Multistep prediction error decomposition in DSGE models: estimation and forecast performance*

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Abstract

DSGE models are of interest because they offer structural interpretations, but are also increasingly used for forecasting. Estimation often proceeds by methods which involve building the likelihood by one-step ahead ($h = 1$) prediction errors. However in principle this can be done using different horizons where $h > 1$. Using the well-known model of Smets and Wouters (2007), for $h = 1$ classical ML parameter estimates are similar to those originally reported. As h extends some estimated parameters change, but not to an economically significant degree. Forecast performance is often improved, in several cases significantly.

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1 Introduction

Forecasting is central to macroeconomic policymaking, especially since the introduction of inflation targeting, which has often been linked to macroeconomic forecasts.[1] Often, attention is focussed on forecasts at several horizons, typically up to two or three years.[2] Policy analysis requires structural models, and the current canonical versions of these are generally dynamic stochastic general equilibrium (DSGE) models. However, until recently it was received wisdom that parsimonious reduced form econometric models are the most appropriate and effective tools for carrying out forecasting, which created a practical tension. Although recent work has suggested that DSGE models can be of use in forecasting (eg Del Negro et al. (2007) or Fawcett et al. (2015)), the record remains mixed (Edge and Gurkaynak (2011)).

The standard approach to forecasting with DSGE models involves linearisation followed by specification in state space form and solution. Standard techniques can be used to estimate the parameters. Subsequently, the estimated model may be used to forecast variables of interest. So estimation is primarily oriented towards obtaining estimates of the structural parameters. But if the aim is also to produce forecasts, a method that takes this into account may be desirable, potentially delivering both estimates of structural parameters necessary for policy making and inference but also good forecast performance.

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[1]See eg Svensson (2005).

[2]Eg, the FRB states that '[t]he FOMC implements monetary policy to help maintain an inflation rate of 2 percent over the medium term.'

One way to approach this is move away from strict maximum likelihood (ML) estimation and instead optimise on an object that is focussed on a vector of multi-step prediction errors. This can be seen as a method of moments (MOM) approach, somewhat akin to cross-validation, widely used in forecasting applications.

Section 2 discusses this amendment, while Section 3 presents the empirical results. Section 4 concludes.

2 Method

As observed above forecasting using DSGE models is routinely carried out with the state space representation of the linearised model. We will focus on this, given by

$$\begin{aligned} y_t &= H\xi_t, \quad t = 1, .., T \\ \xi_t &= C\xi_{t-1} + v_t. \end{aligned}$$

y_t are the observed variables while x_t is an unobserved vector of states that may be estimated using the Kalman filter. In particular the Kalman filter can be used to provide $\hat{\xi}_{t|t-1} = E(\xi_t|Y_{1,t-1})$ and $\hat{\xi}_{t|t} = E(\xi_t|Y_{1,t})$ where $Y_{s,t} = (y_s, ..., y_t)'$. When the parameter matrices H and C are unknown, they can be conveniently estimated using ML based on the prediction error decomposition. Once the parameters are obtained the state space model can be used to produce forecast for any desired horizon. The prediction error is normally assumed to be a one-step ahead error. However, this is not necessary, and may not be optimal when multi-step forecasts are of interest. Consequently we consider an estimation method based on an ML objective function for a vector of prediction errors given by $\upsilon_{1,h,t} = (\upsilon_{1,t}, .., \upsilon_{h,t})'$ where $\upsilon_{h,t} = y_t - \hat{y}_{t|t-h}$ and $\hat{y}_{t|t-h} = E(y_t|Y_{1,t-h})$.

From Hamilton (1994) we know that the h-period-ahead forecast vector using the Kalman filter is given by

$$\begin{aligned}
\begin{bmatrix} \hat{y}_{t+h/t} \\ \hat{y}_{t+h-1/t} \\ \vdots \\ \hat{y}_{t/t} \end{bmatrix} &= \begin{bmatrix} H' & 0 & \cdots & 0 \\ 0 & H' & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & H' \end{bmatrix} \begin{bmatrix} \hat{\xi}_{t+h/t} \\ \hat{\xi}_{t+h-1/t} \\ \vdots \\ \hat{\xi}_{t/t} \end{bmatrix} \\
\begin{bmatrix} \hat{y}_{t+h/t} \\ \hat{y}_{t+h-1/t} \\ \vdots \\ \hat{y}_{t/t} \end{bmatrix} &= (I_{h+1} \otimes H') \begin{bmatrix} \hat{\xi}_{t+h/t} \\ \hat{\xi}_{t+h-1/t} \\ \vdots \\ \hat{\xi}_{t/t} \end{bmatrix} \\
\begin{bmatrix} \hat{y}_{t+h/t} \\ \hat{y}_{t+h-1/t} \\ \vdots \\ \hat{y}_{t/t} \end{bmatrix} &= (I_{h+1} \otimes H') \begin{bmatrix} F^h \\ F^{h-1} \\ \vdots \\ I \end{bmatrix} \hat{\xi}_{t/t} \\
\hat{Y}^h_{t,t+h} &= (I_{h+1} \otimes H') \tilde{F} \hat{\xi}_{t/t}. \qquad (2.1)
\end{aligned}$$

(2.1) is used to obtain an expression about the h-step ahead forecast error vector:

$$\begin{aligned}
\begin{bmatrix} y_{t+h} \\ y_{t+h-1} \\ \vdots \\ y_t \end{bmatrix} - \begin{bmatrix} \hat{y}_{t+h/t} \\ \hat{y}_{t+h-1/t} \\ \vdots \\ \hat{y}_{t/t} \end{bmatrix} &= (I_{h+1} \otimes H') \tilde{F} \left(\xi_t - \hat{\xi}_{t/t}\right) + \begin{bmatrix} I & F & \cdots & F^h \\ 0 & I & \cdots & F^{h-1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I \end{bmatrix} \begin{bmatrix} \upsilon_{t+h} \\ \upsilon_{t+h-1} \\ \vdots \\ \upsilon_t \end{bmatrix} \\
Y_{t,t+h} - \hat{Y}^h_{t,t+h} &= (I_{h+1} \otimes H') \tilde{F} \left(\xi_t - \hat{\xi}_{t/t}\right) + \Gamma V. \qquad (2.2)
\end{aligned}$$

This may be now used to derive the MSE

$$E\left(Y_{t,t+h}-\hat{Y}^{h}_{t,t+h}\right)\left(Y_{t,t+h}-\hat{Y}^{h}_{t,t+h}\right)' = \left(I_{h+1}\otimes H'\right)\tilde{F}E\left(\xi_t-\hat{\xi}_{t/t}\right)\left(\xi_t-\hat{\xi}_{t/t}\right)'\tilde{F}'\left(I_{h+1}\otimes H\right) + \Gamma EVV'\Gamma'$$

$$E\left(Y_{t,t+h}-\hat{Y}^{h}_{t,t+h}\right)\left(Y_{t,t+h}-\hat{Y}^{h}_{t,t+h}\right)' = \left(I_{h+1}\otimes H'\right)\tilde{F}P_{t/t}\tilde{F}'\left(I_{h+1}\otimes H\right) + \Gamma\left(I_{h+1}\otimes Q\right)\Gamma' \quad (2.3)$$

where $\hat{\xi}_{t/t}$ and $P_{t/t}$ are the updated one-step Kalman filter estimate and its covariance matrix. Given expressions (2.1) and (2.3) the likelihood is easily derived, given a normality assumption, since

$$Y_{t,t+h} \sim N\left(\mu_t, \Sigma_t\right) \quad (2.4)$$

$$\mu_t = \left(I_{h+1}\otimes H'\right)\tilde{F}\hat{\xi}_{t/t} \quad (2.5)$$

$$\Sigma_t = \left(I_{h+1}\otimes H'\right)\tilde{F}P_{t/t}\tilde{F}'\left(I_{h+1}\otimes H\right) + \Gamma\left(I_{h+1}\otimes Q\right)\Gamma'. \quad (2.6)$$

Thus this approach estimates parameters using a vector of prediction errors for different horizons, rather than the standard ML built from the one-step ahead errors. As discussed in the introduction, it may be seen as a MOM approach akin to cross-validation. Schorfheide (2005) adopts a similar approach, defining a loss function in terms of prediction errors, in the context of parameter estimation of misspecified models.

The question is then whether this is practically useful, and in the next section we use a benchmark DSGE model to evaluate our approach.

3 Results

3.1 The model

We apply our modified estimation method to the model described in Smets and Wouters (2007), which is an extension of a small-scale monetary RBC model with sticky prices. It contains additional shocks and frictions, including sticky nominal price and wage settings with backward inflation indexation, investment adjustment costs, fixed costs in production, habit formation in consumption and capital utilisation. It features seven exogenous shocks that drive the stochastic dynamics. The foundations are derived from the decisions of different agents by solving intertemporal optimisation problems. Consumers supply labour, choose their consumption, hold bonds and make investment decisions; intermediate goods producers are in a monopolistically competitive market and cannot adjust prices at each period; and final goods producers buy intermediate goods, package them and resell them to consumers in a perfectly competitive market. In addition, there is a labour market with a similar structure: there are labour unions with market power that buy the homogeneous labour from households, differentiate it, set wages and sell it to the labour packers, who package it and resell it to intermediate goods producers in a perfectly competitive environment. Finally, there is a central bank that follows a nominal interest rate rule, adjusting the policy instrument in response to deviations of inflation or output from their target levels and a government that collects lump-sum taxes (or grants subsidies) which appear in the consumer's budget constraint and whose spending appears in the model as one of the seven exogenous shocks.

3.2 Estimates

The model is first log-linearised around its steady state and trended variables detrended with a deterministic trend. It is estimated using seven macroeconomic quarterly time series for the United States as observables. These variables are those used in Smets and Wouters (2007), namely output growth, consumption growth, investment growth, inflation, wages growth, hours and the interest rate. Similarly to Ireland (2004), Fernandez-Villaverde and Rubio-Ramirez (2008) and Ireland (2013) (among others) all the structural parameter estimates discussed below are obtained using only the likelihood of the model (and not the likelihood weighted by a prior distribution of the structural parameter vector).

Although our estimation sample is five years shorter than that used by Smets and Wouters (2007) and we use no prior information, the parameter estimates are remarkably similar to those they report.[3] This is particularly so for the parameters that govern the behaviour of the exogenous states variables and those that control the steady-state values of inflation, hours and productivity growth. On the other hand, the parameters responsible for the model's endogenously generated inertia (such as habit formation, Calvo probabilities, degree of wage indexation and investment adjustment cost) are generally somewhat larger than the estimates reported by Smets and Wouters.

Results for all horizons are reported in Table 1. As the forecast estimation horizon increases, the shock processes' estimated parameters remain largely constant. No obvious regularities can be seen in these variations as the horizon increases. More variation occurs in the structural parameters although in most cases the changes are not dramatic. Exceptions include ρ_{ga} where the coefficient rises an order of magnitude at $h = 7$ and ι_p where it falls dramatically at the same horizon. ρ_R also has a low value at that horizon. As we show below, these changes improve the model's forecasting performance, but it is not easy to associate that improvement with particular parameter changes.

One possible concern is that we are finding local optima. Our ML approach might be loosely interpreted as estimation under a flat prior distribution. We have noted that the estimates are similar to the Bayes estimates under the informative prior used by Smets and Wouters (2007). Nevertheless, Del Negro and Schorfheide (2013) illustrate that changing the prior for the steady state inflation rate from Smets and Wouters' to one more diffuse leads to a substantially larger estimate of π^* and a significant deterioration of the forecast performance. Herbst and Schorfheide (2014) estimate the model under a more diffuse, albeit not flat prior, and show that the posterior distribution becomes multi-modal. So it may be that the likelihood has several modes, some quite different from the Smets and Wouter estimates.

In order to examine this, we estimated the model with two sets of starting points. If the results are affected by these choices, then we may be concerned that we are finding local minima. Our estimates reported in table 1 are based on 100 estimations of the model starting from a different point each time and from these 100 report the results that correspond to the highest maximum. But as a check we also carried out the same exercise for just 10 starting values, and therefore 10 estimations) from which we pick the highest maximum. The rational is that a small set of starting values will expose problems of local minima. In fact, the forecast results from this exercise (not reported but available on request) are almost identical. Table 2 report the set of parameter estimates, and comparison of the two tables reveals that most of the estimates are very similar. There are however a few exceptions, such as the time discount parameter transformation and the investment adjustment cost, the latter apparently varying between about 6 and 12 at different horizons. But these changes do not have an effect on the forecasting performance of the model. This is easy to understand, as the changes are not economically significant. For example, the time discount parameter transform estimates imply that β varies trivially, from 0.9960 to 0.9996 between Tables 1 and 2, so it is unsurprising that these changes have a minimal impact on the forecast. Similarly, an investment adjustment cost greater than 5 implies that investment does not respond to Tobin's Q, so it makes almost no difference (especially in the forecasting performance of the model) if it is 6 or 12.

[3] Fernandez-Villaverde and Rubio-Ramirez (2008) observe that flat and informative priors have little impact on estimates in their baseline model.

Table 1: DSGE Parameter Estimates Using Different Estimation Horizon: Sample $1954Q3-1997Q4$: (*Based on 100 Different Starting Values*

Mnemonic	Description	Forecast Estimation Horizon							
		1	2	3	4	5	6	7	8
σ_a	STD Productivity Shock	0.46	0.47	0.49	0.47	0.46	0.47	0.47	0.48
σ_b	STD Preference Shock	0.26	0.27	0.25	0.23	0.27	0.27	0.28	0.28
σ_g	STD Government Spending Shock	0.55	0.54	0.54	0.58	0.54	0.54	0.54	0.54
σ_q	STD Investment Specific Shock	0.49	0.52	0.54	0.58	0.50	0.52	0.50	0.52
σ_R	STD Monetary Policy Shock	0.25	0.23	0.23	0.23	0.23	0.23	0.24	0.23
σ_p	STD Price Markup Shock	0.15	0.16	0.17	0.17	0.16	0.17	0.18	0.17
σ_w	STD Wage Markup Shock	0.15	0.19	0.19	0.19	0.20	0.19	0.19	0.20
ρ_a	Persistence Productivity Shock	0.96	0.98	0.98	0.98	0.99	0.98	0.98	0.98
ρ_b	Persistence Preference Shock	0.35	0.22	0.32	0.45	0.26	0.26	0.22	0.26
ρ_g	Persistence Government Spending Shock	0.91	0.97	0.97	0.90	0.93	0.95	0.94	0.95
ρ_q	Persistence Investment Specific Shock	0.61	0.56	0.53	0.50	0.58	0.58	0.58	0.54
ρ_R	Persistence Monetary Policy Shock	0.30	0.10	0.10	0.11	0.09	0.08	0.09	0.03
ρ_p	Persistence Price Markup Shock	0.87	0.96	0.98	0.92	0.95	0.98	0.99	0.98
ρ_w	Persistence Wage Markup Shock	0.94	0.96	0.95	0.98	0.98	0.95	0.97	0.98
θ_p	MA Coefficient Price Markup Shock	0.77	0.88	0.92	0.84	0.87	0.91	0.93	0.91
θ_w	MA Coefficient Wage Markup Shock	0.83	0.93	0.92	0.94	0.95	0.92	0.93	0.96
S'	Investment Adjustment Cost	6.11	9.12	9.03	8.40	11.80	8.59	10.49	10.49
σ_C	Intertemporal Substitution Elasticity	1.03	1.37	1.36	1.14	1.14	1.29	1.16	1.15
h	Habit Formation	0.82	0.80	0.79	0.84	0.88	0.82	0.86	0.86
ξ_w	Probability of Resetting Wage	0.84	0.90	0.89	0.83	0.85	0.89	0.85	0.90
σ_L	Labour Supply Elasticity	2.45	2.84	1.42	1.87	2.44	2.32	1.70	3.47
ξ_p	Probability of Resetting Price	0.79	0.74	0.74	0.75	0.72	0.71	0.71	0.72
ι_w	Wage Indexation	0.44	0.32	0.33	0.32	0.41	0.37	0.30	0.43
ι_p	Price Indexation	0.28	0.04	0.05	0.08	0.08	0.05	0.04	0.03
ψ	Utilisation Adjustment Cost	0.22	0.20	0.10	0.21	0.32	0.21	0.19	0.11
Φ	Production Fixed Cost	1.81	1.80	1.67	1.94	1.95	1.77	1.77	1.67
γ_π	Inflation Policy Reaction	1.76	1.93	2.23	2.21	2.38	2.21	1.97	2.12
γ_R	Interest Policy Smoothing	0.81	0.85	0.87	0.86	0.87	0.87	0.84	0.87
γ_y	Output Gap Policy Reaction	0.11	0.08	0.13	0.14	0.12	0.12	0.08	0.11
$\gamma_{\Delta y}$	Output Gap Growth Policy Reaction	0.22	0.17	0.19	0.16	0.18	0.18	0.18	0.18
$\bar{\pi}$	Steady State Inflation	0.77	0.93	0.82	0.91	0.91	0.91	0.93	0.92
$100\left(\beta^{-1}-1\right)$	Time Discount	0.40	0.06	0.04	0.10	0.08	0.07	0.06	0.07
$\bar{L}$	Steady State Hours	1.02	1.04	1.19	0.53	0.52	0.82	0.71	0.48
$\bar{\gamma}$	Productivity Growth	0.46	0.50	0.50	0.42	0.44	0.49	0.50	0.49
ρ_{ga}	Government Spending and Productivity Correlation	0.05	0.53	0.55	0.11	0.51	0.58	0.58	0.55
α	Capital Production Share	0.22	0.21	0.20	0.18	0.21	0.21	0.20	0.19

Table 2: DSGE Parameter Estimates Using Different Estimation Horizon: Sample $1954Q3 - 1997Q4$: (*Based on 10 Different Starting Values*

Mnemonic	Description	Forecast Estimation Horizon							
		1	2	3	4	5	6	7	8
σ_a	STD Productivity Shock	0.48	0.47	0.47	0.47	0.46	0.45	0.47	0.50
σ_b	STD Preference Shock	0.28	0.26	0.23	0.28	0.24	0.23	0.26	0.25
σ_g	STD Government Spending Shock	0.57	0.56	0.59	0.56	0.58	0.59	0.54	0.59
σ_q	STD Investment Specific Shock	0.51	0.49	0.47	0.51	0.42	0.49	0.51	0.46
σ_R	STD Monetary Policy Shock	0.26	0.23	0.24	0.25	0.24	0.22	0.24	0.24
σ_p	STD Price Markup Shock	0.14	0.15	0.15	0.15	0.16	0.18	0.18	0.15
σ_w	STD Wage Markup Shock	0.14	0.19	0.18	0.15	0.17	0.17	0.19	0.19
ρ_a	Persistence Productivity Shock	0.96	0.98	0.97	0.96	0.98	0.98	0.98	0.98
ρ_b	Persistence Preference Shock	0.36	0.30	0.42	0.35	0.41	0.39	0.29	0.35
ρ_g	Persistence Government Spending Shock	0.91	0.94	0.93	0.92	0.94	0.92	0.94	0.93
ρ_q	Persistence Investment Specific Shock	0.60	0.59	0.63	0.58	0.69	0.56	0.56	0.61
ρ_R	Persistence Monetary Policy Shock	0.29	0.14	0.29	0.27	0.24	0.21	0.09	0.16
ρ_p	Persistence Price Markup Shock	0.88	0.96	0.90	0.90	0.92	0.88	0.99	0.91
ρ_w	Persistence Wage Markup Shock	0.94	0.96	0.92	0.95	0.95	0.96	0.97	0.97
θ_p	MA Coefficient Price Markup Shock	0.75	0.88	0.81	0.81	0.85	0.81	0.93	0.81
θ_w	MA Coefficient Wage Markup Shock	0.82	0.92	0.84	0.84	0.88	0.88	0.94	0.91
S'	Investment Adjustment Cost	6.18	9.99	6.78	6.64	7.46	9.39	10.86	9.24
σ_C	Intertemporal Substitution Elasticity	1.03	1.27	1.19	0.97	1.16	1.17	1.20	1.15
h	Habit Formation	0.81	0.84	0.79	0.81	0.82	0.86	0.85	0.84
ξ_w	Probability of Resetting Wage	0.84	0.86	0.82	0.82	0.81	0.83	0.86	0.85
σ_L	Labour Supply Elasticity	2.42	2.15	2.05	2.84	1.78	3.01	1.58	2.53
ξ_p	Probability of Resetting Price	0.79	0.70	0.80	0.77	0.77	0.81	0.70	0.77
ι_w	Wage Indexation	0.43	0.31	0.46	0.43	0.52	0.37	0.29	0.40
ι_p	Price Indexation	0.28	0.10	0.17	0.28	0.19	0.13	0.04	0.13
ψ	Utilisation Adjustment Cost	0.22	0.21	0.19	0.30	0.23	0.23	0.20	0.16
Φ	Production Fixed Cost	1.78	2.12	1.88	1.82	2.03	2.13	1.78	1.82
γ_π	Inflation Policy Reaction	1.74	1.88	1.88	1.88	1.86	1.63	1.86	1.99
γ_R	Interest Policy Smoothing	0.81	0.85	0.83	0.82	0.83	0.86	0.84	0.85
γ_y	Output Gap Policy Reaction	0.11	0.08	0.10	0.12	0.10	0.09	0.09	0.10
$\gamma_{\Delta y}$	Output Gap Growth Policy Reaction	0.21	0.17	0.15	0.21	0.18	0.13	0.17	0.17
$\bar{\pi}$	Steady State Inflation	0.77	1.13	0.95	0.76	0.89	0.89	0.99	0.90
$100\left(\beta^{-1}-1\right)$	Time Discount	0.40	0.12	0.17	0.45	0.19	0.17	0.06	0.16
$\bar{L}$	Steady State Hours	1.00	0.55	0.58	1.36	0.76	1.07	0.33	0.84
$\bar{\gamma}$	Productivity Growth	0.46	0.45	0.46	0.46	0.45	0.45	0.48	0.45
ρ_{ga}	Government Spending Productivity Correlation	0.05	0.17	0.07	0.05	0.08	0.07	0.52	0.09
α	Capital Production Share	0.22	0.21	0.18	0.22	0.18	0.19	0.20	0.18

3.3 Forecasts

We then carry out three forecasting exercises. In the first, the model is estimated over 1954Q3-1997Q4 and forecasts are produced over 1998Q1-2007Q4. For the second, we consider a period that includes the financial crisis and so estimate over 1954Q3-2000Q4; forecasts are produced over 2001Q1-2010Q2. Finally, we consider a recursive rolling-sample forecasting exercise where we first produce forecasts at 1998Q1 having estimated the model over 1954Q3-1997Q4, and then sequentially advance the estimation in one-period steps and produce successive forecasts to 2010Q2.

We use RMSFE and two-sided Diebold-Mariano tests (Diebold and Mariano, 1995) with p-value $= 0.05$ to evaluate forecasting performance compared to a standard DSGE forecast as a benchmark. The charts report performance for each of the seven variables relative to the benchmark where the multi-step horizon h is in the range 2 to 8. The criteria are unity for the RMSFE charts (so that numbers below one favour the multi-step method) and the critical values from the DM tests (so that outcomes below the lower negative value reject equality in favour of the multi-step method and that above the positive value in favour of the benchmark). These are evaluated at forecast horizons $h = 1$, 2, 4, 8 and 12.

For the pre-crisis period (Figures 1 and 2) the multi-step forecast outperforms the benchmark in most cases, even for forecast horizon $h = 1$, and in many cases by large margins. Moreover, and unusually for empirical contests such as this, a high proportion of the outcomes are significant. For the period including the crisis (Figures 3 and 4), the results are more mixed and there are fewer significant outcomes, but the multi-step method remains the best performer. For the rolling forecast (Figures 5 and 6), the multi-step forecast again tends to outperform the benchmark, but significantly so in fewer cases. It might be hypothesised that performance at horizon h would be best if the same horizon h were used when optimising, but in fact this is not the case.

These improved results raise the question of why the method outperforms the standard approach. The immediate answer is that as discussed above we have effectively introduced a moment which introduces a multi-step cross-validation element. But from an accounting point of view, the parameter estimates show no special regularities that might help us.[4]

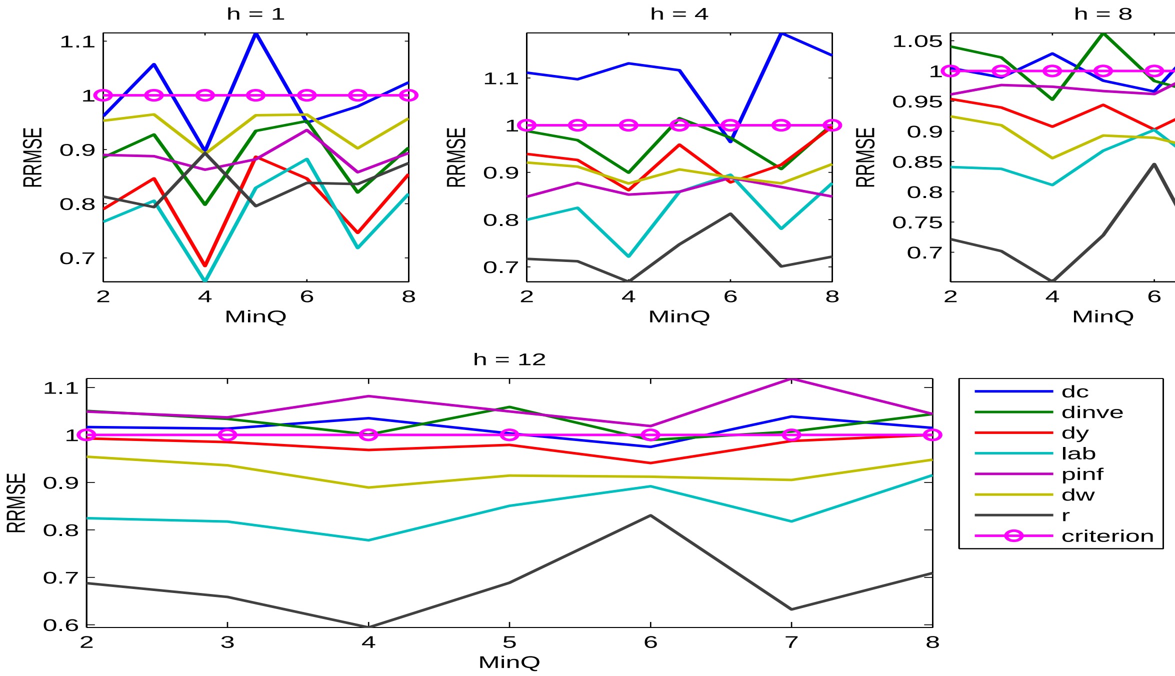
The other way in which performance may improve is from an improved estimate of the state. Intuitively, the quality of the Kalman smoother estimates of the current state increase as the forecast horizon increases as this is associated with a larger number of cross-equation restrictions from the model. A simple Monte Carlo experiment supports this intuition. We carry out the following exercise:

- 200 data points are simulated from the true data generation process.
- Some noise is added to the observed variables (we examine noise to signal ratios of 0.5, 1.0 and 2.0).
- We obtain an estimate of the state vector of the economy *via* the Kalman smoother for one, four and eight steps ahead.
- The Mean Square Forecast Error (between the actual state observations and the smoothed estimates) is calculated for the last 50 periods.
- The exercise is repeated 1000 times.

Table 3 supports our intuition. As the forecast horizon used for estimation increases the filter obtains increasingly and substantially better estimates of the unobserved state of the economy by exploiting the cross-equation restrictions implied by the model. Furthermore, as the noise to signal ratio increases these restrictions become even more important. Finally, if the evaluation is carried out for the entire sample then our procedure delivers

[4] Although relative to Smets and Wouters' method with informative priors, our results using no prior information have larger parameters related to aspects of persistence, which may aid forecasting. But as we have observed that does not systematically rise with the horizon.

Figure 1: Relative RMSFE over period 1998Q1-2007Q4



The figures report the relative RMSFE compared to a standard DSGE forecast at horizons h= 1, 4, 8 and 12 for the variables consumption dc, investment $dinve$, output dy, labour supply (log hours) lab, inflation $pinf$, wages dw and the federal fund rate r where d indicates a log difference. The *criterion* lines indicate the upper or lower 95% confidence bounds of the two-sided DM test statistic. The horizontal axis shows the value of the prediction-error horizon in the range 2 to 8 used in the minimand $MinQ$.

an enormous improvement. This is because the cross-equation restrictions helps the filter to estimate the state at the start of the sample with more precision.

Table 3: Nowcasting Evaluation

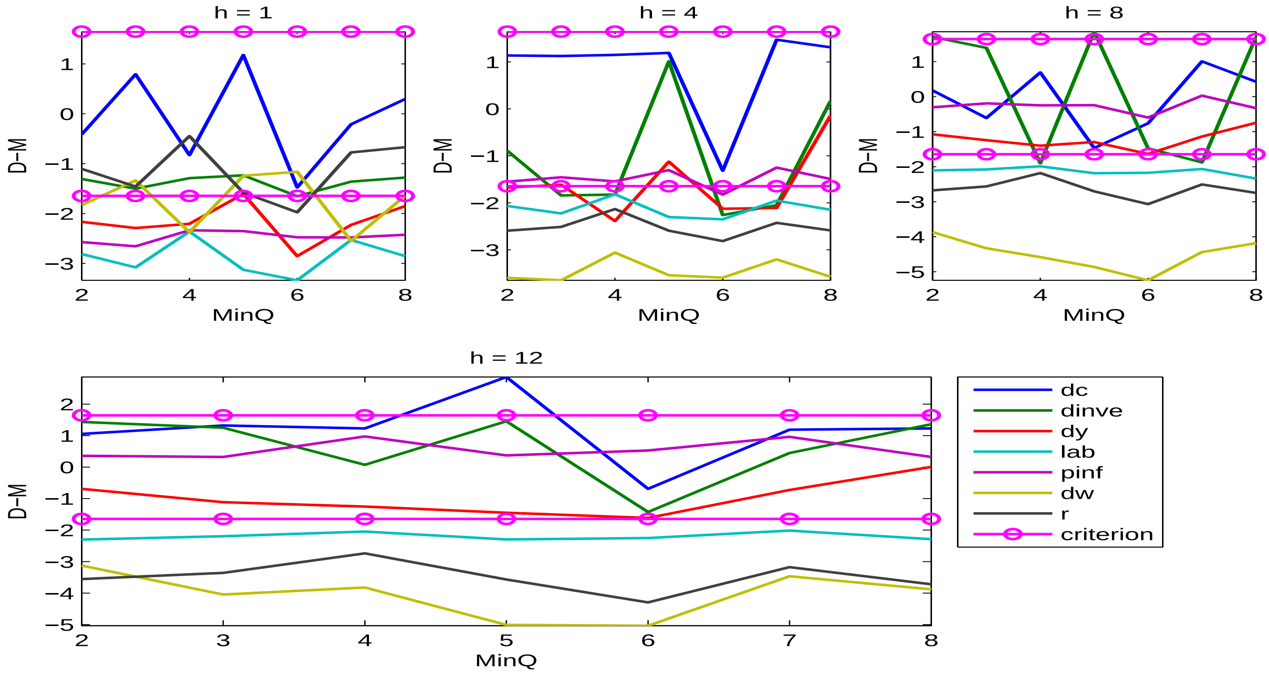
Noise to Signal Ratio	Relative Mean Square Forecast Error	
	$\frac{MSFR(h=4)}{MSFR(h=1)}$	$\frac{MSFR(h=8)}{MSFR(h=1)}$
0.5	0.991	0.975
1.0	0.987	0.968
2.0	0.932	0.894

If we expand the evaluation periods from 50 to 200 periods the performance of our procedure increases dramatically. Again, this is because these additional cross-equation restrictions are particularly useful with regard to estimation of the initial observations of the state vector.

4 Conclusions

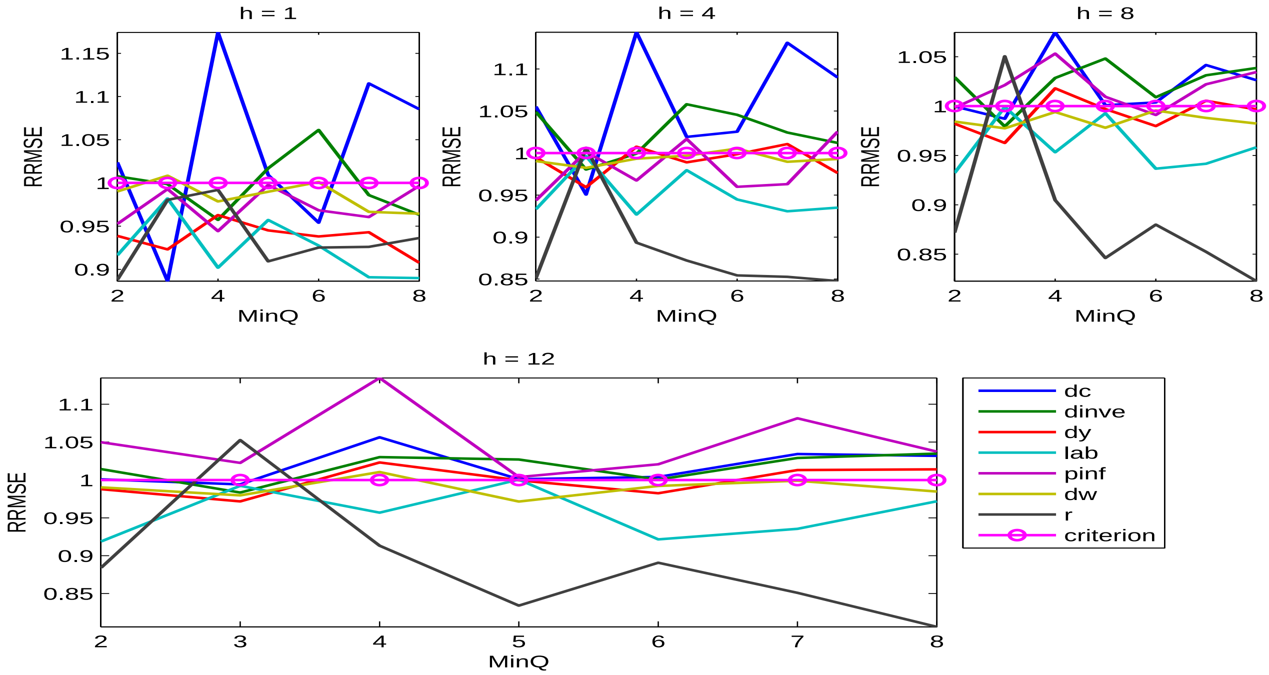
Evidence is mounting that DSGE models, valued for their structural interpretation, may also be useful for forecasting. But forecast performance at policy-relevant horizons is not incorporated in estimation, except to the extent that the one-step ahead forecast error is typically used to build the likelihood. A natural exercise is therefore to use multi-step prediction errors in estimation. This is applied to the standard Smets and Wouters (2007) model of the US economy. Bayesian computational methods are applied, but without using prior information, so the results may be seen as classical in spirit, akin to a method of moments estimator. The parameters are largely similar to those reported in Smets and Wouters (2007), despite being applied without informative priors and over a different sample. Over both the pre-crisis and post-crisis periods and when

Figure 2: Diebold-Mariano test statistics over period 1998Q1-2007Q4



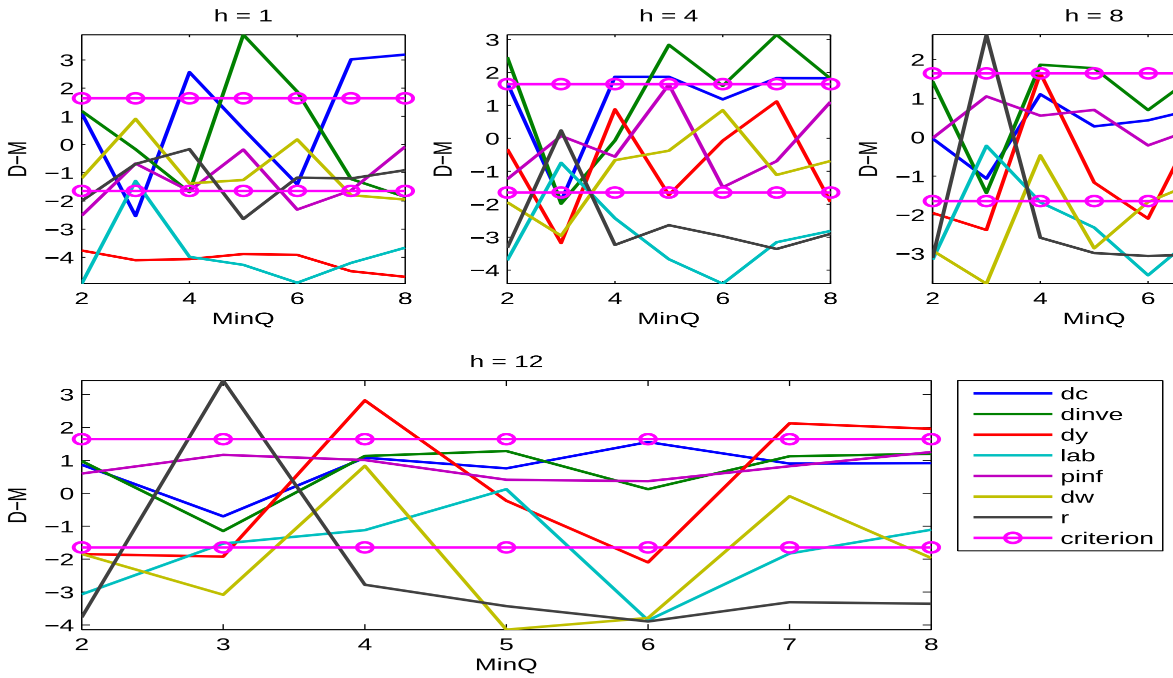
Notes as for Figure 1.

Figure 3: Relative RMSFE over period 2001Q1-2010Q2



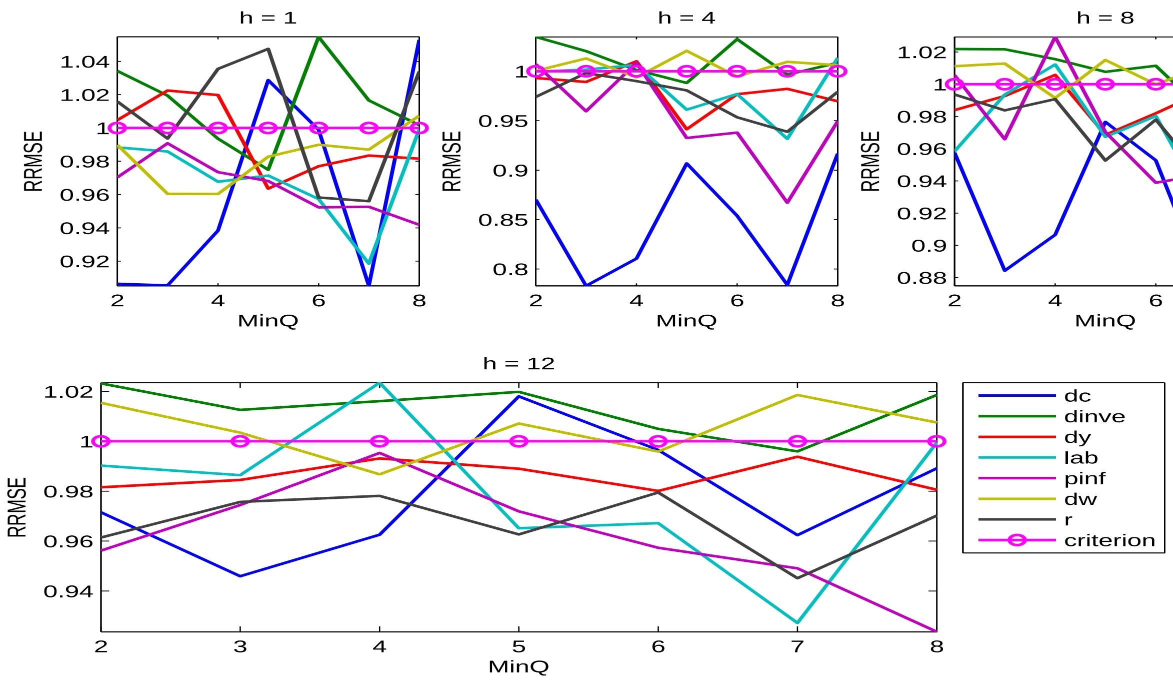
Notes as for Figure 1.

Figure 4: Diebold-Mariano test statistics over period 2001Q1-2010Q2



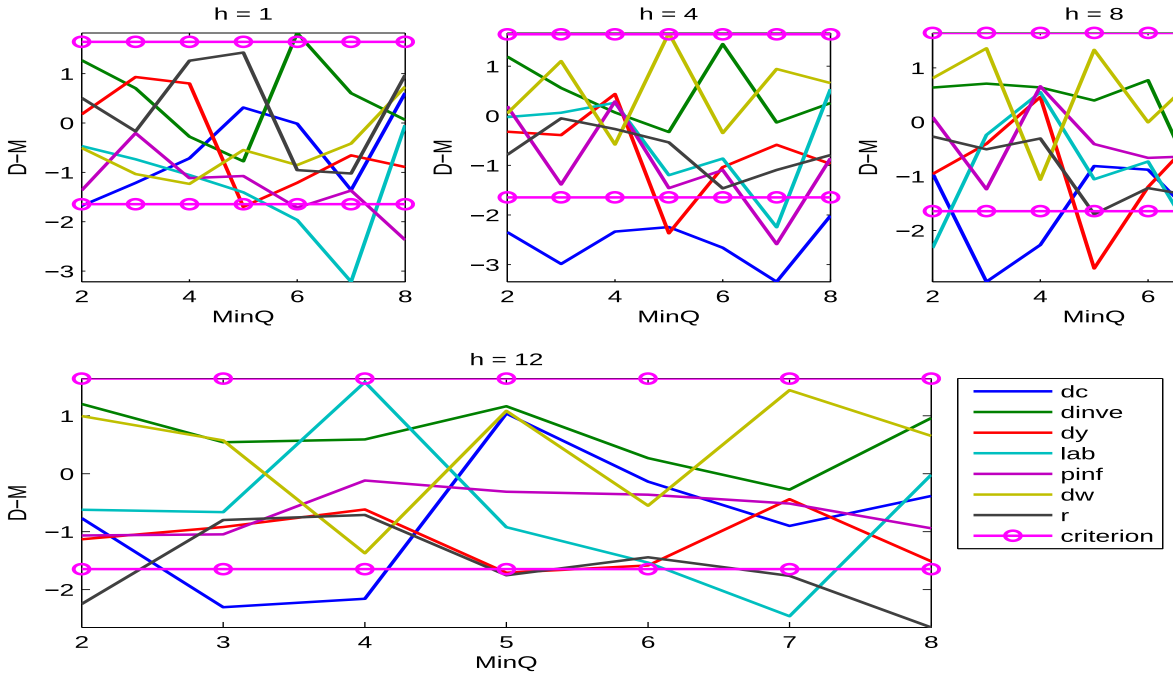
Notes as for Figure 1.

Figure 5: Relative RMSFE for rolling forecasts over period 1998Q1-2010Q2



Notes as for Figure 1.

Figure 6: Diebold-Mariano test statistics for rolling forecasts over period 1998Q1-2010Q2



Notes as for Figure 1.

implemented in recursive mode, the multi-step approach at horizons up to $h = 8$ in the majority of cases improve RMSFE for most variables relative to the standard method ($h = 1$), and in many cases significantly so. Only rarely does the standard one-step approach outperform the new approach.

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