Forward Guidance without Common Knowledge

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Outline

Motivation

- Standard macro analysis assumes **REE** and **complete info**
- By imposing perfect coordination, we might **"overstate"**
  - responsiveness of **forward-looking** expectations
  - potency of **GE effects**
  - ability of PM to influence economic outcomes
- This "bias" in our predictions increases with **horizon** of GE effects
  - we should doubt predictions that rest on long GE loops
  - forward guidance is an example

Forward Guidance Puzzle

- Context: A NK Economy at the ZLB
- Policy Question: **forward guidance** & (backloading) fiscal stimuli
- Answer: mainly driven by **GE** effects from inflation and income
  - GE **quantitatively large**
  - GE **explodes with horizon**
  - **PE** effects decreases with horizon

Main Findings

- Key step: recast IS and NKPC as Dynamic Beauty Contests
- Key insight: removing Common Knowledge $\Longrightarrow$
  - anchors expectations of y and π
  - attenuates GE feedback loops (both within and across two blocks)
  - attenuation larger the longer these loops
- Implications:
  - lessen forward guidance puzzle
  - offer rationale for front-loading fiscal stimuli

Related Literature

Part I: Higher-order uncertainty in macroeconomics

- Morris and Shin (1998, 2000, 2002), Woodford (2003), Angeletos and Pavan (2007), Angeletos and La'O (2009), Nimark (2011), etc
  - Angeletos and Lian (2016): chapter in *Handbook of Macroeconomics*

Part II: Forward guidance

- Different micro foundations:
  - McKay et al.(2016a,b), Del Negro et al. (2015)
- Deviate from rational expectations:
  - Schmidt & Woodford (2015), Farhi & Werning (2016), Gabaix (2016)
- We maintain **micro-foundations** & **rational expectations**
- Complementary: Wiederholt (2015)

Companion Paper:

- Dampening General Equilibrium (Angeletos and Lian, 2017)

Outline

Context

- Continuum of consumers/firms
- Consumer maximizes utility

$$\mathscr{U}_0 = \sum_{t=0}^{+\infty} \beta^t \left(\log c_{i,t} - \frac{1}{1+\varepsilon} n_{i,t}^{1+\varepsilon} \right),$$

s.t. budget constraint

$$c_{i,t} + s_{i,t} = a_{i,t} + w_{i,t} n_{i,t} + e_{i,t},$$
$$a_{i,t} = R_{t-1} s_{i,t-1} / \pi_t.$$

- Incomplete markets in the sense of no risk-sharing
  - but no liquidity constraints & work with log-linearized system
  - aggregates dynamics replicate textbook NK under CK

Firms

- Final goods produced by a competitive sector

$$y_t = \left(\int_0^1 \left(y_t^j \right)^{\frac{\varsigma - 1}{\varsigma}} dj \right)^{\frac{\varsigma}{\varsigma - 1}}$$

- Each variety j produced by a monopolistic firm

$$y_t^j = l_t^j$$

- Nominal rigidity: Calvo
  - fraction $1 - \theta$ changes price each period

Information and Equilibrium Concept

- "Fundamentals"
  - interest rate path (focus), discount rate, government spending
- Complete info: (Common Knowledge of fundamentals)
  - all (current) agents share the same information
  - allows uncertainty about future
  - but rules out all *higher-order* uncertainty
    - uncertainty about other (current) agents' beliefs and actions
- Incomplete info: (Remove CK of fundamentals)
  - Noisy private signals $\Rightarrow$ *higher-order* uncertainty
- **This paper:** maintain REE and remove CK of *future* fundamentals
  - compare with CK outcome
  - always maintain perfect knowledge of *current* fundamentals

Euler/IS *WITH* Common Knowledge

$$y_t = -E_t[r_{t+1}] + E_t[y_{t+1}]$$

- Key implication: $y = f$ (expected path of r)
  - this implication is robust to borrowing constraints
  - even though the aggregate Euler equation itself is different

Euler/IS *WITHOUT* Common Knowledge

$$y_t = -\left\{\sum_{k=1}^{+\infty} \beta^{k-1} \bar{E}_t[r_{t+k}]\right\} + (1-\beta)\left\{\sum_{k=1}^{+\infty} \beta^{k-1} \bar{E}_t[y_{t+k}]\right\}$$

- Dynamic beauty contest among consumers
  - follows from PIH and $y = c$
  - modern version of Keynesian income multiplier
- Key implication: $y \neq$ f(expected path of r)
  - instead, response of y to news about path of r hinges on HOB
- Why no recursive?
  - Law of iterated expectation **do not hold** for $\bar{E}_t[\cdots]$

NKPC *WITH/WITHOUT* Common Knowledge

$$\pi_t = mc_t + \beta E_t [\pi_{t+1}]$$

vs

$$\pi_t = mc_t + \left\{ \sum_{k=1}^{+\infty} (\beta\theta)^k \bar{E}_t^f [mc_{t+k}] \right\} + \frac{1-\theta}{\theta} \left\{ \sum_{k=1}^{+\infty} (\beta\theta)^k \bar{E}_t^f [\pi_{t+k}] \right\}$$

- Dynamic beauty contest among the firms
  - follows from optimal price setting
- Key implication: $\pi \neq$ f(expected path of *mc*)
  - instead, response of π to news about path of *mc* hinges on HOB

Three GE Mechanisms

- **Income multiplier**: $\bar{E}_t [y_{t+k}] \Rightarrow y_t$
- **Pricing complementarity:** $\bar{E}_t^f [\pi_{t+k}] \Rightarrow \pi_t$
- **Inflationary spiral:** interaction the two groups
  - $\bar{E}_t [\pi_{t+k}] \Rightarrow \bar{E}_t [r_{t+k}] \Rightarrow y_t$
  - $\bar{E}_t^f [y_{t+k}] \Rightarrow \bar{E}_t^f [mc_{t+k}] \Rightarrow \pi_t$
- Standard practice: impose CK = maximize all GE effects
- Our paper: relax CK = GE become HOB = attenuate all GE effects

Outline

Dynamic Beauty Contest

- So far: represent the NK model in terms of dynamic beauty contests
  - hint to the role of HOB
- What's next: theory of dynamic beauty contests
  - lack of CK = anchored expectations = **GE attenuation**
  - attenuation **increases with horizon** (*as if* extra discounting)

Dynamic Beauty Contest

- Consider models in which the following Euler-like condition holds:

$$a_t = \theta_t + \left\{ \sum_{k=1}^{+\infty} \gamma^{k-1} \bar{E}_t[\theta_{t+k}] \right\} + \alpha \left\{ \sum_{k=1}^{+\infty} \gamma^{k-1} \bar{E}_t [a_{t+k}] \right\}$$

- θ_t = aggregate fundamental at t
  - a_t = aggregate outcome at t
  - $\alpha > 0$ parameterizes GE feedback loop

Example

- Consumption beauty contest: $\theta_t = -r_t$, $a_t = y_t$
- Inflation beauty contest: $a_t = \pi_t$ and $\theta_t = mc_t$
- Asset pricing: $a_t = p_t$ and $\theta_t =$ dividend

Question of Interest

- **Question:** How a_0 responds news about θ_T
- To facilitate transition with the rest of paper
  - consider the NK setting with rigid price ($\pi_t = 0$)

$$y_t = -R_t - \left\{ \sum_{k=1}^{+\infty} \beta^{k-1} \bar{E}_t [R_{t+k}] \right\} + (1-\beta) \left\{ \sum_{k=1}^{+\infty} \beta^{k-1} \bar{E}_t [y_{t+k}] \right\}$$

- **Question:** How does y_0 responds to news about R_T ?
- Formally:
  - hold R_t (& belief about it) constant for all $t \neq T$
  - treat R_T as a random variable ($\sim N\left(0, \sigma_R^2\right)$)
  - specify information structure about R_T
  - study how y_0 covaries with $\bar{E}_0 [R_T]$
- All results hold for the general dynamic beauty contests as above

The Role of HOB

- By iterating, we can express y_0 as a linear function of
  - 1st-order beliefs: $\bar{E}_0 [R_T]$
  - 2nd-order beliefs: $\bar{E}_0 \left[\bar{E}_\tau [R_T] \right] \ \forall \tau : 0 < \tau < T$
  - 3rd-order beliefs: $\bar{E}_0 \left[\bar{E}_\tau \left[\bar{E}_{\tau'} [R_T] \right] \right] \ \forall \tau, \tau' : 0 < \tau < \tau' < T$
  - and so on, up to beliefs of order T
- With CK, HOB collapse to FOB, and the "usual" predictions apply
- Without CK, we need to understand
  - how much HOB co-move with $\bar{E}_0[R_T]$
  - how much HOB matter in y_0

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Leading Example

- Info structure:
  - Gaussian **private signal** about R_T at 0: $x_i = R_T + \varepsilon_i$,
  - no other info $\tau < T$. R_T becomes known at T
- Implication 1: beliefs constant over time

$$\bar{E}_\tau[.] = \bar{E}_0[.] \ \forall \tau : 0 < \tau < T$$

- Implication 2: a simple exponential structure for HOB

$$\bar{E}_0^h[R_T] = \lambda^{h-1} \bar{E}_0[R_T]$$

where $\lambda \in (0,1]$ is decreasing in the amount of noise

- Key observation (robust to richer info structures):
  - HOB are **anchored** relative to FOB
  - CK obtained as $\lambda \to 1$ and "maximizes" the responsiveness of HOB
- Anchoring HOB as modeling device of limited depth of reasoning

Main Results

$$y_0 = \phi(T)\bar{E}_0[R_T] \quad \text{vs} \quad y_0 = \phi^*(T)E_0[R_T]$$

- Our approach is robust to how much $\bar{E}_0[R_T]$ itself moves

1. Attenuation at any horizon
   - $\beta^{T-1} < \phi < \phi^*$ (ϕ bounded between PE effect and CK counterpart)
   - lower λ CK $\Rightarrow$ ϕ closer to PE effect
   - "CK maximizes GE effect"
2. Attenuation effect increases with horizon
   - ratio ϕ/ϕ^* decreases in T
   - longer horizons = iterating on Euler equation = iterating on beliefs
     - but HOB are more anchored than LOB
     - the more we iterate, the more potent this anchoring
   - it is *as if* the agents discount the future more heavily
3. Attenuation effect grows without limit as $T \to \infty$
   - $\phi/\phi^* \to 0$ as $T \to \infty$, even if $\lambda \approx 1$

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Robustness, Implications, and What's Next

- Results robust to richer information structures
  - exogenous and/or endogenous learning
- As if discounting of future endogenous variables
- Next: translating them to the full NK model:
  - IS: attenuate response of c to news about future real r
  - NKPC: attenuate response of π to news about future mc
  - Deal with caveats:
    - endogeneity of r and mc
    - GE feedback loop between IS and NKPC

Outline

ZLB and Forward Guidance

- Let T index length of liquidity trap and horizon of FG
  - $t \leq T-1$: ZLB binds and $R_t = 0$
  - $t \geq T+\Delta$: "natural level" and $y_t = \pi_t = 0$
  - let $\Delta = 1$ for simplicity
- Forward guidance: policy announcement at $t = 0$ of R_T
  - modeled as $z = R_T + noise$
  - noise captures central banks commitment issues and etc.
- We remove common knowledge of z
  - leading example: noisy private signals about z
- Remark
  - credibility has to do with how much $\bar{E}_0[R_T]$ varies with R_T
  - we focus on how y_0 varies with $\bar{E}_0[R_T]$

The Power of Forward Guidance

- Degree of CK indexed by $\lambda \in (0, 1]$

$$\bar{\mathbb{E}}^h[R_T] = \lambda^{h-1}\bar{\mathbb{E}}^1[R_T]$$

  - consumers vs firms: λ_c vs λ_f
  - CK benchmark nested with $\lambda_c = \lambda_f = 1$

- *Question:* How does y_0 vary with $\bar{E}_0[R_T]$

- *Answer:* There exists a function ϕ such that

$$y_0 = -\phi\left(\lambda_c, \lambda_f; T, \kappa\right) \cdot \bar{E}_0[R_T]$$

  - standard: ϕ^* increases with T and explodes as $T \to \infty$
  - here: ϕ vs ϕ^*

Main Results

- Attenuation for any horizon: $\phi/\phi^* < 1$
  - three GE effects at work:
    1. inside IS: income-spending feedback
    2. inside NKPC: inflation-inflation feedback
    3. across two blocs: inflation-spending feedback
  - all three attenuated; but quantitative bite for (2) and (3)
- Attenuation effect increases with horizon
  - ϕ/ϕ^* decreases in T
  - $\phi/\phi^* \to 0$ as $T \to \infty$, even if $\lambda \approx 1$
  - for λ_c small enough, $\phi \to 0$ in absolute, not only relative to ϕ^*

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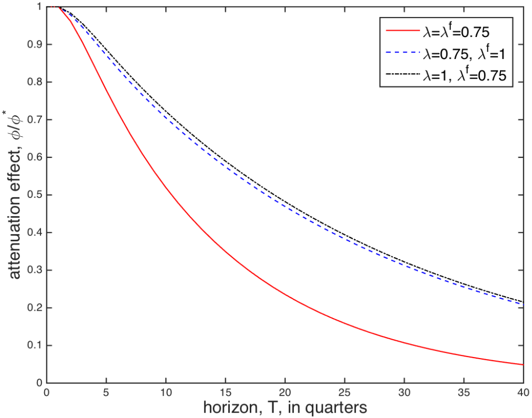
A Numerical Illustration (based on McKay et al.)



- Modest info friction: $\lambda_c = \lambda^f = 0.75$
  - 25% prob that others have failed to hear announcement
- On top of any mechanical effect that first order informational friction

Fiscal Stimuli

- Standard NK under ZLB prediction:
  - fiscal stimuli work because they trigger inflation
  - better to back-load so as to "pile up" inflation effects
- Our twist:
  - such piling up = iterating HOB
  - not as potent when CK assumption is dropped
  - rationale to front-load so as to minimize coordination friction

Discounted Euler Equation and NKPC

- $E_t[x]$: RE conditional on all info. at period t
- Discounted Euler Equation and NKPC for $t < T - 1$

$$
\begin{aligned}
y_t &= \Lambda E_t[y_{t+1}] + \lambda E_t[\pi_{t+1}] \\
\pi_t &= \beta M E_t[\pi_{t+1}] + \kappa m_t y_t + \kappa \mu_t
\end{aligned}
$$

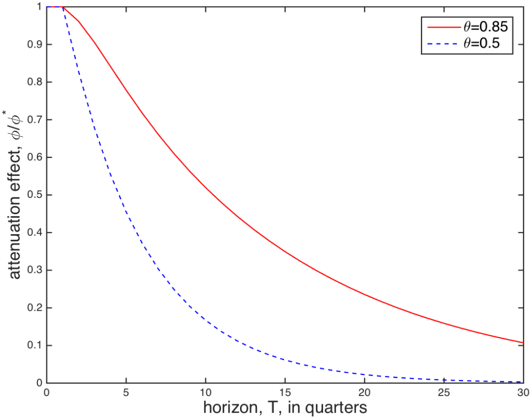
  where $\Lambda, M, m_t \in (0,1)$.

- "As if" result maps heterogenous-agent, incomplete-info model
  - to a fictitious representative-agent, complete-info model
- Individual Euler Equation holds
  - discounting expectations of future endogenous aggregates
  - different from McKay et al. (2016), Werning (2015) & Gabaix (2016)

discount

Paradox of Flexibility

- Standard model: effect of FG increases with price flexibility
  - but is due to GE effect: "inflationary/deflationary" spiral
- Without CK: GE dampened
  - dampening increases with price flexibility



Empirical Support

- Andrade et. al (2016): Survey of Professional Forecasters
- After Fed's date-based forward guidance
  - a drop in the mean forecasts of nominal interest rates
  - an increase in disagreement of future macro conditions
    - inflation and output
  - mean forecasts of future macro conditions barely move

intro

Outline

Companion Paper

- "Dampening GE: from Micro to Macro" (Angeletos and Lian, 2017)
- REE alone $\Rightarrow$ restrict GE in an interval
  - Standard practice (REE+ CK) -> upper bound of the interval
- Lack of CK = GE dampened
- Non-REE variants often, but not always, attenuate GE
  - level-k, Tatonnment, Cobweb, Sparsity, ε-equilibrium
  - Lack of CK = a structured way to relax REE
- Connection to empirical work a la Mian-Sufi
  - reduce GE = reduce gap between micro and macro elasticities

Conclusion

- Forward-looking expectations crucial in modern macro
- By assuming CK with REE, hardwire a certain kind of perfection in
  - how economic agents to coordinate their expectations
  - maximize policy makers abilities to steer economy
- Remove CK helps accommodate a realistic friction
  - alleviate forward guidance puzzle
- Insights and the techniques may find additional applications
  - fiscal multipliers
  - demand driven business cycles

Outline

Shocks

- Shocks to markups
  - μ_t^j at the firm level
  - μ_t at the aggregate level
- Shocks to wages
  - $w_t^j = w_t u_t^j$ at the firm level
  - $w_{it} = w_t \xi_{it}$ at the household level
- Monetary policy to be specified
- Modeling role of shocks: limit aggregation of information

main

Understanding Discounted Euler Equation

- Individual Euler equation always holds

$$c_{i,t} = E_{i,t}\left[c_{i,t+1}\right] + R_t - E_{i,t}\left[\pi_{t+1}\right]$$

- With complete information $E_{i,t}\left[c_{i,t+1}\right] = E_t\left[c_{i,t+1}\right]$ thus

$$\int E_{i,t}\left[c_{i,t+1}\right] di = E_t\left[c_{t+1}\right]$$

- Together with market clearing gives the dynamic IS equation

$$y_t = E_t\left[y_{t+1}\right] + R_t - E_t\left[\pi_{t+1}\right]$$

- Without CK, frictions in predict future income and inflation

$$\int_0^1 E_{i,t}\left[c_{i,t+1}\right] di = \Lambda E_t[c_{t+1}]$$

  - "discounted Euler Equation" main