}}{E_t((V_{pc,t+1})^{1-\theta})^{\frac{1}{1-\theta}}}\right)^{\sigma-\theta}, \tag{9}$$

where $V_{pc,t}$ is the value per capita, $V_{pc,t} = V_t/L_t$.

The resource constraint reads

$$C_t + X_t = Y_t, \tag{10}$$

where $C_t = L_t c_{pc,t}$ is total consumption. Note that X_t are investment expenses measured in consumption good units.

The equilibrium of this economy is $\left\{c_{pc,t}, C_t, X_t, K_t, Y_t, R^K_{t+1}, M_{t+1}, V_{pc,t}, V_t\right\}$ that solve the system of equations (1)-(10), given the exogenous processes $\left\{L_t, Z_t, Q_t, S_t, \chi_{t+1}, \psi_{t+1}\right\}$. As is well known, such a model admits in general no closed form solution. Many authors build their intuition by solving for the nonstochastic steady-state. While useful, this obviously requires to abstract from macroeconomic risk, This makes it difficult to understand the role that macroeconomic risk may play, leading most authors to rely on numerical approximations. We will show, in contrast, that for an interesting special case, our model can be solved easily for a "risky balanced growth path".

3.2 Risky balanced growth

We make two simplifying assumptions. First, to obtain a balanced growth path, we make the usual assumption that population L_t, total factor productivity Z_t, and investment-specific technical progress Q_t all grow at (possibly different) constant rates, so that $L_{t+1}/L_t = 1+g_L$, $Z_{t+1}/Z_t = 1+g_Z$, $Q_{t+1}/Q_t = 1+g_Q$. Second, we assume that the productivity shock and capital quality shock are equal:

$$\chi_{t+1} = \psi_{t+1}.$$

In that case, it is straightforward to verify that there is an equilibrium which has the following structure:

$$\begin{aligned} X_t &= T_t S_t x^*, \\ Y_t &= T_t S_t y^*, \end{aligned}$$

and similarly for C_t and V_t, while for capital we have $K_t = T_t S_t Q_t k^*$.Here the lower case starred values denote constants; S_t is the stochastic trend defined in equation (2) corresponding to the accumulation of past productivity/capital quality shocks χ_t; and T_t is a common deterministic trend defined as:

$$T_t = L_t Z_t^{\frac{1}{1-\alpha}} Q_t^{\frac{\alpha}{1-\alpha}},$$

which growth rate is denoted g_T. Finally, the stochastic discount factor is

$$M_{t+1} = \beta \left(1 + g_L\right) \left(1 + g_T\right)^{-\sigma} e^{-\theta \chi_{t+1}} E(e^{(1-\theta)\chi_{t+1}})^{\frac{\theta-\sigma}{1-\theta}},$$

and we can hence easily calculate all objects of interest in the model.

Figure 5 presents an example of the time series produced by the model. The equilibrium corresponds to a "balanced growth path", but one where macroeconomic risk still affects decisions and realizations. Specifically, the realization of χ_{t+1} affects S_{t+1} and hence X_{t+1}, Y_{t+1}, etc., while the effect of risk, on the other hand, is reflected in the constants x^*, c^*. These constants, which are outcomes of optimization, will be solved in closed form in the next section. The bottom line is that the "big ratios" such as I_t/Y_t, Π_t/Y_t, $\Pi_t/(K_t/Q_t)$, etc. are constant, as in the standard Kaldor calculations, but now incorporate risk.[6] This result holds regardless of the probability distribution of χ_{t+1}.

The treatment of deterministic trends is completely standard. What is less standard is that the model allows a common stochastic trend to affect equally all variables, which generates great tractability. If $\psi_{t+1} = 0$, a permanent productivity shock χ_{t+1} leads to a transition as the economy adjusts its capital stock to the newly desired level, before eventually reaching the new steady-state. By assuming $\chi_{t+1} = \psi_{t+1}$, this transition period is eliminated because the capital stock "miraculously" adjusts by the correct amount. This simplifies the solution of the model because agents' expectations of future paths are now easy to calculate.[7,8] The capital quality shock is also important if the economy is to generate a significant equity premium, for it makes the return on capital volatile rather than bounded below by $1 - \delta$.

3.3 Model implications

This section presents model implications for the "big ratios" and other key moments of interest along the risky balanced growth path. We will present the Euler equation, which leads to a standard user

[6] If the economy starts off the balanced growth path, because its capital stock is either too low or too high, it will converge over time to the balanced growth path.

[7] Since we will not study the actual responses to χ_{t+1} shocks, there is little loss in this simplification: what is key for us is that agents regard the future as uncertain, and that bad realizations of χ_{t+1} will have reasonable consequences (e.g. a low return on capital), which lead agents ex-ante to adjust their choices (e.g. capital accumulation).

[8] This argument (formulated in Gabaix (2011) and Gourio (2012)) can be applied to larger models; for instance see Gourio, Kashyap and Sim (2018) or Isore and Szczerbowicz (2018) for new keynesian models with disaster risk.

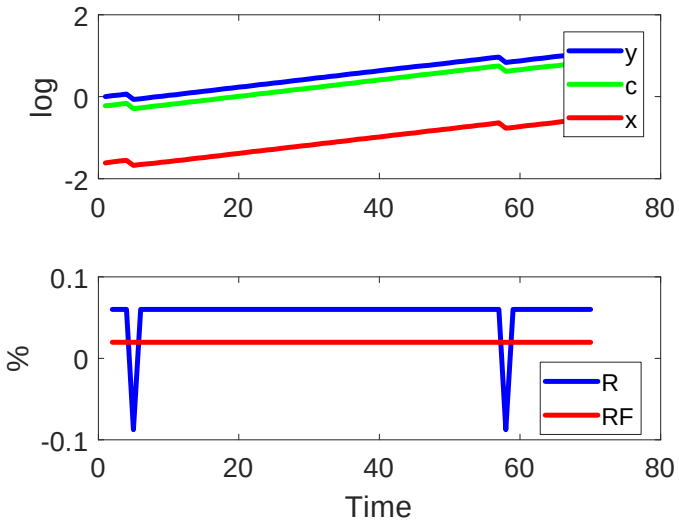

Figure 5: The figure presents an example of the time series generated by the model. Top panel: output, consumption and investment (in log); bottom panel: return on capital and risk-free rate. In this example, the economy is affected by two realizations of χ shocks, at $t = 4$ and $t = 57$.

cost calculation, then discuss valuation ratios and rate of returns. But first, note that the deterministic trend growth rate of GDP is the growth rate of T, which satisfies the standard formula

$$1 + g_T = (1 + g_Q)^{\frac{\alpha}{1-\alpha}} (1 + g_L)(1 + g_Z)^{\frac{1}{1-\alpha}}, \tag{11}$$

where α is the Cobb-Douglas parameter, g_Q the rate of growth of investment-specific technical progress, g_L is population growth, and g_Z is productivity growth.

It is useful to define the composite parameter

$$\beta^* = \beta(1 + g_L)(1 + g_T)^{-\sigma} \times E(e^{(1-\theta)\chi_{t+1}})^{\frac{1-\sigma}{1-\theta}}, \tag{12}$$

and its rate of return version $r^* = 1/\beta^* - 1$, which satisfies

$$r^* \simeq \rho - g_L + \sigma g_T + \sigma \frac{1 - 1/\sigma}{1 - \theta} \log E(e^{(1-\theta)\chi_{t+1}}), \tag{13}$$

where $\rho = 1/\beta - 1$. The parameter r^* will turn out to be, in equilibrium the expected return on capital, and to be a "sufficient statistic" to solve for the "big ratios" (i.e., we do not need to know ρ, θ, or the distribution of χ, but only r^*.)

3.3.1 Capital accumulation

To solve the model, we use the Euler equation (7), which along the risky balanced growth path reads

$$\frac{1}{\beta^*} = \left(\frac{\alpha}{\mu} Q^* \left(\frac{k^*}{\overline{N}} \right)^{\alpha - 1} \frac{1}{1 + g_Q} + \frac{1 - \delta}{1 + g_Q} \right), \tag{14}$$

where Q^* is the "level" of Q_t, i.e. $Q_t = Q^*(1 + g_Q)^t$. This equation pins down the capital-labor ratio, and it generalizes the familiar condition of the neoclassical growth model. We can rewrite this (approximately) as the equality of the user cost of capital and marginal revenue:

$$r^* + \delta + g_Q \approx \frac{\alpha}{\mu} Q^* \left(\frac{k^*}{N^*} \right)^{\alpha - 1}, \tag{15}$$

Equation (15) directly shows how higher market power or a higher required risky return lower the desired capital-labor ratio.

To calculate the other big ratios, first note that K_t/Q_t is the capital stock, evaluated at current cost. The capital-output ratio is obtained from equation (15) as:

$$\frac{K_t/Q_t}{Y_t} \approx \frac{\alpha}{\mu} \frac{1}{r^* + \delta + g_Q}, \tag{16}$$

and the investment-capital ratio is

$$\frac{X_t}{K_t/Q_t} \approx g_Q + g_T + \delta, \tag{17}$$

which reflects the familiar balanced growth relation. Last, the investment-output ratio is obtained by combining equations (17) and (16)

$$\frac{X_t}{Y_t} \approx \frac{\alpha}{\mu} \frac{g_T + \delta + g_Q}{r^* + \delta + g_Q}. \tag{18}$$

3.3.2 Income Distribution

The labor share (in gross value added) is, using equation (4):

$$s_L = \frac{w_t N_t}{Y_t} = \frac{1-\alpha}{\mu}, \tag{19}$$

and hence the measured capital share is $s_K = 1 - s_L = \frac{\mu+\alpha-1}{\mu}$. This capital share can be decomposed into a pure profit share, that rewards capital owners for monopoly rents, and a true capital remuneration share, corresponding to rental payments to capital, i.e. $s_K = s_\pi + s_C$, with

$$s_\pi = \frac{\mu - 1}{\mu}, \tag{20}$$

and

$$s_C = \frac{\alpha}{\mu}. \tag{21}$$

3.3.3 Valuation ratios

The firm value is simply the present discounted value of the dividends $D_t = \Pi_t - X_t$. In equilibrium, as we will see, this value equals the value of installed capital plus monopolistic rents. Formally, the ex-dividend firm value P_t satisfies the standard recursion,

$$P_t = E_t \left(M_{t+1} \left(P_{t+1} + D_{t+1} \right) \right).$$

Given that the equilibrium is *iid*, the price-dividend ratio is constant, and satisfies the familiar Gordon formula:

$$\frac{P^*}{D^*} = \frac{1}{\frac{1}{\beta^*(1+g_T)} - 1} \approx \frac{1}{r^* - g_T}. \tag{22}$$

Tobin's Q is

$$\frac{P_t}{K_t/Q_t} \approx 1 + \frac{\mu - 1}{\alpha} \frac{r^* + \delta + g_Q}{r^* - g_T}. \tag{23}$$

Because we do not incorporate adjustment costs, Tobin's Q equals one when there is no market power, i.e. $\mu = 1$.

3.3.4 Rates of Return

We now compare three benchmark rate of returns in this economy: the risk-free rate, the return on equity, and the profitability of capital, which is often used in macroeconomics as a proxy for the marginal product of capital. The gross risk-free rate (which can be priced even though it is not traded in equilibrium) is

$$R^f = \frac{1}{E(M_{t+1})} = \frac{E\left(e^{(1-\theta)\chi_{t+1}}\right)}{\beta^* E\left(e^{-\theta\chi_{t+1}}\right)},$$

which we can rewrite as the net risk-free rate, i.e. $r^f = R^f - 1$:

$$r_f \approx r^* + \log E\left(e^{(1-\theta)\chi_{t+1}}\right) - \log E\left(e^{-\theta\chi_{t+1}}\right). \tag{24}$$

The average profitability of capital can be inferred, as in Gomme, Ravikumar and Rupert (2011) and Mulligan (2002), as the ratio of (measured) profits to the stock of capital. This can be calculated either gross or net of depreciation. For instance, in gross terms, we have

$$\begin{aligned} MPK &= \frac{\Pi_t}{K_t/Q_t}, \\ &= \frac{\mu+\alpha-1}{\alpha}\left(r^* + \delta + g_Q\right). \end{aligned} \tag{25}$$

Conceptually, this MPK excess the risk-free rate for three reasons; first, it is gross of both physical and economic depreciation; second, it incorporates profit rents; third, it is risky. We can decompose the spread between the MPK and the risk-free rate to reflect these different components:

$$MPK - r_f = \delta + g_Q + \frac{\mu-1}{\alpha}\left(r^* + \delta + g_Q\right) + r^* - r_f. \tag{26}$$

A main goal of our empirical analysis is to evaluate the importance of these different components in the data.

The expected equity return is defined as

$$E\left(R_{t+1}\right) = E\left(\frac{P_{t+1} + D_{t+1}}{P_t}\right),$$

and it is easy to show using equation (22) that

$$E(R_{t+1}) = \frac{1}{\beta^*} E(e^{\chi_{t+1}}). \tag{27}$$

In the case where $E\left(e^{\chi_{t+1}}\right) = 1$, which we will enforce in our applications, the gross expected return on equity is exactly $1/\beta^*$, and the net return is r^*.

Finally, the equity risk premium is obtained by combining equations (24) and (27):[9]

$$ERP = \frac{E\left(R_{t+1}\right)}{R_{f,t+1}} = \frac{E\left(e^{-\theta\chi_{t+1}}\right) E(e^{\chi_{t+1}})}{E\left(e^{(1-\theta)\chi_{t+1}}\right)}.$$

3.3.5 Distributional assumptions for χ_{t+1}

The expressions for the key moments of interest involve expectations of the macro shocks χ_{t+1}. It is useful to spell out these expectations in some interesting special cases. Technically, following Martin

[9] The same risk premium also applies to the return on capital R^K_{t+1} : $ERP = E(R^K_{t+1})/R^f$ where R^K_{t+1} is defined in equation 8.

(2013), we recognize that we can rewrite these moments using the moment-generating function, defined for $x \in \mathbb{R}$ as $\phi(x) = E\left(e^{x\chi_{t+1}}\right)$. In particular, defining $\widehat{\beta} = \beta(1+g_L)(1+g_T)^{-\sigma}$, we have:

$$\log \beta^* = \log \widehat{\beta} + \frac{1-\sigma}{1-\theta} \log \phi(1-\theta),$$

$$\log R^f = -\log \widehat{\beta} - \frac{\theta-\sigma}{1-\theta} \log \phi(1-\theta) - \log \phi(-\theta),$$

$$\log ERP = \log \phi(1) + \log \phi(-\theta) - \log \phi(1-\theta).$$

While in our paper we focus on the case where χ is a rare disaster, nothing in our analysis precludes using a different distribution. One that is particularly tractable is the lognormal case, i.e. χ is normal with mean μ_χ and variance σ_χ^2. In particular, setting $\mu_\chi = -\sigma_\chi^2/2$, an increase in σ_χ is a pure risk shock (i.e. in the sense of second order stochastic dominance). In that case, we have $\phi(x) = e^{-x(1-x)\frac{\sigma_\chi^2}{2}}$, and hence

$$\log \beta^* = \log \widehat{\beta} - (1-\sigma)\theta\frac{\sigma_\chi^2}{2}, \tag{28}$$

$$\log R^f = -\log \widehat{\beta} - (1+\sigma)\theta\frac{\sigma_\chi^2}{2}, \tag{29}$$

$$\log ERP = \theta\sigma_\chi^2. \tag{30}$$

These formulas capture the usual effect of risk aversion and the quantity of risk on the ERP and the risk-free rate, but are now valid in a production economy, and furthermore β^* links macroeconomic risk to macroeconomic variables such as the capital-output ratio as discussed above.[10]

In our application we will assume that$\chi_{t+!}$ follows a three-point distribution, i.e.

$$\begin{aligned}
\chi_{t+1} &= 0 \text{ with probability } 1-2p, \\
\chi_{t+1} &= \log(1-b) \text{ with probability } p, \\
\chi_{t+1} &= \log(1+b_H) \text{ with probability } p,
\end{aligned}$$

where b and b_H and are chosen so that $E\left(e^{\chi+1}\right) = 1$. The second state is a "disaster": output and consumption fall permanently by a factor $1-b$. The third state is a "windfall" or "bonanza" state that offsets the mean effect of the disaster. One could also use a more traditional two-point process, without the third state, which would then not satisfy $E\left(e^{\chi+1}\right) = 1$, and in that case a change in p would have

[10] Another tractable case is the compound Poisson process. Suppose that for $j \geq 0$,

$$\Pr(\chi_{t+1} = -jb) = \frac{\lambda^j}{j!}e^{-\lambda},$$

i.e. instead of at most a single disaster realization per period, there are potentially several of these shocks, and that the number of shocks follows a Poisson distribution, with intensity $\lambda \approx p$. (Because p is small, this compound Poisson process case is very close quantitatively to the simple binomial case, but leads to somewhat more elegant formulas.) The moment generating function is $\phi(x) = e^{\lambda\left(e^{-xb}-1\right)}$, and the objects of interest are:

$$\begin{aligned}
\log \beta^* &= \log \widehat{\beta} + \frac{1-\sigma}{1-\theta}\lambda\left(e^{-(1-\theta)b}-1\right), \\
\log R^f &= -\log \widehat{\beta} + \frac{\theta-\sigma}{1-\theta}\lambda\left(e^{-(1-\theta)b}-1\right) - \lambda\left(e^{\theta b}-1\right), \\
\log ERP &= \lambda\left(e^{-b}+e^{\theta b}-e^{-(1-\theta)b}-1\right).
\end{aligned}$$

It is straightforward to extend this calculation to the case of random size of shocks b, as in Kilic and Wachter (2017).

both a first moment and second moment effect. In any event, because our estimation approach does not rely on the specific process for χ_t, the purpose of this assumption is simply to map the implied risk into a more tangible measure, the change in p.

3.4 Comparative statics

We now use the expressions developed in the previous section to illustrate some key comparative statics of the risky balanced growth path. These comparative statics are useful to understand identification of our model. Most of the parameters have the usual effects; we will focus on parameters that are often absent from the neoclassical growth model, or parameters that play an important role in our empirical results.

3.4.1 Effect of risk

The effect of higher risk on macroeconomic variables is mediated through β^*. The cleanest thought experiment is to consider a shift of the distribution of the shock χ in the sense of second-order stochastic dominance. This would reduce the quantity $E\left(\chi^{1-\theta}\right)^{\frac{1}{1-\theta}}$, and hence lead to a lower β^* if and only if $\sigma < 1$ i.e. the intertemporal elasticity of substitution is greater than unity. A lower β^* in turn leads to lower investment-output and capital-output ratios according equations (18) and (**??**), and a higher profit-capital ratio according to equation (25). The logic is that risk deters investment in that case, leading to less capital accumulation. This reduction in the supply of capital increases the marginal product of capital given a stable capital demand. Higher risk decreases the price-dividend ratio according to the Gordon growth formula (22) because the equity premium $r^* - r_f$ is increasing in risk. Risk has no effect on the labor share or long-term growth (though higher risk creates a level effect on capital and GDP). The spread between the MPK and the risk-free rate is increasing in risk (equation 26).

3.4.2 Effect of savings supply

A higher discount factor β has the same exact effects as a decrease in risk (provided that the intertemporal elasticity is greater than unity) since its effects are mediated through β^*. Indeed, the one moment which is not affected identically by both measures is the risk-free rate, which is affected directly by β^* but also by risk (θ, χ). Hence, higher savings supply (which one might interpret for instance as an increase in longevity, in the usual interpretation of the dynastic model) leads to higher capital accumulation, higher investment-output ratio and a lower marginal product of capital (reflecting higher supply of capital with stable demand), and a higher price-dividend ratio, while the risk-free rate falls. The spread between the MPK and the risk-free rate, shown in equation (26) is little affected by β, only affecting the quantity of rents through r^*. The equity risk premium $r^* - r_f$ is independent of β.

3.4.3 Effect of market power

One potentially important factor that has been invoked to explain of the trends we document is market power. In our model, an increase in μ has no effect on long-term growth, the risk-free rate, or the price-dividend ratio (it has a level effect on GDP), but it has a powerful effect on other variables.

Higher markups reduce both the labor share and the "true capital share", that is, the share of national income devoted to the fair (risk-adjusted) compensation of capital, but increases the pure profit share. According to equations (18) and (**??**), higher market power reduces investment-output and capital-output ratios, as firms have less incentive to build capacity. The spread between the MPK and the risk-free rate is increasing in market power (equation (26)).

3.4.4 Effect of trend growth

Trend growth g_T - which can traced back to productivity growth, population growth, or investment-specific technical growth – affects β^* but also affects independently the ratios of interest. Higher growth generally increases the investment-capital and investment-output ratios and increases the risk-free rate and valuation ratios, while the effect on profitability ratios depends on the exact source of growth.

4 Accounting framework

This section describes our empirical approach and discusses our identification procedure.

4.1 Methodology

We use a simple method of moment estimation. In the interest of clarity and simplicity, we perform an exactly identified estimation with 9 parameters and 9 moments. In a first exercise, we estimate the model separately over our two samples: 1984-2000 and 2001-2016. We then discuss which parameters drive variation in each moment. In a second exercise, we estimate the model over 11-year rolling windows, starting with 1950–1961, and ending with 2006-2016. In all cases, we fit the model risky balanced growth path to the model moments. In doing so, we abstract from business cycle shocks, in line with our focus on longer frequencies.[11]

The moments we target are motivated by the observations in introduction and in the first section:

(M1) the measured gross profitability Π/K;

(M2) the measured gross capital share Π/Y;

(M3) the investment-capital ratio I/K;

(M4) the risk-free rate R^f;

(M5) the price-dividend ratio PD;

(M6-M8) the growth rates of population, total factor productivity, and investment prices;

(M9) the employment-population ratio.

As we will see, these moments will lead to a clear identification of our nine parameters, which are:

(P1) the discount factor β;

(P2) the probability of an economic crisis or "disaster" p;

(P3) the markup μ;

[11] This exercise involves some schizophrenia, because our model assumes that parameters are constant, even though they are estimated to change over time; and when parameters change, the model would exhibit some transitional dynamics, which we abstract from for now; see Section 6. Further, the agents inside our model do not understand that parameters might change, let alone anticipate some of these changes.

(P4) the depreciation rate of capital δ;

(P5) the Cobb-Douglas parameter α;

(P6-P8) the growth rates of total factor productivity g_Z, investment-specific progress g_Q, and population g_L;

(P9) the labor supply parameter $\overline{N}$.

The choice of moments is motivated, of course, by the questions of interest - explaining the joint evolution of interest rates, profitability, investment, valuation, and trend growth - but also by the clarity with which these moments map into estimated parameters. For instance, note that given that we target Π/K, Π/Y and I/K (and that we have taken care to construct these moments in a consistent manner), the model will mechanically match the evolution of the investment-output ratio I/Y or the capital-output ratio K/Y. Hence, we could have taken I/Y as a target moment, which would have lead to the exact same estimates and implications, but the identification is clearer with I/K. Beyond this, some changes in identification strategy are possible however; for instance, one could target the price-earnings ratio instead, or GDP growth per worker; these yield quite similar results.[12]

There are three parameters which we do not estimate. First, the elasticity of intertemporal substitution is not identified separately from β along our risky balanced growth path. We use an elasticity equal to 2, as in Gourio (2012). Changing this values has no impact on the fit of the model or the estimated parameters, except that it affects the estimated value for β, and some counterfactuals. We discuss this in Section 6 in more detail.

Second, the coefficient of risk aversion and the size of the macroeconomic shock ("disaster") are not identified separately from the probability of a disaster. Indeed, it is possible to fit our model assuming a constant disaster probability, but time-varying risk aversion, rather than the opposite. Or it is possible to fit the model with a time-varying size of disaster rather than a time-varying probability. We use a coefficient of risk aversion equal to 12, and a disaster size $b = 0.15$ (significantly lower than in the disaster literature pioneered by Barro (2006)). Here too, changing these values has no impact on the fit of the model or the estimated parameters, except in that it affects the estimated value for the disaster probability p.

4.2 Identification

In this section we provide a heuristic discussion of identification. The identification is *almost* recursive. In a first step, some parameters are obtained directly as their counterparts are assumed to be directly observed: population growth, investment price growth (the opposite of g_Q), and the employment-population ratio. The growth rate g_Z is chosen to match measured total factor productivity.[13] One hence obtains g_T, the trend growth rate of GDP, given by equation (11). The depreciation rate δ is next chosen to match I/K according to equation (17), which is the familiar balanced growth relation:

[12] We have also verified that taking into account the mismeasurement of TFP (i.e. the labor share used for measuring TFP is not $1-\alpha$ owing to market power) does not have a significant effect on our conclusion.

[13] This step requires, however, a value for the parameter α, which is why we say that the identication is "almost" recursive. This caveat does not affect much the identification "in practice".

$$\frac{I}{K} \simeq \delta + g_Q + g_T$$

The model then uses the Gordon growth formula (22) to infer the expected return on risky assets, r^* given the observed PD ratio:

$$\frac{P^*}{D^*} \simeq \frac{1}{r^* - g_T}.$$

The value of r^*, as noted in the previous section, is a "sufficient statistic" for the big ratios - there is no need to know separately β, risk aversion, or actual risk. Our approach hence is to use the traditional Gordon growth formula - which holds in our standard neoclassical framework - to deduce the required return on capital from the PD ratio, given the growth rate.

There are finally two parallel steps. On the one hand, one can deduce the risk premium, and hence the actual risk, by using the risk-free rate; on the other, one can deduce α and μ. First, using equation (24), we have

$$r_f = r^* + \log E\left(e^{(1-\theta)\chi_{t+1}}\right) - \log E\left(e^{-\theta\chi_{t+1}}\right) m$$

which is one equation in one unknown (p).Economically, this amounts to inferring the level of risk that justifies the observed risk premium $r^* - r_f$. (It is this step that requires us to take a stand on the distribution of χ_t and on whether it is p or θ which is varying.)

Second, the parameters α and μ are chosen to match the profit share of output and the ratio of profits to capital given the other parameters, i.e. using the equations (19) and (26). Specifically, denote by W the wedge between the marginal product of capital and its value if there was no market power, i.e.

$$W = \frac{MPK}{r^* + \delta + g_Q}.$$

Then, the model finds μ given W and the observed labor share LS :

$$\frac{\mu - 1}{\mu} = \frac{W - 1}{W}(1 - LS), \tag{31}$$

and then finally obtains α as

$$1 - \alpha = \mu\,(1 - LS)\,. \tag{32}$$

Intuitively, equation (31) infers market power by the magnitude of the discrepancy between the MPK and the frictionless user cost of capital. The parameter α is then obtained to fit the observed labor share.

Our procedure is hence closely related to the approach of Barkai (2016) but differs in a crucial point - we bring information from asset prices (the PD ratio) to infer the expected return on risky investment, i.e. to infer a risk premium, which then affects the user cost of capital. Barkai (2016) simply uses a treasury rate or corporate bond yield to construct the user cost.

5 Empirical Results

We first compare the two subsamples, then we contrast the results with more standard macroeconomic approaches which do not entertain a role for risk, and finally we present results over rolling windows in a long sample.

Parameter name	Symbol	Estimates		
		1984-2000	2001-2016	Difference
Discount factor	β	0.955	0.967	0.012
Markup	μ	1.079	1.146	0.067
Disaster probability	p	0.034	0.065	0.031
Depreciation	δ	2.778	3.243	0.465
Cobb-Douglas	α	0.244	0.243	-0.000
Population growth	g_N	1.171	1.101	-0.069
TFP growth	g_Z	1.298	1.012	-0.286
Invt technical growth	g_Q	1.769	1.127	-0.643
Labor supply	$\overline{N}$	0.623	0.608	-0.015

Table 2: The table reports the estimated parameters in our baseline model for each of the two subsamples, 1984-2000 and 2001-2016, and the change between subsamples.

5.1 Comparison of two subsamples

Table 2 shows the estimated parameters for each subsample and the change of parameters between subsamples. Overall, our results substantiate many of the narratives that have been advanced and that we mention in the introduction. The discount factor β rises by 1.2 point, reflecting higher savings supply. Market power increases significantly, by 6.7 points. Technical progress slows down and labor supply falls (relative to population). The model also estimates a significant increase in macroeconomic risk (the probability of a crisis), which goes from 3.4% per year to 6.5% per year. On the other hand, there is only moderate technological change: depreciation increases, reflecting the growing importance of high-depreciation capital such as computers, but the Cobb-Douglas parameter remains fairly stable. This stability of the production function is an interesting result. Overall, the model gives some weight to four of the most popular explanations (β, μ, p, gs). But exactly how much does each story explain?

Table 3 provides one answer. By construction, the model fits perfectly all nine moments in each subsample using the nine parameters . We can decompose how much of the change in each moment between the two subsamples is accounted for by each parameter. Because our model is nonlinear, this is not a completely straightforward task; in particular, when changing a parameter from first subsample value to second subsample value, the question is at which value to evaluate the other parameters (e.g., the first or second subsample value). If the model were linear, or the changes in parameters small, this would not matter, but such is not the case here (in particular for the price-dividend ratio, which is highly nonlinear). In this table, we simply report the average over all possible combinations of parameters, taken from first or second subsample.[14]

[14] Formally, let $\Theta^a = \left(\theta_1^a, ...\theta_K^a\right)$ and $\Theta^b = \left(\theta_1^b, ...\theta_K^b\right)$ denote the parameter vectors in subsample a and b respectively, and consider a model moment which is a function of the parameters: $m = f(\Theta)$. The change in m due to parameter

Overall, we see that the decline in the risk-free rate of 3.1% (314bps) is explained mostly by two factors, higher perceived risk p, and higher savings supply β, with lower growth playing only a moderate role.[15] Why does the model not attribute all the change in the risk-free rate to savings supply? Simply because it would make it impossible to match other moments, in particular the PD ratio. Even as it is, if only the change in savings supply were at work, the PD ratio would increase by nearly 32 points. The model attributes offsetting changes to risk and growth, explaining in this way that the PD ratio increased only moderately over this period despite the lower interest rates.

Similarly, profitability would decrease by about 2 points if the change in β was the only one at work - all rate of returns ought to fall if the supply of savings increases. The model reconciles the stable profitability with the data by inferring higher markups and higher risk. Overall, we see how the model needs multiple forces to account for the lack of changes observed in some ratios. The higher capital share, is attributed entirely to higher markups, as capital-biased technical change appears to play little role.

We can now use these model estimates to understand the evolution of some other moments; these are reported in 4. First, as we discussed in Section 3 (equation 26), the spread between the marginal product of capital and the risk-free rate can be decomposed in three components:

$$MPK - r_f = \delta + g_Q + \frac{\mu - 1}{\alpha}(r^* + g_Q + \delta) + r^* - r_f,$$

where the three components are depreciation ($\delta + g_Q$), rents, and risk ($r^* - r_f$). We can calculate this decomposition in the model using the estimated parameters. The table reveals that depreciation changed little overall - faster physical depreciation is offset by slower economic depreciation - but the rents and risk components both rise by about two percentage points. (An alternative way to decompose the change in spread is to read, in the first row, the decomposition of the change in spread due to each

$i \in \{1...K\}$ is defined as

$$\Delta_i = \frac{1}{N_\sigma} \sum_{\theta_{-i} \in \sigma} f(\theta_i^b, \theta_{-i}) - f(\theta_i^a, \theta_{-i}),$$

where θ_{-i} denotes the $K-1$ dimensional parameter vector excluding the i-th parameter, and σ is the set of all combinations of θ_{-i} such that each of the $K-1$ component is either the respective component of θ^a or of θ^b, and N_σ is the number of elements of σ, which equals 2^{K-1}. By construction, $\sum_{i=1}^{K} \Delta_i = f(\Theta^b) - f(\Theta^a)$ accounts exactly for the model implied change in the moment, which, because the model fits the target moments perfectly, accounts also exactly for the change in the data: $f(\Theta^b) - f(\Theta^a) = m^b - m^a$.

In table 16 in appendix, we report the upper and lower bounds when consider all possible combinations of other parameters. This provides a way to bound the importance of each factor.

Conceptually, our approach is equivalent to consider all the possible *orders* in which the parameters could be changed from the initial sample to the final sample, and averaging over all these orders. Of course, there is no direct economic meaning to the ordering.

An alternative approach is to simply fix all the parameters, except the one we are interested in, at their initial (or final) value. The results are similar overall, except for the PD ratio which is highly nonlinear. Moreover, the results are not perfectly additive in that case, leading us to prefer this method. We thank Sam Schulhofer-Wohl for this suggestion.

[15] This conclusion does depend somewhat on our assumed intertemporal elasticity of substitution, as we discuss in detail below.

	Target Moment			Contribution of each parameter to change in moment								
	1984-00	2001-16	Diff.	β	μ	p	δ	α	g_N	g_Z	g_Q	$\overline{N}$
Gross profitability	14.01	14.89	0.88	-1.94	2.76	0.76	0.68	0.00	0.05	-0.29	-1.15	-0.00
Measured cap. share	29.89	33.99	4.10	-0.00	4.13	-0.00	0.00	-0.03	-0.00	0.00	-0.00	-0.00
Risk-free rate	2.79	-0.35	-3.14	-1.25	0.00	-1.62	0.00	-0.00	0.03	-0.19	-0.10	0.00
Price-dividend ratio	42.34	50.11	7.78	31.89	0.00	-13.34	0.00	-0.02	-2.82	-5.13	-2.80	0.00
Investment-capital	8.10	7.23	-0.88	0.00	-0.00	0.00	0.47	-0.00	-0.07	-0.39	-0.88	-0.00
Growth of TFP	1.10	0.76	-0.34	-0.00	-0.14	0.00	0.00	-0.00	-0.00	-0.26	0.06	0.00
Growth of invt. price	-1.77	-1.13	0.64	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.64	0.00
Growth population	1.17	1.10	-0.07	0.00	0.00	0.00	0.00	0.00	-0.07	0.00	0.00	0.00
Employment-pop.	62.3	60.8	-1.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.50

Table 3: The table reports the target moments in each of the two subsamples 1984-2000 and 2001-2016, as well as the change between samples, and the contribution of each parameter to each change in moment. See text for details.

parameter change; this yields a similar answer, as the increases in μ and in p account for the bulk of the increase in the spread).

We also report the model implied equity return and equity premium. While not a direct target, we estimate a sizeable equity premium, of nearly 5 percent per year in the recent sample. (This premium assumes no leverage; see section 6 for a discussion of leverage.) More interestingly, the premium increased by around 2 percentage points since 2000. In total, equity returns have fallen slightly because the decline in the risk-free rate is larger than this increase in the equity premium.

Another implication of the model is the income distribution between labor, capital, and rents. The approach taken here is that we observe accurately the payments to labor in the data, and cannot easily split the remainder between capital and profits. In the model, we can study the decomposition and how it changes between the two subsamples. The nearly 4 point decline in the labor share is accompanied by an even larger increase in the profit share, by nearly 5 points, so that the capital share actually declines slightly.

Finally, we can use the model to see the effect of these changes on macroeconomic variables - for instance the capital-output or investment-output ratios. On one hand, higher savings supply pushes investment up leading to more capital accumulation. For instance, the change in β would push the investment-output ratio up by over 2 percentage points, while in the data it fell. On the other hand, rising market power and rising risk push investment down. Our model hence accounts for the coexistence of low investment and low interest rates. Note also that higher depreciation also requires more investment along the balanced growth path, while lower growth implies less investment. The model hence produces a fairly nuanced decomposition for the evolution of this ratio.

Last, we can ask what is the effect of each parameter on the level of GDP or investment.[16] For instance, higher market power discourages capital accumulation and reduces output. It is easy to show that the elasticity of GDP to markups in this model is $-\alpha/(1-\alpha)$, or -0.32 for our estimate. Given estimated markups rise by 6.2 percent (=6.7/1.079), the effect on GDP is about -0.32×6.2, or about minus two percentage points (-1.95% in our table). Here too, there are several counteracting factors, however, which imply that the overall level effect on GDP is small (-0.30%). In particular, higher savings supply and lower economic depreciation lead to higher capital accumulation, while higher risk leads to lower capital accumulation. Investment is more negatively affected by the changes, with a level effect of about minus 5 percentage points, owing largely to markups and risk, but also to lower growth and a lower employment-population ratio.

5.2 Comparison with macroeconomic approaches

It is interesting to compare our results with alternative procedures followed by macroeconomists. Indeed, our empirical exercise is essentially the calibration of the "steady-state" of a very bare-bone DSGE model. Any DSGE model writer faces the same issues we do in fitting these key moments of interest.

Indeed, real business cycle modelers are aware of a trade-off between fitting the capital-output ratio and the risk-free interest rate. Since these models target the labor share, the discrepancy precisely reflects the gap between the MPK (profitability, or profit-capital ratio) and the risk-free interest rate. Often, modelers reject short-term Treasury interest rates as measures of the rate of return on capital, noting that these securities have special safety and liquidity attributes, which are not explicitly modeled.[17] In terms of our framework, this approach involves rejecting the use of the risk-free rate as a target moment, and setting risk to zero. Furthermore, many models have traditionally abstracted from aggregate market power, setting $\mu = 1$,[18] and have not explicitly targeted the price-dividend ratio. The assumptions lead to a well-defined exactly identified exercise with seven moments and seven parameters, which is an alternative to our approach. The last two columns of Table 5 present the results from this exercise, which we call the "macro-without-markups" approach. (We infer the liquidity premium required to explain the discrepancy between the MPK and the risk-free rate.) This approach leads to a much higher value of α and "explains" the decline of the labor share by an increase of α. The decline of the Treasury rate, and the growing gap between the MPK and this rate, are fully accounted for by a very large, and growing, liquidity premium.

A more ambitious approach is to incorporate market power and target the risk-free rate, while still omitting the PD ratio from the list of targets and risk from the potential parameters. This is also a well-

[16] By level of GDP we mean y^*, i.e. the level of GDP once the proper deterministic and stochastic trends have been removed. We abstract from the growth effects - e.g., a higher g_Z or g_Q has the mechanical effect of steepening the overall path of GDP.

[17] See for instance Campbell et al. (2017) for a presentation of the Chicago Fed DSGE model, which, based on Fisher (2015), introduces a liquidity wedge that accounts for the discrepancy between the risk-free rate and the rate of return of capital.

[18] New Keynesian models are an important exception, but market power is often set on a priori basis in these studies (e.g., a markup of 15%), and profits are offset in steady-state by fixed costs.

	Model implied moment			Contribution of each parameter								
	1984-99	2000-16	Change	β	μ	p	δ	α	g_N	g_Z	g_Q	N
A. MPK-RF spread												
Total spread	11.22	15.24	4.02	-0.68	2.76	2.39	0.68	0.00	0.02	-0.10	-1.05	-0.00
- Depreciation	4.55	4.37	-0.18	0.00	0.00	0.00	0.47	0.00	0.00	0.00	-0.64	0.00
- Market power	3.39	5.55	2.17	-0.60	2.73	0.24	0.21	0.00	0.02	-0.09	-0.35	0.00
- Risk premium	3.15	5.23	2.08	-0.05	0.00	2.14	0.00	-0.00	0.00	-0.01	-0.00	0.00
B. Rate of returns												
Equity return	5.85	4.90	-0.96	-1.25	0.00	0.56	0.00	-0.00	0.03	-0.19	-0.10	0.00
Equity premium	3.07	5.25	2.18	0.00	0.00	2.18	0.00	0.00	0.00	0.00	0.00	0.00
Risk-free rate	2.79	-0.35	-3.14	-1.25	0.00	-1.62	0.00	-0.00	0.03	-0.19	-0.10	0.00
C. Income distribution												
Share Labor	70.11	66.01	-4.10	0.00	-4.13	0.00	0.00	0.03	0.00	0.00	0.00	0.00
Share Capital	22.59	21.24	-1.35	0.00	-1.33	0.00	0.00	-0.03	0.00	0.00	0.00	0.00
Share Profit	7.30	12.76	5.46	0.00	5.46	0.00	0.00	0.00	0.00	0.00	0.00	0.00
D. Macroeconomic variables												
K/Y	2.13	2.28	0.15	0.30	-0.13	-0.12	-0.11	-0.00	-0.01	0.05	0.18	0.00
I/Y	17.28	16.50	-0.78	2.26	-1.03	-0.90	0.23	-0.02	-0.22	-0.52	-0.59	0.00
Detrend Y (% chg)	–	–	-0.30	4.30	-1.95	-1.70	-1.52	-0.07	-0.12	0.65	2.56	-2.45
Detrend I (% chg)	–	–	-4.95	17.67	-8.02	-6.98	-0.13	-0.20	-1.43	-2.45	-0.96	-2.45

Table 4: The table reports some moments of interest calculated in the model using the estimated parameter values for each of the two subsamples 1984-2000 and 2001-2016, as well as the change between samples, and the contribution of each parameter to each moment change.

	Baseline approach			Macro-with-markups			Macro-without-markups		
	1984-00	2001-2016	Diff.	1984-00	2001-2016	Diff.	1984-00	2001-2016	Diff.
β	0.955	0.967	0.012	0.978	1.007	0.028	0.920	0.908	-0.012
μ	1.079	1.146	0.067	1.165	1.330	0.166	1	1	0
p	0.034	0.065	0.031	–	0	0	0	0	0
δ	2.778	3.243	0.465	2.778	3.243	0.465	2.778	3.243	0.465
α	0.244	0.243	-0.000	0.183	0.122	-0.061	0.299	0.340	0.041
g_P	1.171	1.101	-0.069	1.171	1.101	-0.069	1.171	1.101	-0.069
g_Z	1.298	1.012	-0.286	1.544	1.358	-0.187	1.074	0.738	-0.335
g_Q	1.769	1.127	-0.643	1.769	1.127	-0.643	1.769	1.127	-0.643
N	0.623	0.608	-0.015	0.623	0.608	-0.015	0.623	0.608	-0.015

Table 5: The table reports the estimated parameters in each of the two subsamples 1984-2000 and 2001-2016 in our baseline model, in the macro model with markups, and in the macro model without markups.

posed exercise with 8 moments and 8 parameters, which we call the "macro-with-markups" approach. In this case, the spread between the MPK and the risk-free rate must reflect depreciation or rents. Intuitively, this approach assumes that the risk-free rate can be used to infer the cost of capital, and hence rents are deduced as a residual. The approach is conceptually quite similar to Barkai (2016), though we present it in a slightly more structural framework. The results are shown in the middle two columns of table 5. There are a number of differences between these results and our baseline results. First, the level of markups is much higher, and the increase in markups is much stronger (16.6 points instead of 6.7 points). Second, the increase in markups is so large that the model requires a sharp decline in α (from 0.18 to 0.12) to keep the labor share from falling too much. This estimate suggests that technical progress has been biased towards labor over the past thirty years - a somewhat surprising conclusion. On the other hand, this model also implies that β rose significantly. We will show some further differences for a longer sample below.

Table 6 presents the implications of these different "calibrations". Notably, our approach offers a balanced view where markups and risk increases explain the rising spread, while the macro model without markups accounts all of it with an unmodeled liquidity premium and the macro model with markups accounts for all of it with rising market power. Note, as a result, that the macro model with markups implies a sharp decline of the level of GDP, by about 8 percentage points. Moreover, the share of income going to capital falls sharply, in opposite to the share of profits which surges. Interestingly, the macro model without markups predict an increase in the level of GDP relative to trend, owing to the lower economic depreciation and higher incentives for capital accumulation.

	Baseline approach			Macro-with-markups			Macro-without-markups		
	1984-00	2001-2016	Diff.	1984-00	2001-2016	Diff.	1984-00	2001-2016	Diff.
A. MPK-RF spread									
Total spread	11.22	15.24	4.02	11.22	15.24	4.02	11.22	15.24	4.02
Depreciation component	4.55	4.37	-0.18	4.55	4.37	-0.18	4.55	4.37	-0.18
Market power component	3.39	5.55	2.17	6.58	10.89	4.30	0	0	0
Risk premium component	3.15	5.23	2.08	0	0	0	0	0	0
Liquidity component	0	0	0	0	0	0	6.67	10.87	4.20
B. Rate of returns									
Equity return	5.85	4.90	-0.96	2.79	-0.35	-3.14	2.63	-0.47	1.10
Equity risk premium	3.07	5.25	2.18	0	0	0	0	0	0
Risk-free rate	2.79	-0.35	-3.14	2.79	-0.35	-3.14	2.63	-0.47	1.10
C. Income distribution									
Share Labor	70.11	66.01	-4.10	70.11	66.01	-4.10	70.11	66.01	-4.10
Share Capital	22.59	21.24	-1.35	15.75	9.17	-6.58	29.89	33.99	4.10
Share Profits	7.30	12.76	5.46	14.14	24.82	10.69	0	0	0
D. Macroeconomic variables									
K/Y	2.13	2.28	0.15	2.13	2.28	0.15	2.13	2.28	0.15
I/Y	17.28	16.50	-0.78	17.28	16.50	-0.78	17.28	16.50	-0.78
Detrend Y (% chg)			-0.30			-8.00			7.77
Detrend I (% chg)			-4.95			-12.65			3.12

Table 6: The table reports some moments of interest calculated in the baseline model, in the macro model with markups, and in the macro model without markups, using the estimated parameter values for each of the two subsamples 1984-2000 and 2001-2016, as well as the change between samples.

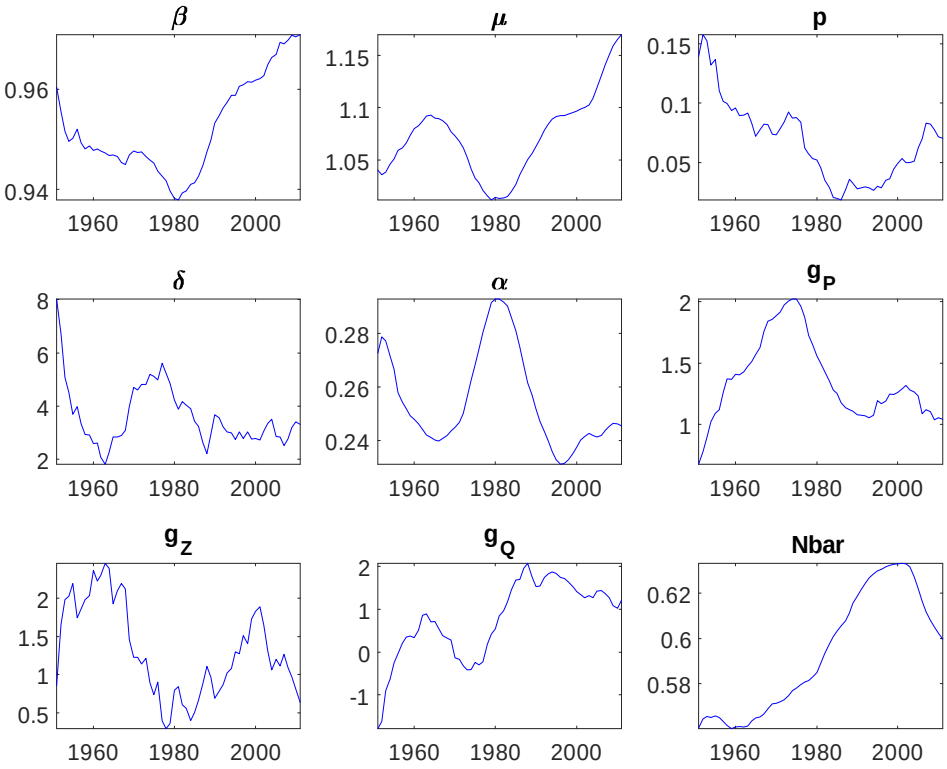

Figure 6: This figure plots the estimated parameters for each year. The target moments are the local moving average over the 11 surrounding years.

5.3 Rolling estimation

An alternative approach to fitting the model is to estimate it using rolling windows rather than two subsamples. In this spirit, figure 6 presents the estimated parameters when we estimate the model each year using a 11–year centered moving average to calculate the target moments. (That is, we target the smooth lines shown in Section 2 in figure 4.) We start our analysis in 1950 to avoid World War II.[19] As noted above, this calculation assumes that agents are myopic, in the sense that they believe that the currently observed target moments will be constant forever, and it abstracts from transitional dynamics.

We find a U shape in the parameter β (savings supply) and in macroeconomic risk p. Markups also have a U shape but also an initial increase in the 1950s and 1960s. The capital parameter α has an increase in the late 1970s which is later reversed. Figure 7 compares the evolution of our parameters β, μ to the parameters estimated using the "macro with markups" approach. Our estimated parameters are significantly more stable over time.

We can then use these rolling estimates to study the income distribution, the return spread $MPK - R^f$, and their drivers. Figure 8 presents the share of pure profits, the true capital share, and the sum of the two for each year. By construction, the total equals one minus the labor share, and matches the data exactly.

The figure shows that the share of pure profits is estimated to have risen in the 1960s, then falling in the 1970s and rising since 1980. Inversely, the capital share fell, then rose and fell. This picture reflects

[19] We thank Matthew Rognlie for proposing (and executing) this exercise in his discussion at the NBER Summer Institute.

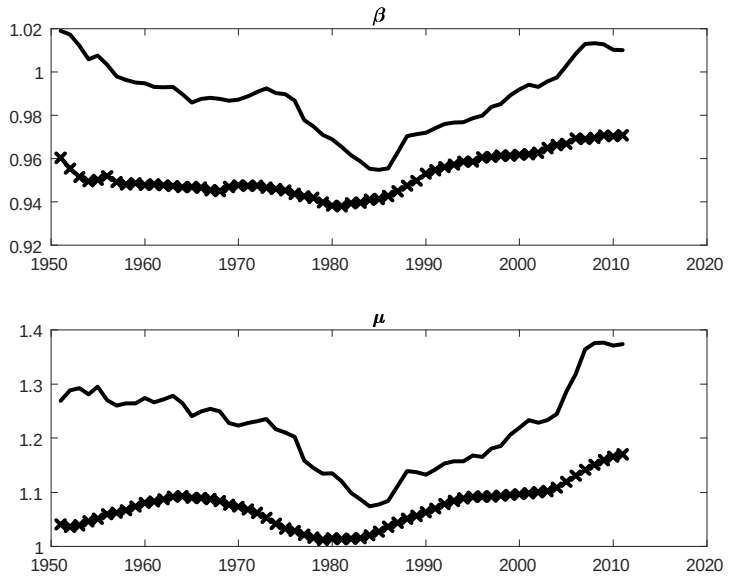

Figure 7: The figure plots the estimated β and μ over rolling windows for the baseline model (starred line) and for the macro approach (full line).

the puzzling pattern of U shape in profits and inverse U shape in α emphasized by Karabarbounis and Neiman (2018). However, we find it interesting that the U shape is significantly less strong with our estimation strategy than if one follows the macro with markups strategy. Karabarbounis and Neiman (2018) note that the strong negative correlation between the interest rate and the capital share, and the strong positive correlation between the interest rate and the profit share, are suggestive of measurement problems in the cost of capital. Figure 9 shows the capital share and the pure profit share implied by the two estimations. There is clearly less volatility of the macro-finance estimates.

Figure 10 presents the $MPK - R^f$ spread and its three subcomponents: economic and physical depreciation, rents, and risk. The spread falls in the 1970s before rising in the 1980s. The depreciation component moves, if anything, in opposite direction to the spread, and hence does not help explain its movements. Rents are estimated to fall then rise, and so does risk. The empirical success here is that the risk premium - which is estimated without looking at the MPK, but rather by single-mindedly observing the PD ratio - helps explain some of this variation.

Figure 11 again compares these results to those obtained with the more standard macro estimation. Both estimation approaches infer the same depreciation component. The macro approach attributes none of the spread to risk by construction and hence infers a large and highly volatile rent (or profit) component. Finally, figure 12 depicts the implied risk-free rate, equity return and equity risk premium. The risk-free rate exactly matches our data target by construction. The equity premium mimics the evolution of p depicted in figure 6.

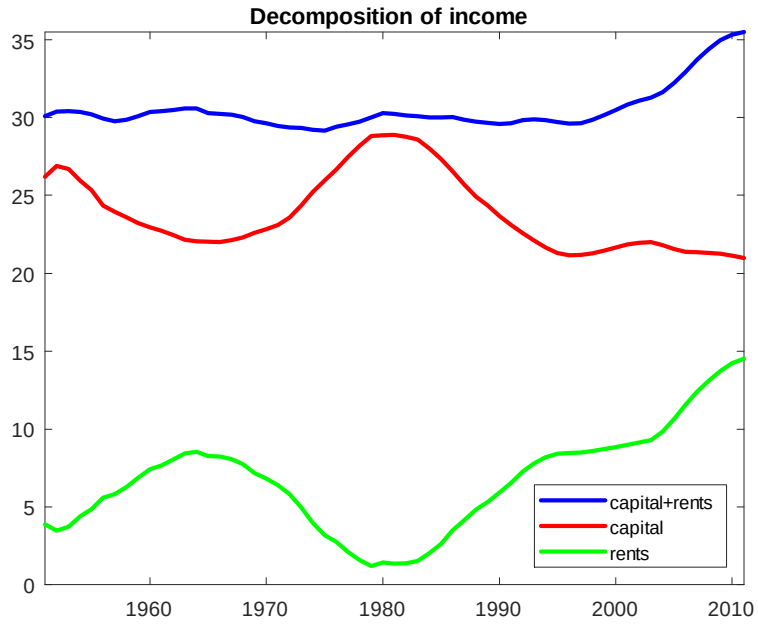

Figure 8: This figure presents the model-implied distribution of income, using the parameters estimated in each year using the rolling window estimation. The labor share is one minus the sum of capital and rents.

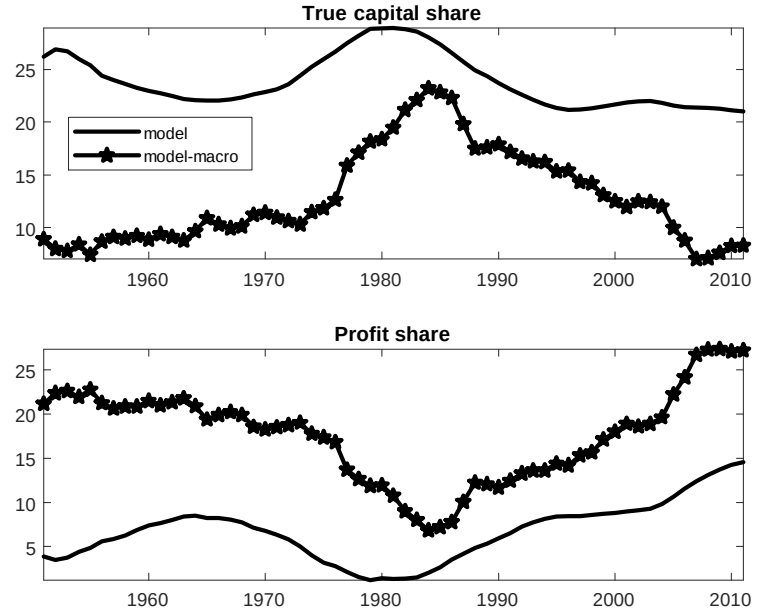

Figure 9: This figure presents the distribution of income, using the parameters estimated at each point in time, for both the macro and macro-finance (baseline) estimations. Top panel: true capital share; Bottom panel: profit share; the lines with stars correspond to the macro estimation, and the full lines to the macro-finance (baseline) estimation.

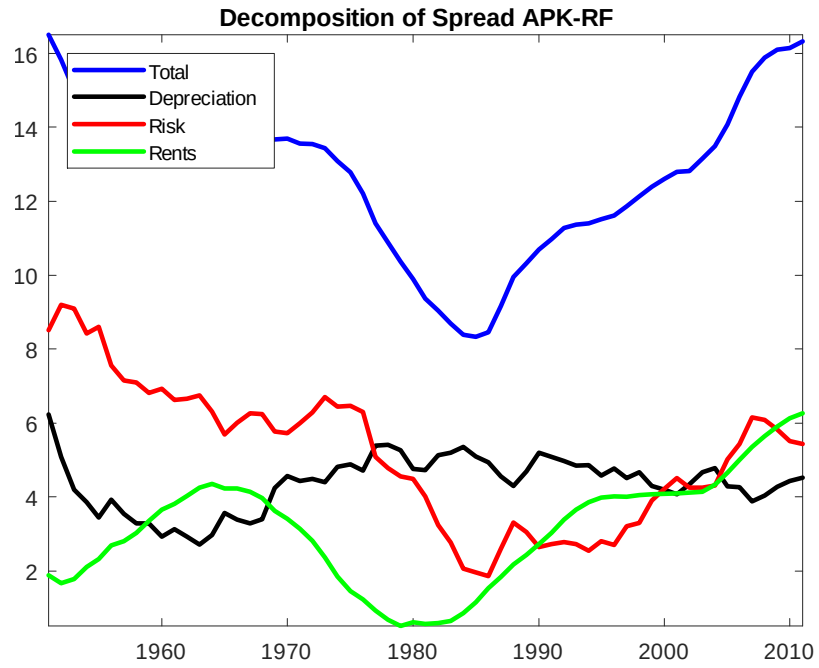

Figure 10: This figure presents the model-implied spread between the average product of capital and the risk-free rate, and the three components which explain this wedge: depreciation, rents, and risk, using the estimated parameters for each year.

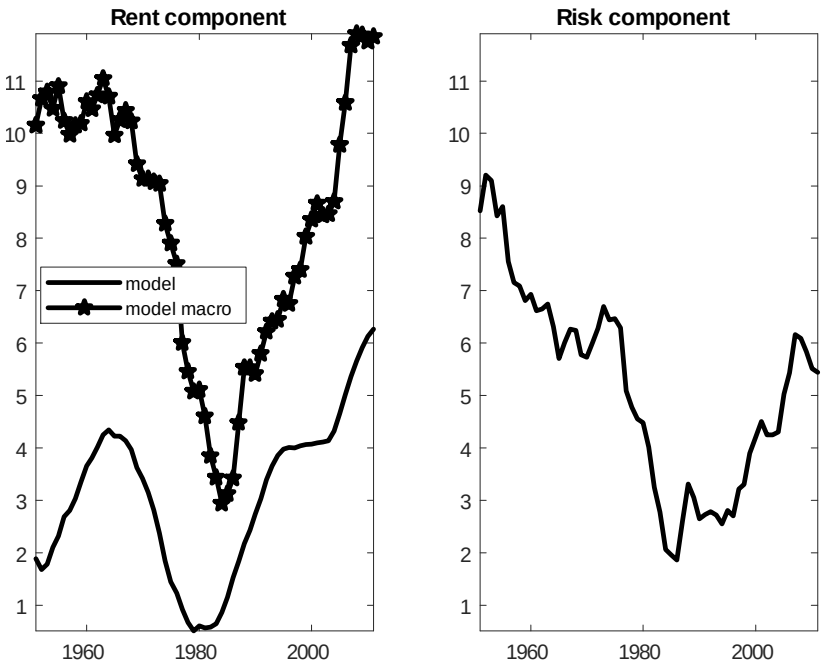

Figure 11: This figure presents the three components of the model-implied spread between the average product of capital and the risk-free rate, for both the baseline (macro-finance) calibration and the macro calibration. Left panel: rent (profit) component; middle panel: risk component; right panel: depreciation component (which is the same across the two calibrations).

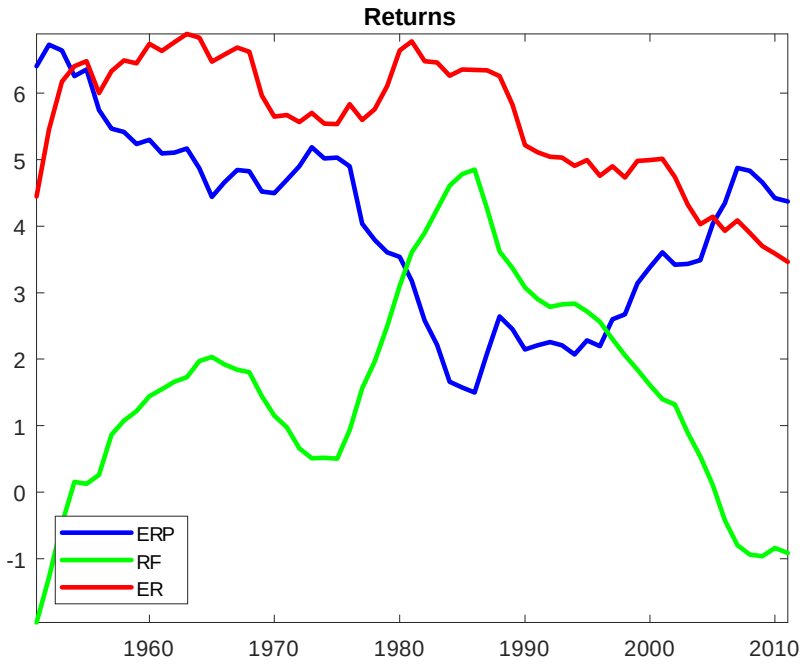

Figure 12: This figure shows the model-implied risk-free rate, expected equity return, and equity risk premium. (By construction, the risk-free rate matches the data.)

6 Extensions and robustness

This section presents some extensions of our baseline framework. We first analyze how financial leverage, the intertemporal elasticity of substitution, or liquidity demand for Treasury securities affect our results. We then study the role of capital mismeasurement generally, and of intangible capital more specifically. Finally, we present an example to evaluate the importance of transitional dynamics.

6.1 Leverage

Our model calculations assume an all-equity financed firm. In reality corporations are leveraged, which may affect in particular the price-dividend ratio, which we use as an input in our estimation strategy. In this section, we propose a simple approach to bound the effect of leverage. To take this into account, we assume a Modigliani-Miller world where corporate leverage has no effect on real quantities, and only affects prices and dividends. We assume corporate debt is fully risk-free. We then adjust the price-dividend ratio of the model given an exogenous leverage decision which we take directly from the data.[20] We then re-estimate the model and obtain the results shown in the third set of columns in tables 7 and 8.[21]

Qualitatively, the findings are quite similar to those of the model without leverage: β, μ and p all go up, and are important contributors to the observed changes in the risk-free rate, profitability, and

[20] Specifically, we use the SP500 data and define leverage as short-term debt plus long-term debt less cash, divided by market value of equity.

[21] As an alternative approach, one can adjust the r^* from the model directly to account for leverage, noting that the r^* identified by the model from the PD ratio is actually $(1+\omega)r^* - \omega r^f$ where ω is the observed debt-equity ratio. This approach yields nearly identical results to the one where we adjust the PD ratio directly.

	Baseline			IES=0.5			Leverage			AA rate as RF		
	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.
β	0.955	0.967	0.012	0.987	0.976	-0.012	1.002	0.995	-0.006	0.995	0.982	-0.013
μ	1.079	1.146	0.067	1.079	1.146	0.067	1.106	1.191	0.084	1.079	1.146	0.067
p	0.034	0.065	0.031	0.034	0.065	0.031	0.021	0.044	0.023	0.012	0.043	0.031
δ	2.778	3.243	0.465	2.778	3.243	0.465	2.778	3.243	0.465	2.778	3.243	0.465
α	0.244	0.243	-0.000	0.244	0.243	-0.000	0.224	0.214	-0.010	0.244	0.243	-0.000
g_P	1.171	1.101	-0.069	1.171	1.101	-0.069	1.171	1.101	-0.069	1.171	1.101	-0.069
g_Z	1.298	1.012	-0.286	1.298	1.012	-0.286	1.378	1.096	-0.282	1.298	1.012	-0.286
g_Q	1.769	1.127	-0.643	1.769	1.127	-0.643	1.769	1.127	-0.643	1.769	1.127	-0.643
N	0.623	0.608	-0.015	0.623	0.608	-0.015	0.623	0.608	-0.015	0.623	0.608	-0.015

Table 7: The table reports the estimated parameters in each of the two subsamples 1984-2000 and 2001-2016 in the baseline model, in the model with IES=0.5, in the model with financial leverage, and in the model estimated with a different interest rate target (AA).

the price-dividend ratio. However, the role of risk is somewhat smaller than in our baseline version. The logic is clear from the Gordon formula: with leverage, the change in r^* required to account for the change in valuation ratio is smaller. (Going in the other direction, however, is that leverage has declined somewhat from the first sample to the second one.) In particular for the spread decomposition $MPK - R^f$ in table 8, the share of the spread due to risk is smaller (2.08 and 3.81 percentage points in the first and second sample respectively). However, the share of the increase in the spread due to risk remains substantial. Moreover, in terms of the implied equity premium, the increase is actually similar, because leverage now amplifies the variation in r^*.Working against us here is that we have assumed that corporate debt pays the same return as the risk-free asset; in reality, corporate debt yields are higher than Treasuries yields, which would reduce the adjustment to the PD ratio.

6.2 Intertemporal elasticity of substitution

We have assumed an elasticity of substitution equal to 2 in our baseline estimation. As noted above, the IES is not identified given that the model generates *iid* growth rates for all macroeconomic variables. Indeed, the model identification obtains the same r^* regardless of the IES, and hence its value only affects the discount factor β according to equation (13). This can be verified in tables 7 and 8 where we present parameter estimates for an elasticity equal to 0.5. Our conclusions that risk and market power increased are hence completely unaffected by this assumption. However, changing the IES does affect the counterfactual scenarios studied above; for instance the effect of an increase in risk on investment depends heavily on the assumed IES. Table 17 in appendix shows the decompositions providing the role of each parameter In that case, for instance, the role of growth factors in driving the interest rate is larger, while the role of β is now opposite (β is estimated to fall). Moreover, the sensitivity of the

	Baseline			IES=0.5			Leverage			AA rate as RF	
	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.	1984-00	2001-16
A. MPK-RF spread											
Total spread	11.22	15.24	4.02	11.22	15.24	4.02	11.22	15.24	4.02	9.32	13.80
– Depreciation	4.55	4.37	-0.18	4.55	4.37	-0.18	4.55	4.37	-0.18	4.55	4.37
– Market power	3.39	5.55	2.17	3.39	5.55	2.17	4.47	6.99	2.52	3.39	5.55
– Risk premium	3.15	5.23	2.08	3.15	5.23	2.08	2.08	3.81	1.73	1.25	3.79
B. Rate of returns											
Equity return	5.85	4.90	-0.96	5.85	4.90	-0.96	5.77	4.84	-0.93	5.88	4.84
Equity premium	3.07	5.25	2.18	3.07	5.25	2.18	2.99	5.19	2.20	1.19	3.75
Risk-free rate	2.79	-0.35	-3.14	2.79	-0.35	-3.14	2.79	-0.35	-3.14	4.69	1.09
C. Income distribution											
Share Labor	70.11	66.01	-4.10	70.11	66.01	-4.10	70.11	66.01	-4.10	70.11	66.01
Share Capital	22.59	21.24	-1.35	22.59	21.24	-1.35	20.26	17.96	-2.30	22.59	21.24
Share Profits	7.30	12.76	5.46	7.30	12.76	5.46	9.62	16.03	6.40	7.30	12.76
D. Macroeconomic variables											
K/Y	2.13	2.28	0.15	2.13	2.28	0.15	2.13	2.28	0.15	2.13	2.28
I/Y	17.28	16.50	-0.78	17.28	16.50	-0.78	17.28	16.50	-0.78	17.28	16.50
Detrend Y (% chg)	–	–	-0.30			-0.30			-1.88		
Detrend I (% chg)	–	–	-4.95			-4.95			-6.52		

Table 8: The table reports some moments of interest calculated in the baseline model, in the model with IES=0.5, in the model with financial leverage, and in the model estimated with a different interest rate target (AA), using the estimated parameter values for each of the two subsamples 1984-2000 and 2001-2016, as well as the change between samples.

risk-free rate to uncertainty is now larger. Perhaps counter-intuitively, higher risk and lower growth raise the PD ratio because of their strong effect on the risk-free rate.

6.3 Role of liquidity premium

As a risk-free rate proxy in the data, we use the one year nominal Treasury rate (minus median SPF inflation expectations). One concern is that our model abstracts from the liquidity premium which makes this rate especially low. (It is not a priori obvious that this liquidity premium has risen over time in a way that would explain the rising spread between MPK and RF.) To gauge the role of the liquidity premium, we instead use as a risk-free rate proxy the rate on AAA/AA corporate bonds, less inflation expectations. This is a longer maturity rate for securities which furthermore do not possess the same unique liquidity attributes as a US Treasury. We then repeat our estimation. The rightmost columns of tables 7 and 8 show the results. Given the identification provided by the model, changing the risk-free rate does not affect β^* (or r^*), which is the relevant statistic for macroeconomic decisions. Hence, it does not affect the estimates for the markup μ, the Cobb-Douglas parameter α, etc. However, the different risk-free rate target will affect the value of β and the amount of risk identified by the model, and their respective changes. Indeed, we see that the estimated β does not increase across samples. The level of risk estimated is smaller, but crucially, our model still estimates that risk increased significantly between the two samples. Our conclusion about the relative importance of risk and markups is not affected by this change in target, suggesting that liquidity considerations do not play a very large role in these trends.

6.4 Capital mismeasurement

One natural explanation for the rising spread $MPK - RF$ is that K is mismeasured, and in particular is increasingly underestimated by the BEA analysts, who traditionally focus on tangible assets. To get a sense of how much mismeasurement of capital matters, we present a simple approach in this section. We then estimate a more detailed model of intangible accumulation in the next section. We are interested in two questions: first, can a plausible amount of mismeasurement explain the rising spread? Second, is this rising mismeasurement also consistent with the other observed features of the data?

In this section, we simply assume that the BEA measures only a fraction λ of total investment. When $\lambda = 1$, there is no mismeasurement, and our baseline model is correct. When $\lambda < 1$ however, this mismeasurement of investment affects our target moments, and hence possibly our parameter estimates. Denote with a superscript m the measured values of the model variables.[22] Measured investment is $x^m = \lambda x$, and hence along the balanced growth path $k^m = \lambda k$. Moreover, GDP and the profit share are now under-estimated since the unmeasured investment $(1-\lambda)x$ is treated as an intermediate input by BEA accountants. As a result, measured GDP is $y^m = y - (1-\lambda)x$. Measured profits equal measured GDP less labor compensation, or $\pi^m = \pi - (1-\lambda)x$. The profit share is hence underestimated as

$$\frac{\pi^m}{y^m} = \frac{\pi - (1-\lambda)x}{y - (1-\lambda)x} < \frac{\pi}{y}.$$

[22] We do the algebra for detrended variables, but one can obviously also apply the same adjustments to the level variables.

However, dividends are correctly measured since the unmeasured investment reduces both profits and investment: $d = \pi - x = \pi^m - x^m$. Hence, the asset price is unaffected by measurement error (even if investors do not observe intangible investment).

It is easy to extend our formula 26:

$$MPK - r_f = \delta + g_Q + \frac{\mu - 1}{\alpha}(r^* + \delta + g_Q) + r^* - r_f + \frac{1 - \lambda}{\lambda}\frac{d}{k}, \tag{33}$$

and mismeasurement ($\lambda < 1$) now adds an additional component to the measured spread, consistent with basic intuition.

How large does the mismeasurement need to be to explain the rising spread? First, note that the measured ratio $d/(\lambda k)$ is fairly small, around 7.5% in the second sample (and 6% in the first sample), according to our data moments. Hence, to increase the spread by 2 percentage points (or about half of the increase in the spread observed during our sample, and about the same as what is explained by risk premia or markups according to our baseline results), the model requires λ to go from 1 (perfect measurement) to $\lambda = 0.73$, a 27% underestimation of investment. This mismeasurement would reduce measured GDP by 4.4% and the profit share by 4 percentage points. One tension, hence, is that rising intangibles lead to a measured labor share going up rather than down, as in the data.

To evaluate more precisely how this mismeasurement affects our results, we estimate three versions of our baseline model corresponding to different assumptions about mismeasurement. In the first version, mismeasurement is constant at 10% in both samples ($\lambda = 0.9$). In the second version, mismeasurement starts at 10% in the first subsample then rises to 20% in the second subsample. In the third version, mismeasurement starts at 10% then rises to 30%. These numbers are largely illustrative; note however that the share in capital of measured "intangibles", that is intellectual property products, is about 6% recently. We hence assuming that the unmeasured stock of intangible capital is larger than the measured stock, and has been rising importantly over the past 15 years.

Table 9 reports the parameter estimates and table 10 reports the implied moments corresponding to different scenarios. There are a few interesting results. First, all parameters are completely unaffected, except for μ and α. In particular, the increase in β and in risk are not affected by these assumptions. Second, when mismeasurement is constant at 10%, the model has similar implications to our baseline model (the level of α is higher and the level of μ lower, but the changes between two subsamples are nearly identical). Third, the estimated increase in markup is smaller when there is an increase in mismeasurement. For instance, with a mismeasurement rising to 30% of capital, the markup rises by only 4.1 points instead of 6.6 points when mismeasurement is constant and 6.7 points in the baseline. This is intuitively consistent with the simple formula (33): with more mismeasurement, there is less of a gap between the MPK and the risk-free rate to explain. The other implication is that the estimated α rises. This is because the labor share rises with mismeasurement; to offset this, the model needs an increase in capital-biased technical change, i.e. α.

Overall, in our most generous calibration, the rising mismeasurement explains 1.65 point increase in the wedge, the markup now only 0.47 point, and the risk premium 2.08 points. Of course, the magnitude of the mismeasurement is difficult to ascertain. But it is interesting that incorporating realistic mismeasurement would reduce further the implied markup, while leaving the role of risk unaffected.

	Baseline			Constant bias: 10%			Rising bias: 10% to 20%			Rising bias: 10% to 30%		
	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.
β	0.955	0.967	0.012	0.955	0.967	0.012	0.955	0.967	0.012	0.955	0.967	0.012
μ	1.079	1.146	0.067	1.070	1.136	0.066	1.070	1.125	0.055	1.070	1.111	0.041
p	0.034	0.065	0.031	0.034	0.065	0.031	0.034	0.065	0.031	0.034	0.065	0.031
δ	2.778	3.243	0.465	2.778	3.243	0.465	2.778	3.243	0.465	2.778	3.243	0.465
α	0.244	0.243	-0.000	0.264	0.263	-0.000	0.264	0.287	0.023	0.264	0.315	0.051
g_P	1.171	1.101	-0.069	1.171	1.101	-0.069	1.171	1.101	-0.069	1.171	1.101	-0.069
g_Z	1.298	1.012	-0.286	1.217	0.956	-0.262	1.217	0.889	-0.328	1.217	0.809	-0.408
g_Q	1.769	1.127	-0.643	1.769	1.127	-0.643	1.769	1.127	-0.643	1.769	1.127	-0.643
N	0.623	0.608	-0.015	0.623	0.608	-0.015	0.623	0.608	-0.015	0.623	0.608	-0.015

Table 9: The table reports the estimated parameters in each of the two subsamples 1984-2000 and 2001-2016 in the baseline model and in the model with mismeasured capital, for different values of the mismeasurement parameters, using the estimated parameter values for each of the two subsamples 1984-2000 and 2001-2016, as well as the change between samples.

6.5 A model with intangible accumulation

We now extend our model to incorporate intangible capital explicitly. We will use our estimation framework to examine how the presence of intangible capital affects our results. The extended model makes the following changes compared to the baseline model. First, the production function is now a Cobb-Douglas over both tangible and intangible capital, with respective shares α_T and α_U :

$$Y_t = Z_t K_{T,t}^{\alpha_T} K_{U,t}^{\alpha_U} (S_t N_t)^{1-\alpha_T-\alpha_U}.$$

Second, tangible and intangible capitals are separately accumulated, and subject to potentially different rates of depreciation and of technical progress:

$$\begin{aligned} K_{T,t+1} &= \left((1-\delta_T) K_{T,t} + Q_{T,t} X_{T,t}\right) e^{\chi_{t+1}}, \\ K_{U,t+1} &= \left((1-\delta_U) K_{U,t} + Q_{U,t} X_{U,t}\right) e^{\chi_{t+1}}. \end{aligned}$$

Note our assumption that both types of capital are equally risky, i.e. have the same exposure to the macroeconomic shock χ_{t+1}. Relatively little is known about the relative riskiness of tangible and intangible capital, leading us to make this assumption. Finally, the resource constraint is modified to $C_t + X_{T,t} + X_{U,t} = Y_t$.

In terms of matching this model to data, we will consider as "tangible" all capital except intellectual property products (IPP), that is, tangible is the sum of residential, equipment and structures. We will assume, similar to the previous section, that measured IPP investment is a fraction λ of true intangible investment:

$$X_{U,t}^{obs} = \lambda X_{U,t},$$

	Baseline			Constant bias: 10%			Rising bias: 10% to 20%			Rising bias: 10% to 30%		
	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.
A. MPK-RF spread												
Total spread	11.22	15.24	4.02	11.22	15.24	4.02	11.22	15.24	4.02	11.22	15.24	4.02
– Depreciation	4.55	4.37	-0.18	4.55	4.37	-0.18	4.55	4.37	-0.18	4.55	4.37	-0.18
– Market power	3.39	5.55	2.17	2.80	4.79	1.99	2.80	4.03	1.23	2.80	3.27	0.47
– Risk premium	3.15	5.23	2.08	3.15	5.23	2.08	3.15	5.23	2.08	3.15	5.23	2.08
– Mismeasurt	0	0	0	0.72	0.85	0.13	0.72	1.61	0.89	0.72	2.37	1.65
B. Rate of returns												
Equity return	5.85	4.90	-0.96	5.85	4.90	-0.96	5.85	4.90	-0.96	5.85	4.90	-0.96
Equity premium	3.07	5.25	2.18	3.07	5.25	2.18	3.07	5.25	2.18	3.07	5.25	2.18
Risk-free rate	2.79	-0.35	-3.14	2.79	-0.35	-3.14	2.79	-0.35	-3.14	2.79	-0.35	-3.14
C. Income distribution												
Share Labor	70.11	66.01	-4.10	68.79	64.82	-3.97	68.79	63.39	-5.40	68.79	61.65	-7.14
Share Capital	22.59	21.24	-1.35	24.63	23.17	-1.46	24.63	25.49	0.87	24.63	28.33	3.71
Share Profits	7.30	12.76	5.46	6.58	12.01	5.43	6.58	11.11	4.53	6.58	10.02	3.44
D. Macroeconomic variables												
K/Y	2.13	2.28	0.15	2.13	2.28	0.15	2.13	2.28	0.15	2.13	2.28	0.15
I/Y	17.28	16.50	-0.78	17.28	16.50	-0.78	17.28	16.50	-0.78	17.28	16.50	-0.78
Detrend Y (% chg)			-0.30			-0.04			7.88			18.53
Detrend I (% chg)			-4.95			-4.60			12.87			34.08

Table 10: The table reports some moments of interest calculated in the baseline model and in the model with mismeasured capital, for different values of the mismeasurement parameters, using the estimated parameter values for each of the two subsamples 1984-2000 and 2001-2016, as well as the change between samples.

	$\lambda = 1$			$\lambda = 2/3$			$\lambda = 1/2$			$\lambda = 1/4$		
	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.
β	0.955	0.967	0.012	0.955	0.967	0.012	0.955	0.967	0.012	0.955	0.967	0.012
μ	1.078	1.141	0.063	1.075	1.136	0.060	1.073	1.131	0.058	1.063	1.114	0.051
p	0.034	0.062	0.028	0.034	0.062	0.028	0.034	0.062	0.028	0.034	0.062	0.028
δ_T	1.792	2.585	0.794	1.792	2.585	0.794	1.792	2.585	0.794	1.792	2.585	0.794
α_T	0.210	0.199	-0.011	0.207	0.195	-0.012	0.203	0.190	-0.013	0.191	0.174	-0.017
g_P	1.171	1.101	-0.069	1.171	1.101	-0.069	1.171	1.101	-0.069	1.171	1.101	-0.069
g_Z	0.994	0.715	-0.280	0.984	0.684	-0.300	0.973	0.652	-0.321	0.919	0.509	-0.410
g_{QT}	1.781	0.809	-0.972	1.781	0.809	-0.972	1.781	0.809	-0.972	1.781	0.809	-0.972
α_U	0.034	0.048	0.014	0.050	0.070	0.020	0.065	0.091	0.026	0.123	0.167	0.044
δ_U	22.875	23.797	0.922	22.875	23.797	0.922	22.875	23.797	0.922	22.875	23.797	0.922
g_{QU}	1.710	2.150	0.440	1.710	2.150	0.440	1.710	2.150	0.440	1.710	2.150	0.440
$\overline{N}$	0.623	0.608	-0.015	0.623	0.608	-0.015	0.623	0.608	-0.015	0.623	0.608	-0.015

Table 11: The table reports the estimated parameters in the model with intangibles, for each of the two subsamples 1984-2000 and 2001-2016, as well as the change between samples.

and hence along the balanced growth path we also have $K_{U,t}^{obs} = \lambda K_{U,t}$. The same points made in the previous subsection about the mismeasurement of GDP, profits, and the labor share apply. We estimate this model for a fixed λ, finding the same parameters as the baseline model, plus α_U, δ_U, and the growth rate of Q_U, using similar moments as the baseline model. (Here mismeasurement rises over time not because λ is changing but because intangibles are growing faster than other types of capital.) Specifically, we use the growth rates of investment prices in both tangible and intangible capital, the ratio of measured profits to tangible capital and the ratio of profits to intangible capital, and finally the ratio of tangible investment to tangible capital, and of intangible investment to intangible capital.

Table 11 presents the estimated parameters for different values of λ, and table 12 presents the model implications.

First, note that the estimated α_U is small with no mismeasurement, corresponding to the share of IPP capital in total capital (about 6% lately). However, α_U is estimated to rise from 3.4% to 4.8%. The depreciation rate of intangible investment is quite high, over 20%, consistent with the usual estimates. This high depreciation is precisely the reason why the share of IPP in the capital stock is small, despite a fairly large share in investment (about 25% lately). Finally, there is progress in the technology to make IPP, but it is slower than for equipment.

Similar to the simple analysis with mismeasurement, we find that (i) the model without mismeasurement behaves quite similarly to the baseline model; (ii) higher mismeasurement has no effect on most parameters except μ, α_T, and α_U. Specifically, more mismeasurement leads to lower estimated markups, lower α_T, and higher α_U. Here too, rising intangibles reduce the role of the markup story while preserving the risk story.

	$\lambda = 1$			$\lambda = 2/3$			$\lambda = 1/2$			$\lambda = 1/4$		
	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.	1984-00	2001-16	Diff.
A. Spread MPK-RF												
Spread	11.95	16.19	4.24	11.95	16.19	4.24	11.95	16.19	4.24	11.95	16.19	4.24
B. Rates of Returns												
Equity return	5.86	4.73	-1.13	5.86	4.73	-1.13	5.86	4.73	-1.13	5.86	4.73	-1.13
Equity premium	3.07	5.08	2.01	3.07	5.08	2.01	3.07	5.08	2.01	3.07	5.08	2.01
Risk-free rate	2.79	-0.35	-3.14	2.79	-0.35	-3.14	2.79	-0.35	-3.14	2.79	-0.35	-3.14
C. Income Distribution												
Labor	70.11	66.01	-4.10	69.12	64.75	-4.37	68.15	63.53	-4.62	64.53	59.09	-5.44
Tangible cap.	19.52	17.48	-2.04	19.24	17.14	-2.10	18.97	16.82	-2.15	17.96	15.64	-2.32
Intangible cap.	3.14	4.19	1.05	4.64	6.16	1.52	6.10	8.06	1.96	11.55	14.99	3.44
Profits	7.24	12.33	5.09	7.01	11.95	4.95	6.79	11.59	4.81	5.96	10.27	4.32
D. Macroeconomic variables (detrended, % change)												
Y			-4.36			-5.16			-5.67			-6.09
IT			-4.36			-4.65			-4.68			-3.31
IU			-14.73			-15.52			-16.03			-16.45
True IU			25.92			25.12			24.61			24.20

Table 12: The table reports some moments of interest calculated in the model with intangible capital, for different values of the mismeasurement parameters, using the estimated parameter values for each of the two subsamples 1984-2000 and 2001-2016, as well as the change between samples.

6.6 Transitional Dynamics

Our calculations so far assume that the economy remains along its "risky balanced growth path". However, if the model parameters such as the discount factor or markup change, the economy will experience a transition before it reaches its new balanced growth path. This transition may affect our estimation results.

To evaluate the importance of this "bias", we estimated the model, taking into account the transitional dynamics. Specifically, we make the following assumptions. We use the baseline version of the model and assume that the economy starts in 1992 in balanced growth with the parameters that we estimate over the first sample.[23] We then assume that the nine parameters change linearly over 24 years (to end in 2016) from the value we estimated in the first sample to a final value that we will estimate, and which may not be our estimate for the second sample.

We then calculate the transitional dynamics for this economy using a standard shooting method. A key issue is agents' expectations. With perfect foresight, the model cannot fit the data, because agents see the lower interest rates coming, which leads to a boom in the price-dividend ratio. (Furthermore, the long-term interest rate would fall significantly more than the short rate, unlike what we see in the data.) We hence assume myopic expectations: each period, agents observe the new values of the parameters, and they assume (incorrectly, at least for the first 24 years) that these parameters will remain constant forever.[24]

We then numerically find the final parameters such that, when calculating the transition, this procedure yields an average time series for our targets (over the period 2001-2016) that matches what we measured in the data. We are able to find a close, though not perfect, match. Figure 13 presents the path obtained for parameter values and figure 14 the path for the moments targeted (we abstract here from parameters that map directly into moments). As can be seen in figure 14, the model moments, averaged over periods 10-26 (i.e. 2001-2016), match reasonably well the target moments for the second sample (in red). The more surprising result is in figure 13, where we see that the parameter values estimated in this way are quite similar to these obtained in the simple baseline model, which assumes balanced growth. To see this, note that the blue line, averaged over periods 10-26, is economically quite similar to the red line (results from the baseline model). The one exception is δ, which now falls slightly instead of rising.

We view this results as suggesting that, at least in the myopic case, perhaps not much is lost by focusing on the risky balanced growth path. (This conclusion might not hold true for all models, in particular with intangibles.)

[23] We use 1992 to take into account that these parameters are estimated over 1984-2000.

[24] Agents consequently make investment choices that would, eventually, lead to converge to a new steady-state corresponding to today's parameter values. However, the next period, new parameter values (unexpectedly) arrive, leading to new choices and a revised transition path. This process continues until the parameters are indeed constant, and the economy then converges to its final steady-state.

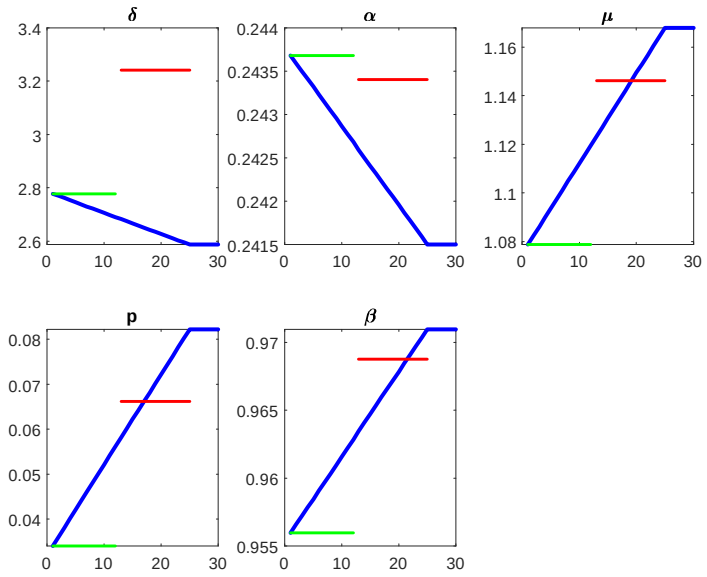

Figure 13: This figure plots the estimated path for the parameters using the transitional dynamics method. The green and red lines denote the values estimated in the baseline approach in the first and second sample.

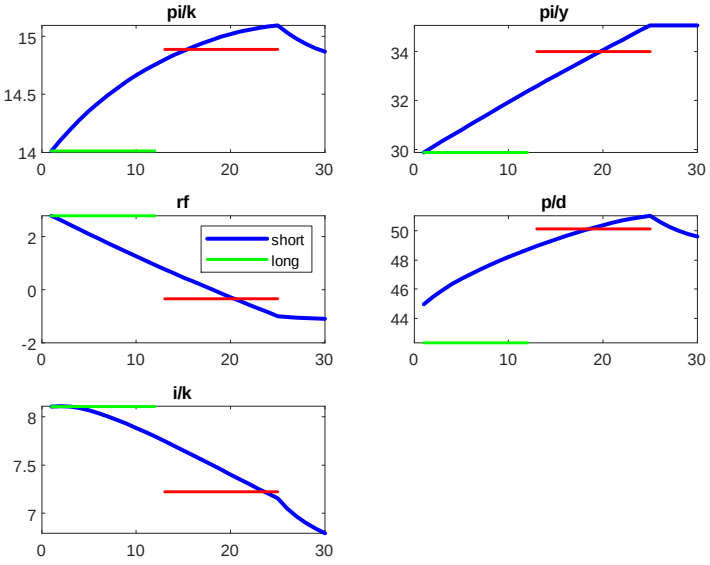

Figure 14: This figure plots the estimated path for the target moments using the transitional dynamics method. The green and red lines denote the values targeted in the baseline approach. In our approach, the red line is the target which the blue line (averaged over periods 10-26) attempts to mimick.

7 Other evidence on market power, risk premia and intangibles

Our empirical results show that rising risk premia and rising market power appear to be two of the significant drivers of some of the macro-finance trends we focus on, and intangibles have a potential contribution as well. In this section we step outside of the model and present some simpler and independent evidence for these two phenomena. We also discuss some related estimates presented by other researchers, which tend to support our conclusions.

7.1 Some empirical estimates of the equity risk premium

Estimating the equity premium is notoriously difficult, even retrospectively. Using realized excess equity returns is essentially pointless over short samples, because returns are noisy,[25] and because an increase in the risk premium may lead, by itself, to lower realized returns. But methods that use standard forecasting return regressions have also been found to be very unstable; Goyal and Welch (2006) argue that none of them outperforms the simple mean out-of-sample. Here we follow a few approaches which have been shown to be somewhat more successful empirically.

Our first approach is simply to use the static Gordon growth formula, which states that the price-dividend ratio is the inverse of the difference between the return on the asset and the dividend growth rate:

$$\frac{P}{D} = \frac{1}{R - G},$$

where R is the expected equity return, which can be decomposed into $R = R^f + EP$, with R^f risk-free and EP the equity premium, and G is the growth rate of dividends. This approach can be used at any point in time, given the observed PD and R^F and given an assumption about G going forward.

Our second approach builds on Fama and French (2002) who argue that, if the dividend-yield or earnings-yield are stationary, as they ought to be, one can advantageously estimate the mean of P_{t+1}/P_t by D_{t+1}/D_t or E_{t+1}/E_t (which are less volatile). As a result, they suggest estimating

$$ERP = E\left(\frac{D_{t+1}}{P_t}\right) + E\left(\frac{D_{t+1}}{D_t}\right) - E(R^f),$$

which amounts to the Gordon growth formula, or replacing dividend growth with earnings growth,

$$ERP = E\left(\frac{D_{t+1}}{P_t}\right) + E\left(\frac{E_{t+1}}{E_t}\right) - E(R^f).$$

This approach is best thought as applying to a long-sample average.

Our third approach follows Campbell and Thompson (2008) who show how combining the current dividend yield and the return on book equity can be used to create a real-time estimate of the equity premium:

$$ERP = \frac{D}{E}\frac{E}{P} + \left(1 - \frac{D}{E}\right) ROE,$$

and where they suggest smoothing the payout ratio D/E, earnings-price ratio E/P, and the return on book equity ROE to reduce the effect of influential but transitory observations.

[25] For instance, suppose a researcher has a sample of 16 years (as we do) and that the excess equity return has a mean of 8% with a volatility of 16%. The 95% confidence interval for the mean excess equity return is $[0\%, 16\%]$. It is clearly impossible to detect a change of the equity premium of even several percentage point based solely on realized returns.

	Arithmetic average			Geometric average		
	1984–00	2001-16	Change	1984–'00	2001-'16	Change
Real dividend growth	2.03	4.7	2.67	2.04	8.21	6.17
Real earnings growth	6.22	16.97	10.75	10.25	12.06	1.81
Return on book equity	10.94	10	-.94	10.7	9.4	-1.3
D/P	2.78	1.92	-.86	–	–	–
D/E	.49	.47	-.02	–	–	–
E/P	5.74	4.69	-1.05	–	–	–
ERP Gordon	.87	5.56	**4.69**	1.91	9.16	**7.25**
ERP Fama-French Earnings	2.43	4.78	**2.35**	4.61	8.66	**4.05**
ERP Campbell-Thompson	1.47	4.11	**2.64**	1.84	3.65	**1.81**
ERP Gordon - w. variance adj	–	–	–	2.43	8.26	**5.83**
ERP Fama-French Earnings - w. var. adj.	–	–	–	4.81	10.3	**5.49**
ERP Campbell-Thompson - w. var. adj.	–	–	–	2.31	5.56	**3.25**

Table 13: The table reports estimates of the equity premium for the samples 1984-2000 and 2001-2016. See text for details.

These formulas can be applied either using arithmetic averages or using geometric averages. We report both below in table 13, though we like Campbell and Thompson's recommendation to use the geometric averages. We then incorporate an adjustment of 1/2 the variance of stock returns to produce an estimate of arithmetic equity premium.

The key observation from table 13 is that, while the estimates of the equity premium are clearly different across models and methods, most calculations suggest that the ERP increased from the first sample to the second sample. Specifically, all nine estimates in bold are positive, ranging from 1.8% to 7.2%. This reflects that valuation ratios increased moderately, while earnings or dividend growth increased more significantly, and the risk-free rate fell. (We take the risk-free rate to be the 10 year Treasury yield minus SPF inflation expectations over the next 10 year.)

Figure 15 presents graphically estimates of the equity risk premium for each of the three approaches, obtained over centered 11-year rolling windows. We smooth the estimates using a 3-year moving average. Here too, the exact numbers vary quite a bit across models, but all models suggest some increase over the past 15 years or so. (A particular difficulty is how one deals with the very low corporate earnings in 2008 or 2009, which affect the FF-earnings model significantly, leading to the extreme arithmetic implication in the middle panel.)

7.2 Other measures of macroeconomic risk

We now discuss other existing estimates of the equity risk premium. It is probably helpful, before reviewing this research, to clarify what we think is a common misconception. The empirical literature

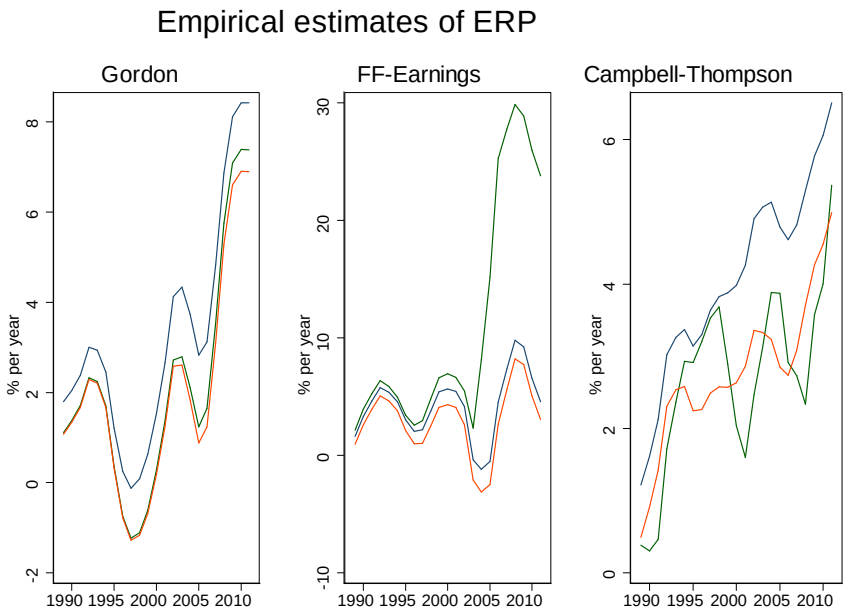

Figure 15: Empirical estimates of the equity risk premium. Left panel: Gordon growth model using dividends; middle panel: Fama-French model using earnings growth instead of dividend growth; right panel: Campbell-Thompson estimates. Red line = arithmetic average; Green = geometric; Blue = geometric + variance adjustment.

on the predictability of equity returns has emphasized the role of valuation ratios such as the price-dividend or price-earnings ratios. A standard finding in this literature (e.g., Cochrane (2013)) is that high valuation ratios forecast low equity returns, but not (significantly) lower short-term interest rates, so that the variation must reflect a change in the equity premium. One might then naturally assume that the current fairly high valuation ratios imply a low equity premium. However, this conclusion is premature, and we believe, incorrect, for two reasons. First, the valuation ratios are not especially large, as we showed earlier. Second, the current period differs from historical episodes because the interest rate fell dramatically. The assumption that the risk-free rate is stable does not seem adequate in light of the observed changes over the past two decades.

We now discuss other evidence on the changes in the equity premium. Duarte and Rosa (2015) provides an exhaustive survey of the different methods that can be used to estimate the equity premium in real time. They distinguish between different methods based on variants of the Gordon growth model, on predictive regressions, and on cross-sectional regressions. Overall, the conclusion is that the equity premium has risen, in line with our findings. Campbell and Thompson (2008; see updated results) propose a method to estimate the equity premium in real time. Their estimate also shows a small increase after 2000. Using a very different methodology, based on a MLE estimation of a structural model, Avdis and Wachter (2015) reach a fairly similar conclusion. Another important contribution is Martin (2015) who uses an ingenious argument to provide, under a weak condition, a lower bound on

	Mean			Differences			
	1984-00	2001-2016	2001-16 ex GFC				
	(1)	(2)	(3)	(2)-(1)	SE	(3)-(1)	SE
spread Gilchrist-Zakrajsek	1.5	2.54	2.31	1.04	.24	.81	.16
spread BAA 10y	1.94	2.74	2.61	.80	.19	.67	.15
spread AAA 10y	1.01	1.64	1.61	.63	.13	.60	.12
VIX	18.92	20.22	18.62	1.3	2.27	-.3	1.98
Realized volatility	13.36	17.43	15.34	4.07	2.21	1.98	1.62

Table 14: The table reports the mean of various credit spreads and volatility measures for the samples 1984-2000, 2001-2016, and 2001-2016 excluding the June 2007-June 2009 period. The table also reports the difference between these means and a standard error (calculated using the Newey-West method with 12 monthly lags).

the equity premium based on option data. His lower bound has a very high correlation with the VIX index. The estimate is very elevated during the Global Financial Crisis, and remains at a higher level post GFC than pre GFC. However, his lower bound is quite low in the mid-2000s. If the lower bound has a constant bias with the mean, then this series does not behave like the other estimates we discussed above. However, it is possible that the bias between the lower bound he finds and the true expected equity premium is time-varying.

Table 14 presents evidence on the evolution of alternative measures of risk; the Gilchrist-Zakrajsek spread, the standard BAA and AAA spreads, the VIX index, and stock market realized volatility (using daily data). The table reports the mean in the two samples, as well as the mean in the second sample excluding the GFC period. We see that all these credit spreads have increased between the two samples, and this conclusion is true even excluding the GFC period. Realized volatility is also somewhat higher. The VIX index exhibits little trend (but is only available starting in 1996). These results are consistent with Del Negro et al. (2017) who show that the premia for safe and liquid assets increased over time.[26]

Kozlowski, Veldkamp and Venkateswaran (2017, 2018) also argue that macroeconomic risk has increased, and that this explains the decline of the risk-free rate, as well as the weak economic recovery. Their precise timing is different, however, since they focus on the post-crisis period, and their focus is different, since they are not interested in the implications for the trends that we study.

Our finding that the equity premium increases after 2000 follows a large literature that documented a decline of the equity premium decline during the 1980s and 1990s (Blanchard (1993), Jagannathan and McGrattan (2000), ,Heaton and Lucas (1999), Lettau and Ludvigson (2007)). As we discuss below in more detail, our finding of an increase since 2000 is consistent with other estimates (for instance Duarte (2015) or Campbell and Thompson (2008)), though not all of them (Martin (2015)).

[26] One caveat is that the underlying riskiness of the firms issuing corporate bonds may have changed over time, even within credit ratings.

7.3 Independent evidence on risking markups

A number of recent contributions, using different methods, have found that average markups have been increasing. For example, Barkai (2016) uses aggregate data and implements a user cost approach a la Hall-Jorgenson (1967) to decompose the non-labor share into a true capital share and a profit share. The true capital share is computed by multiplying the capital output ratio by the user cost of capital. The profit share is a residual. The aggregate markup can be directly inferred from the profit share. Because his measure of user cost does not incorporate a meaningful risk premium, Barkai finds that the evolutions of the user cost track those of the interest rate, and so that the user cost has declined substantially over the period 1984-2014. This implies a large decrease in the capital share, a large increase in the profit share, and a large increase in the aggregate markup of about 20% roughly in line with our macro estimation. Caballero et al. (2017) implement a similar approach but allowing for sizeable and variable risk premia, which they estimate have increased over the same time period, resulting in a smaller increase in markups, more in line with our finance estimates.

De Loecker and Eeckhout (2016) use firm-level data and estimate firm-level markups using a production function approach which recovers markups as the ratio of the elasticity of production to a flexible input the the share of that input in revenues, where the former is computed by estimating the production function. The aggregate markup computed as a harmonic sales-weighted average of firm-level markups increases by about 25%. Traina (2018) criticizes the measure of costs used by De Loecker and Eeckhout (2016). Using a broader measure, he finds that the increase in average markups is much smaller. Gutierrez and Philippon (2016) also use firm-level data but they estimate firm-level markups using a user cost approach allowing for sizeable and variable risk premia. They also find a sizeable increase in aggregate markups of about 10% over the period 1984-2014, roughly in line with our finance estimation.

7.4 Rising Intangible Capital

There is a growing literature that recognizes the importance of intangible capital in the US economy. Corrado et al. (2005, 2009) and Nakamura (2010) present estimates of the size of intangible capital. Bhandari and McGrattan (2017) also contribute to this measurement. Koh,Santaeullia-Llopis and Zheng (2015) argue that rising intangibles help explain the evolution of the labor share. Crouzet and Eberly (2018) argue that growing intangibles help explain both the rising market power and lower capital investment.

8 Conclusion

We provide a simple accounting framework that allows decomposing the changes observed over the past 30 years in some key macro-finance trends into "semi-structural" parameters using a fairly clear identification. We say "semi-structural" because, allowing these parameters to vary over time flexibly suggests they are not microfounded and invariant to policy. Yet we find the results useful because deeper explanations have to be consistent with the changes of parameters implied by our approach.

We find that about half of the increase in the spread between the return on private capital and the

risk-free rate is due to rising market power, and half due to rising risk. Technical change plays little role. Higher savings supply and higher risk are the prime proximate contributors to the decline of the risk-free rate. Rising market power help explain the evolution of the capital share, profitability, and capital accumulation, but its contribution is substantially overstated if the model is estimated using a macro approach that abstracts from risk. Finally, taking into account intangibles reduces further the estimated increase in the market power.

One limitation of our approach is that we treat the parameter changes as independent causal factors, but they might actually be driven by common causes; for instance, higher market power might reduce innovation and hence productivity growth, but we treat these as independent. Our analysis also does not incorporate some factors which could help explain the evolution of some of the big ratios that we study. In particular, we abstract from taxes and from agency issues (e.g. external finance or corporate governance frictions) or market incompleteness, that could also give rise to wedges that might vary over time. Our study of transitional dynamics is only scratching at the vast possibilities. Finally, it would be interesting to study these issues taking into account the specific open economy considerations or at least to study these same facts for a variety of countries.

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10 Data appendix

To be added.

11 Model Appendix

The first subsection lists the equations characterizing the equilibrium. The second subsection shows how to solve the model and the moments of interest.

11.1 System of equations characterizing the equilibrum

Utility recursion

$$V_t = L_t \left((1-\beta) c_{pc,t}^{1-\sigma} + \beta E_t \left(V_{t+1}^{1-\theta} \right)^{\frac{1-\sigma}{1-\theta}} \right)^{\frac{1}{1-\sigma}}.$$

Utility per capita

$$V_{pc,t} = \frac{V_t}{L_t} = \left((1-\beta) c_{pc,t}^{1-\sigma} + \beta \left(\frac{L_{t+1}}{L_t} \right)^{1-\sigma} E_t \left(V_{pc,t+1}^{1-\theta} \right)^{\frac{1-\sigma}{1-\theta}} \right)^{\frac{1}{1-\sigma}}.$$

Stochastic discount factor

$$M_{t+1} = \beta \left(\frac{L_{t+1}}{L_t} \right)^{1-\sigma} \left(\frac{c_{pc,t+1}}{c_{pc,t}} \right)^{-\sigma} \left(\frac{V_{pc,t+1}}{E_t((V_{pc,t+1})^{1-\theta})^{\frac{1}{1-\theta}}} \right)^{\sigma-\theta}.$$

Stochastic trend

$$S_{t+1} = S_t e^{\chi_{t+1}}.$$

Production function

$$Y_t = Z_t K_t^{\alpha} (S_t N_t)^{1-\alpha}.$$

Capital accumulation

$$K_{t+1} = \left((1-\delta) K_t + Q_t X_t \right) e^{\chi_{t+1}}.$$

Prices as marginal revenue

$$\begin{aligned}(1-\alpha)\frac{Y_t}{N_t} &= \mu w_t,\\ \alpha\frac{Y_t}{K_t} &= \mu R_t.\end{aligned}$$

Euler equation

$$E_t\left[M_{t+1}R^K_{t+1}\right]=1.$$

Return on capital

$$R^K_{t+1}=\left(R_{t+1}+(1-\delta)\frac{1}{q_{t+1}}\right)q_t e^{\chi_{t+1}}.$$

Resource constraint

$$C_t+X_t=Y_t.$$

11.2 Detrended system and solution

We look for g_T such that $y_t=\frac{Y_t}{S_tT_t}=y^*$, and same for x,c,v,etc. But note that $k_t=\frac{K_t}{S_tT_tQ_t}$ is the capital variable that is stationary (capital grows faster than output). We denote $c_t=\frac{C_t}{S_tT_t}$ total consumption, note that c_t is not $c_{pc,t}$ but rather $c_t=\frac{L_tc_{pc,t}}{S_tT_t}$. Similarly, we denote by $v_t=\frac{V_t}{S_tT_t}$ and note that $V_{pc,t}=\frac{V_t}{L_t}=\frac{T_tS_t}{L_t}v_t$.To find g_T, we use the production function:

$$y^*S_tT_t=Z_t(k^*S_tT_tQ_t)^\alpha N_t^{1-\alpha}S_t^{1-\alpha}$$

hence we find the growth rate of T by taking the ratio of this equation for $t+1$ and t :

$$\frac{T_{t+1}}{T_t}=1+g_T=(1+g_T)^\alpha(1+g_Q)^\alpha(1+g_N)^{1-\alpha}(1+g_Z)$$

leading to

$$1+g_T=(1+g_Q)^{\frac{\alpha}{1-\alpha}}(1+g_N)(1+g_Z)^{\frac{1}{1-\alpha}}.$$

Writing $N_t=N^*(1+g_N)^t$, we also obtain $y^*=k^{*\alpha}\overline{N}^{1-\alpha}$. Next, we detrend the capital accumulation equation:

$$k^*T_{t+1}S_{t+1}Q_{t+1}=\left((1-\delta)k^*T_tS_tQ_t+Q_tx^*S_tT_t\right)e^{\chi_{t+1}}$$

leading to

$$k^*\left((1+g_Q)(1+g_T)-(1-\delta)\right)=x^*.$$

The SDF can be simplified as:

$$\begin{aligned}M_{t+1} &= \beta\left(\frac{L_{t+1}}{L_t}\right)^{1-\sigma}\left(\frac{T_{t+1}S_{t+1}L_t}{T_tS_tL_{t+1}}\right)^{-\sigma}\left(\frac{S_{t+1}}{E_t((S_{t+1})^{1-\theta})^{\frac{1}{1-\theta}}}\right)^{\sigma-\theta},\\ &= \beta\left(\frac{L_{t+1}}{L_t}\right)\left(\frac{T_{t+1}}{T_t}\right)^{-\sigma}\left(\frac{S_{t+1}}{S_t}\right)^{-\theta}\left(E_t\left(\frac{S_{t+1}}{S_t}\right)^{1-\theta}\right)^{\frac{\theta-\sigma}{1-\theta}},\\ &= \beta(1+g_L)(1+g_T)^{-\sigma}\left(e^{\chi_{t+1}}\right)^{-\theta}E(e^{(1-\theta)\chi_{t+1}})^{\frac{\theta-\sigma}{1-\theta}}. \qquad (34)\end{aligned}$$

The Euler equation can be rewritten as:

$$1=E\left[\left(R_{t+1}+(1-\delta)\frac{1}{q_{t+1}}\right)q_te^{\chi_{t+1}}\times\beta(1+g_L)(1+g_T)^{-\alpha}\left(e^{\chi_{t+1}}\right)^{-\theta}E(e^{(1-\theta)\chi_{t+1}})^{\frac{\theta-\sigma}{1-\theta}}\right]$$

or using the definition of $\beta^* = \beta(1+g_L)(1+g_T)^{-\sigma} \times E(e^{(1-\theta)\chi_{t+1}})^{\frac{1-\sigma}{1-\theta}}$,

$$1 = E\left(R_{t+1}q_t + (1-\delta)\frac{Q_t}{Q_{t+1}}\right)\beta^*$$

where along the BGP $R_{t+1}Q_t$ is constant, equal to

$$R_{t+1}Q_t = \frac{\alpha}{\mu}Z_{t+1}K_{t+1}^{\alpha-1}\left(S_{t+1}N_{t+1}\right)^{1-\alpha}Q_t = \frac{\alpha}{\mu}k^{*\alpha-1}N^{*1-\alpha}\frac{Q^*}{1+g_Q}.$$

So in summary, the Euler equation is

$$\frac{1+g_Q}{\beta^*} - 1 + \delta = \frac{\alpha}{\mu}\left(\frac{k^*}{N^*}\right)^{\alpha-1}Q^*.$$

Finally we obtain $c^* = y^* - x^*$. The calculation of the risk-free rate follows immediately from the formula for the SDF (34).

The price-dividend is obtained easily using the standard recursion:

$$\frac{P_t}{D_t} = E_t\left(M_{t+1}\left(\frac{P_{t+1}}{D_{t+1}}+1\right)\frac{D_{t+1}}{D_t}\right),$$

and given the *iid* nature of the model, P_t/D_t is constant along a balanced growth path, so that

$$\frac{P^*}{D^*} = \left(\frac{P^*}{D^*}+1\right)E\left(\frac{D_{t+1}}{D_t}M_{t+1}\right),$$

and moreover

$$E\left(\frac{D_{t+1}}{D_t}M_{t+1}\right) = (1+g_T)\beta^*$$

which leads to equation (22). The price-earnings ratio and Tobin's Q can then be derived from the identities

$$\frac{P_t}{\Pi_t} = \frac{P_t}{D_t}\frac{\Pi_t - X_t}{\Pi_t} = \frac{P_t}{D_t}\frac{1 - X_t/Y_t}{\Pi_t/Y_t}$$

and

$$\frac{P_t}{K_t/Q_t} = \frac{P_t}{D_t}\left(\frac{\Pi_t}{K_t/Q_t} - \frac{X_t}{K_t/Q_t}\right).$$

	Pi/K	Pi/Y	RF	PD	I/K	growth TFP	growth price invt	growth pop.	Emp/Pop
β	0.00	-0.02	-0.20	0.04	-0.00	-0.37	0.11	-1.20	0.00
μ	1.88	0.28	0.00	0.15	-2.48	-0.09	0.09	-0.06	0.00
p	-0.00	0.07	-1.28	-0.07	0.00	1.81	-0.54	1.27	-0.00
δ	0.00	-5.94	0.00	0.00	100.00	-145.15	146.88	-101.77	0.00
α	-1.32	0.88	-0.00	-0.10	1.74	0.06	-0.06	0.04	-0.00
g_P	-0.00	0.00	0.00	0.00	0.00	-0.00	-0.00	100.00	-0.00
g_Z	5.36	0.73	0.00	0.42	-7.09	105.34	-7.05	-1.87	0.00
g_Q	0.00	-0.00	-0.00	0.00	-0.00	0.00	-100.00	0.00	0.00
N	-0.00	-0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	100.00

Table 15: Sensitivity matrix. Here the parameters are redefined with betastar instead of beta.

12 Additional Empirical Results

12.1 Identification

Table 15 reports the (opposite of) moment sensitivity, as suggested by Andrews, Gentzknow and Shapiro (2017). For each parameter (row), it shows the effect of changing each data moment on the parameter. For instance, changing the estimate of the risk-free rate by 1 percentage point leads to a lower β by about 0.20.

12.2 Decomposition: bounds

Table 16 reports the upper bound and lower bound of the effect of each parameter on each moment. This is calculated by consider all possible combinations of parameter values, as explained in the text, footnote 14. For instance, the effect of β on the PD ratio is bounded between 19.50 and 48.43 As can be seen from the table, thh bounds are fairly tight,except for the PD ratio.

12.3 Moment decompositions in low IES case

Table 17 is the analog of table 3 for the model with IES=0.5.

	Pi/K	Pi/Y	RF	PD	I/K	growth TFP	growth price invt	growth pop.	Emp/Pop
β	-2.14	-0.00	-1.27	19.50	-0.00	-0.00	0.00	0.00	0.00
	-1.74	0.00	-1.24	48.43	0.00	0.00	0.00	0.00	0.00
μ	2.29	4.13	0.00	0.00	-0.00	-0.17	0.00	0.00	0.00
	3.22	4.13	0.00	0.00	0.00	-0.12	0.00	0.00	0.00
p	0.68	-0.00	-1.64	-28.11	-0.00	-0.00	0.00	0.00	0.00
	0.84	0.00	-1.61	-5.56	0.00	0.00	0.00	0.00	0.00
δ	0.62	-0.00	0.00	0.00	0.47	-0.00	0.00	0.00	0.00
	0.74	0.00	0.00	0.00	0.47	0.00	0.00	0.00	0.00
α	0.00	-0.03	-0.00	-0.06	-0.00	-0.00	0.00	0.00	0.00
	0.01	-0.02	-0.00	-0.01	-0.00	-0.00	0.00	0.00	0.00
g_P	0.05	-0.00	0.03	-7.83	-0.07	-0.00	0.00	-0.07	0.00
	0.06	0.00	0.04	-1.03	-0.07	-0.00	0.00	-0.07	0.00
g_Z	-0.32	-0.00	-0.19	-13.31	-0.39	-0.27	0.00	0.00	0.00
	-0.26	0.00	-0.19	-1.92	-0.39	-0.25	0.00	0.00	0.00
g_Q	-1.27	-0.00	-0.10	-7.78	-0.88	0.04	0.64	0.00	0.00
	-1.03	0.00	-0.10	-1.02	-0.87	0.08	0.64	0.00	0.00
N	-0.00	-0.00	0.00	0.00	-0.00	-0.00	0.00	0.00	-0.02
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02

Table 16: The table reports for each moment, and for each parameter, a lower bound and an upper bound on the effect of the change in parameter on the moment, where the bounds are obtained by considering all possible combinations of the other parameters (i.e. evaluated at initial or final value).

	β	μ	p	δ	α	g_P	g_Z	g_Q	$\overline{N}$
Gross profitability	-1.94	2.76	0.76	0.68	0.00	0.05	-0.29	-1.15	-0.00
Measured cap. share	-0.00	4.13	0.00	-0.00	-0.03	-0.00	0.00	0.00	-0.00
Risk-free rate	-1.25	0.00	-1.62	0.00	-0.00	0.03	-0.19	-0.10	0.00
Price-dividend ratio	31.89	0.00	-13.34	0.00	-0.02	-2.82	-5.13	-2.80	0.00
Investment-capital	0.00	0.00	0.00	0.47	-0.00	-0.07	-0.39	-0.88	0.00
Growth of TFP	-0.00	-0.14	-0.00	0.00	-0.00	-0.00	-0.26	0.06	0.00
Growth of invt. price	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.64	0.00
Growth population	0.00	0.00	0.00	0.00	0.00	-0.07	0.00	0.00	0.00
Employment-pop.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02

Table 17: The table reports the target moments in each of the two subsamples 1984-2000 and 2001-2016, as well as the change between samples, and the contribution of each parameter to each change in moment, for the model with a low IES. See text for details.