Fractionally Integrated Multivariate Models for Fat-Tailed Realized Covariance Kernels and Returns

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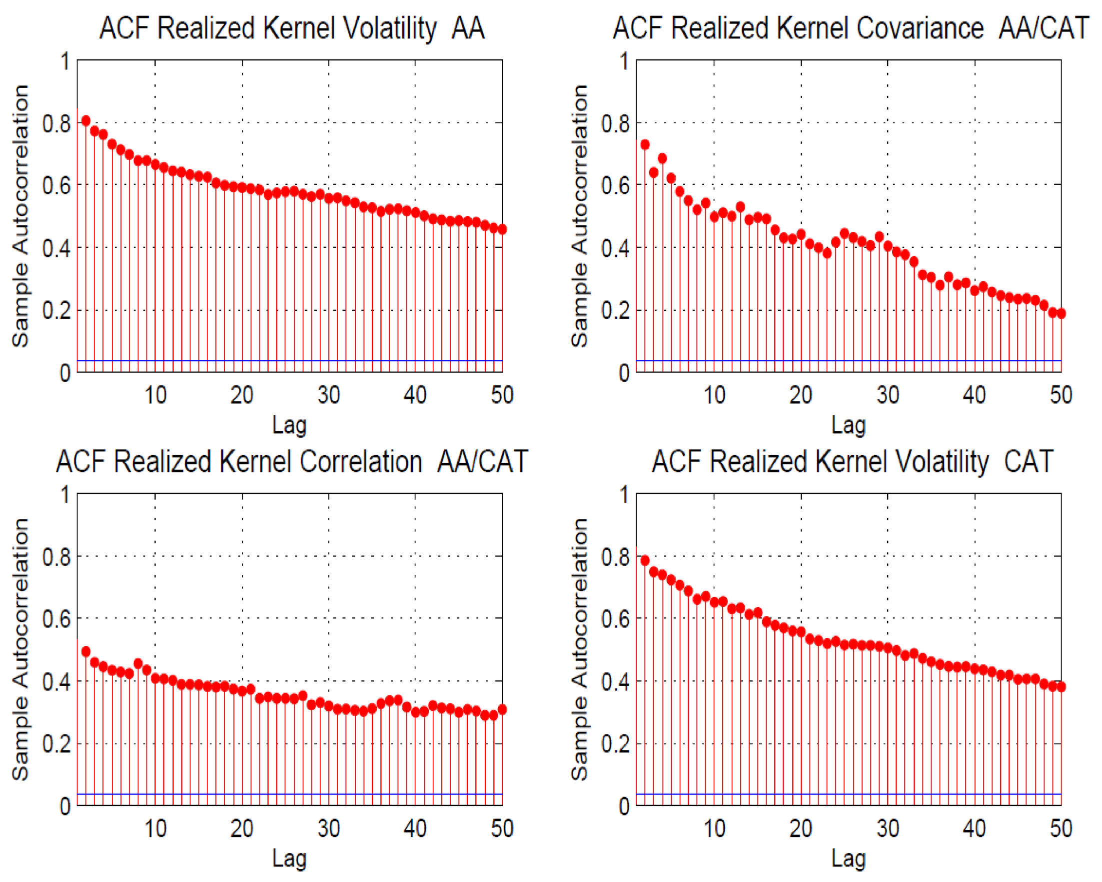
Highlights

- We introduce a new model for multivariate covariance dynamics based on long-memory behavior of daily returns and daily realized covariance kernels
- In addition, the model takes into account fat-tailedness in both returns and realized kernels by assuming a Multivariate Student-t distribution for returns and a matrix-F distribution for realized kernels
- We apply our model on a panel of 15 equities listed at the S&P 500 index from 2001-2012
- The results show the new fractionally integrated model both statistically and economically outperforms recent alternatives such as the Multivariate HEAVY model (Nouraldin et al. 2012) and the Riskmetrics 2006 methodology

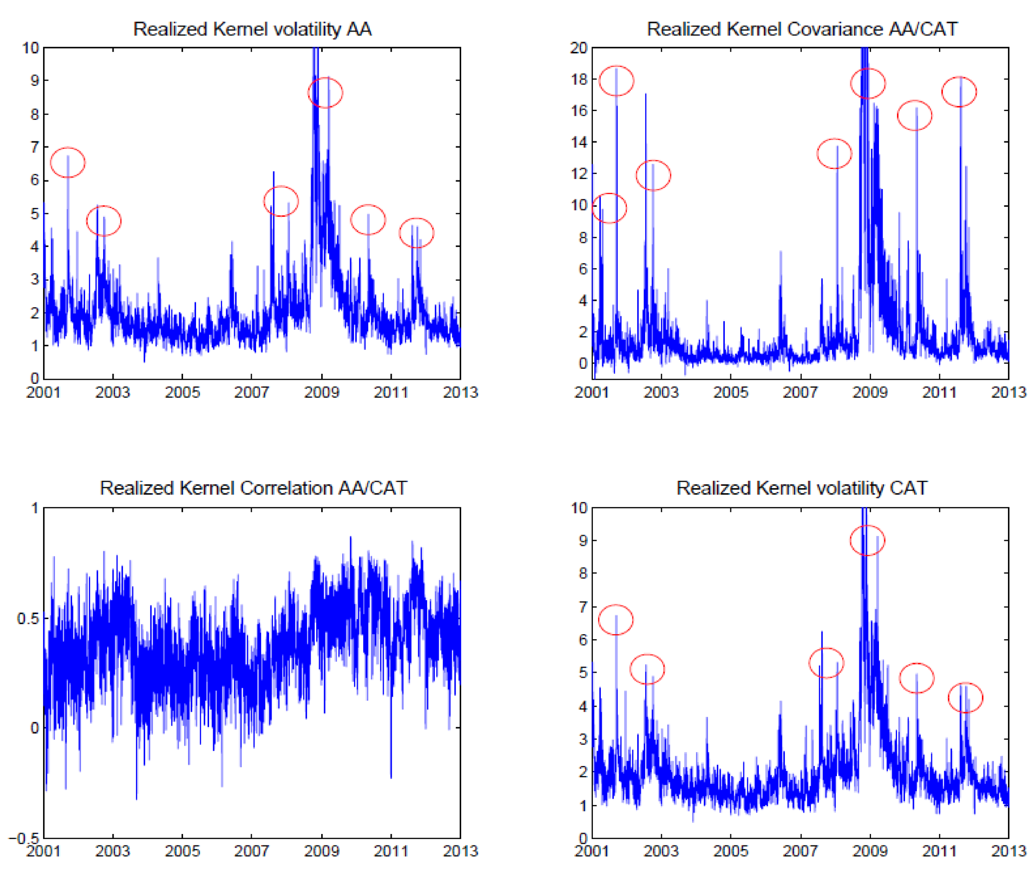
Motivation/Literature

Volatility is persistent. Baillie et al. (1996) introduce the Fractionally Integrated GARCH model (FIGARCH) using returns

Realized measures are highly persistent (Andersen et al. 2001) → HAR model (Corsi, 2009), ARFIMA models (Univariate: Koopman et al. 2005, Multivariate: Chiriac and Voev, 2011)



Important aspect of returns and realized measures: they are fat-tailed and may contain outliers. This has not been taken into account yet by the literature on long-memory volatility models!



Bauer and Vorkink (2011) and Chiriac and Voev (2011) consider the *vech* (of the cholesky decomposition) of the covariance matrix of 5 or 6 assets. Our purpose is to retain the matrix format and consider also dimension 15.

The Multivariate FIGAS model

Our contribution: we connect long memory behavior of both returns and realized measures with their fat-tailedness property by means of the FIGAS tF model. Denote y_t as a vector of k returns, and RK_t as a k x k realized covariance kernel, specified as

$$y_t = \mu + V_t^{1/2}\, z_t, \qquad z_t|\mathcal{F}_{t-1} \sim D_z(0, \mathrm{I}_k),$$
$$RK_t = V_t^{1/2}\, Z_t\, (V_t^{1/2})', \qquad Z_t|\mathcal{F}_{t-1} \sim D_Z(\mathrm{I}_k),$$

where the time-varying conditional covariance matrix is modeled as a FIGAS process:

$$(1-L)^d V_{t+1} = \Omega + B\,(1-L)^d V_t + A\, s_t$$

with L the lag operator and $(1-L)^d$ the fractional difference operator, defined as

$$(1-L)^d = 1 - dL + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 + \ldots,$$

for $d > -1$. Further, A and B are scalars, and s_t denotes the scaled score:

$$s_t = \frac{V_t(\nabla_{y,t} + \nabla_{RK,t})V_t}{\nu_1 + 1}$$

which depends on the partial derivative of the logarithm of the fat-tailed Multivariate Student-*t* (ν_0) and Matrix-*F* (ν_1, ν_2) distribution with respect to V_t:

$$\nabla_{y,t} = \frac{1}{2}V_t^{-1}\left[w_t y_t y_t' - V_t\right]V_t^{-1}$$
$$\nabla_{RK,t} = \frac{\nu_1}{2}V_t^{-1}\left[\frac{\nu_1+\nu_2}{\nu_2-k-1}RK_t\left(\mathrm{I}_k + \frac{\nu_1}{\nu_2-k-1}V_t^{-1}RK_t\right)^{-1} - V_t\right]V_t^{-1}$$

with $w_t = \frac{\nu_0 + k}{\nu_0 - 2 + y_t' V^{-1} y_t}$.

Interpretation of the score:

- Impact of "large values" of $y_t y_t'$ on V_t is downweighted by w_t if density for y_t is fat-tailed (i.e. $1/\nu_0 > 0$)
- Likewise, the inverse term in $\nabla_{RK,t}$ shows that large values of RK_t - measured by $V_t^{-1}RK_t$ - do not automatically lead to substantial changes in the covariance matrix V_t

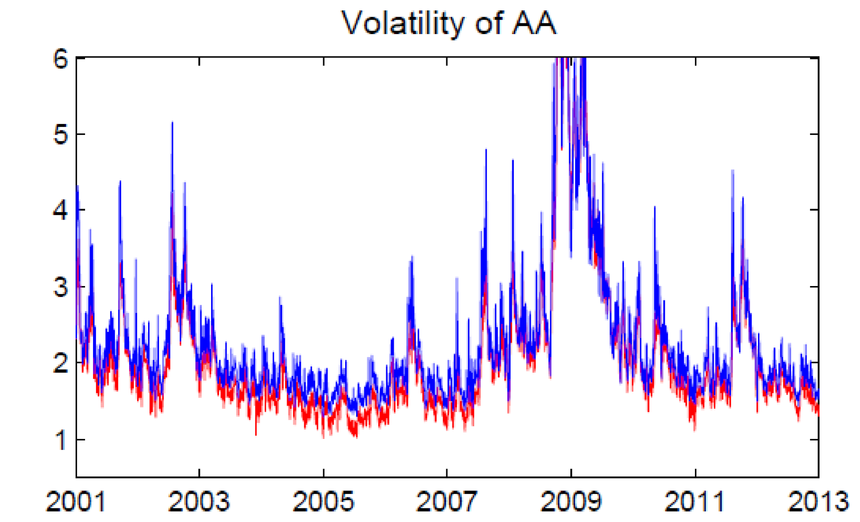
Estimation

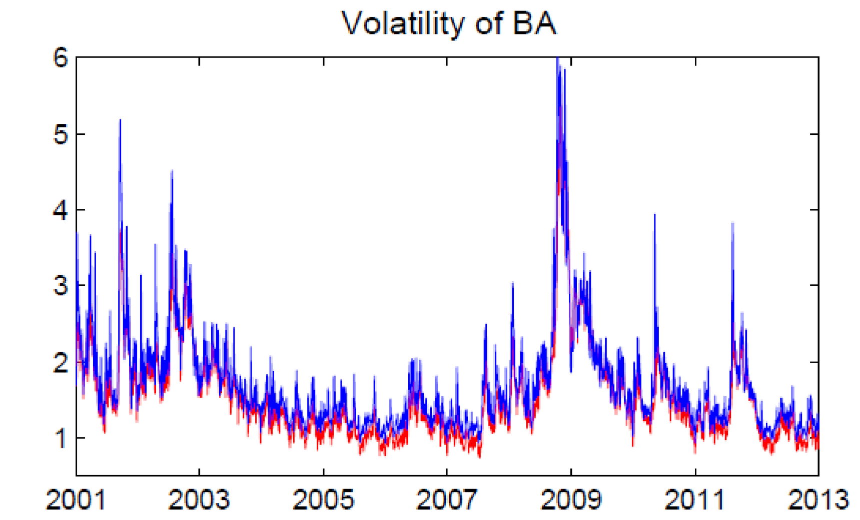
We estimate the FIGAS tF model by Maximum Likelihood and compare our model against the GAS tF (Janus et al. 2014) M-HEAVY (Noureldin et al. 2012) and the Riskmetrics 2006 models.

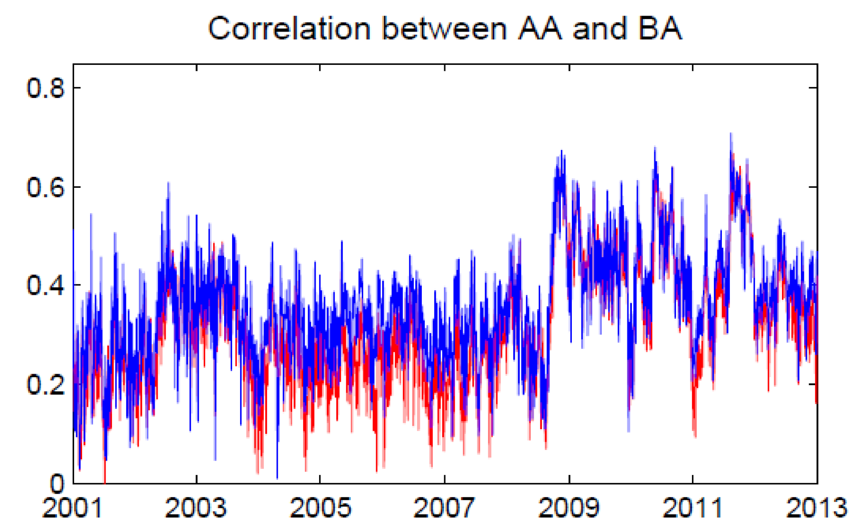
Data: 15 assets from S&P 500, from January 2, 2001 until December 30, 2012 (3017 observations).

	AA/BA/CAT/GE/KO			
Coef.	FIGAS	HEAVY	GAS	RM
A	0.735	0.419	0.619	
	(0.014)	(0.035)	(0.012)	
B	0.999	0.597	0.986	
	(0.001)	(0.033)	(0.001)	
c		0.046		
		(0.006)		
A_M		0.286		
		(0.009)		
B_M		0.698		
		(0.010)		
ν_0	10.37		10.01	
	(0.504)		(0.469)	
ν_1	46.27		46.61	
	(0.925)		(0.911)	
ν_2	36.22		34.97	
	(0.577)		(0.521)	
d	-0.241			
	(0.006)			
$\mathcal{L}_t$	-26,436		-26,474	
$\mathcal{L}_F/\mathcal{L}_W$	-20,788	-45,750	-21,243	
QLIK	7.694	7.806	7.712	51.43

In-sample results







Out-of-sample analysis

- We forecast a 15 x 15 covariance matrix 1,5,10, and 22 steps ahead, based on a MW-approach with T_w=1500
- Statistical application: test on predictive ability between models based on the QLIK loss function and the log-score (i.e. density forecasts)
- Economic application: Global Minimum Variance (GMV) weights

$$\min w_{t+h|t}' V_{t+s|t} w_{t+h|t} \quad \text{s.t.} \quad w_{t+h|t}'\iota = 1.$$

and test on the difference of the ex-post conditional portfolio standard deviation $\sigma_{p,t} = \sqrt{w_{t+h|t}' RK_{t+h} w_{t+h|t}}.$

	1	5	10	22
	mean of log-score			
FIGAS vs HEAVY	43.55	40.93	40.92	42.08
	(49.1)	(36.3)	(24.3)	(13.5)
FIGAS vs GAS	0.78	0.80	1.52	3.00
	(4.1)	(2.0)	(2.7)	(4.3)

	1	5	10	22	1:5	1:10
	QLIK loss function					
FIGAS tF	19.07	20.04	20.75	21.84	43.78	54.65
HEAVY	19.12	20.11	20.93	22.33	43.87	54.84
	(-0.8)	(-0.7)	(-1.3)	(-3.6)	(-0.9)	(-1.5)
GAS tF	19.12	20.06	20.89	22.23	43.79	54.72
	(-2.3)	(-0.3)	(-1.5)	(-3.2)	(-0.2)	(-0.9)
RM 2006	24.58	26.62	29.74	38.20	49.61	61.27
	(-10.1)	(-8.6)	(-8.0)	(-8.4)	(-5.8)	(-4.5)
	Mean of ex-post σ_p					
FIGAS tF	0.688	0.700	0.707	0.718	1.586	2.278
HEAVY	0.690	0.703	0.711	0.723	1.592	2.289
	(-2.6)	(-4.1)	(-4.6)	(-4.5)	(-3.3)	(-4.6)
GAS tF	0.689	0.703	0.711	0.723	1.590	2.286
	(-4.6)	(-5.4)	(-5.5)	(-6.4)	(-3.9)	(-4.0)
RM 2006	0.830	0.817	0.805	0.796	1.872	2.646
	(-15.4)	(-14.5)	(-13.6)	(-12.6)	(-8.2)	(-6.2)