Do Recessions Slow Technology Growth? Evidence From the Firm Level

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The opinions expressed are those of the authors and do not necessarily reflect the views of the Bank of Finland, Deutsche Bundesbank or the Eurosystem.

Motivation

- **Well-established insights from endogenous growth theory** (Romer (1990), Aghion and Howitt (1992), Grossman and Helpman (1991))
  1. Driver of long-run growth = technology growth.
  2. The main determinant of technology growth is investment in innovation.

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  2. The main determinant of technology growth is investment in innovation.
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  - Abstract from modeling technology dynamics endogenously in general equilibrium.
  - $A_t = f(\rho, \epsilon)$, with $\epsilon \sim i.i.d\ N(0, \sigma)$.

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  - $A_t = f(\rho, \epsilon)$, with $\epsilon \sim i.i.d\ N(0, \sigma)$.
- Exogenous technology short-cut implies **substantial assumptions**:
  - Cyclical fluctuations $\nrightarrow$ innovation, technology and TFP.
  - Business cycles = short-term phenomenon, strict dichotomy between cycle and trend.

This paper

Key question: Do firms cut their investment in innovation in a recession?

Insights from previous literature:

- Procyclicality of aggregate innovation and TFP (Barlevy (2007), Fatas (2000)).
- Persistent effects of recessions through drop in technology-enhancing investment
  - Medium-term business cycles (Comin and Gertler (2006))
  - Contractionary demand shocks, innovation and TFP (Jorda et al. (2023), Anzoaetgui et al. (2019))

Mechanism: contraction $\rightarrow$ innovation investment$\downarrow$ $\rightarrow$ technology growth$\downarrow$

- Tentative evidence (aggregated data, theory)
- Challenges: identification, data availability

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What we do: Firm-level evidence on the innovation investment patterns in a crisis (novel, granular data set) + persistent effects of short-run shocks (theoretical analysis).

Results

1. **Elasticity between crisis impact and innovation investment cuts:**
   - Firms which were adversely hit by crisis are significantly more likely to decrease investment in R&D and diffusion.
   - 1% cyclical output drop $\rightarrow$ 0.27% drop in R&D and 0.3% in diffusion investment.

2. **Firm-level innovation adjustment patterns:**
   - Extensive margin: drop in innovation expenditures relative to pre-crisis plans: 25% (R&D) and 20% (diffusion) of firms.
   - Intensive margin: -65% (R&D) and -70% (diffusion) of pre-crisis plans.
   - Large, economically substantial cuts: R&D: 750,000€, diffusion: 954,000€.

3. **Identification of underlying driving shocks:**
   - Role for short-run demand fluctuations for innovation and aggregate supply over at least the medium term.
   - If firms expect problems with demand $\rightarrow$, probability to cut innovation$\uparrow$ by 10pp.
   - Role of financial frictions (amplification; estimates as a lower bound).

Previous literature

Procyclicality of innovation investment: Fatás (2000); Comin and Gertler (2006); Barlevy (2007); Anzoategui et al. (2019).

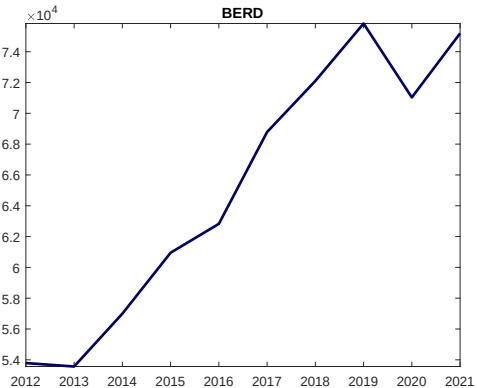
Empirical evidence on long-run effects in TFP through innovation: Ma and Zimmermann (2023), Jordà et al. (2022), Moran and Queralto (2018), Cloyne et al. (2022), Antolin-Diaz and Surico (2022), Ilzetzki (2022).

Models on cycle-trend interaction through hysteresis in TFP: Benigno and Fornaro (2018), Anzoategui et al. (2019), Bianchi et al. (2019), Moran and Queralto (2018), Garga and Singh (2020), Elfsbacka-Schmöller and Spitzer (2021), Fornaro and Wolf (2023).

Micro evidence on long-run effects from financial constraints: Huber (2018); Duval et al. (2020).

Macroeconomic dynamics: Business R&D

Real GDP (Germany, source: FRED)



Business R&D (Germany, source: FRED)

Data

Large, representative sample of firms across sectors and size categories

- Bundesbank Online Panel of Firms: representative monthly survey of firms in Germany
- Innovation module: 5500 firms, 2021Q3
- Full distribution of firms (size, sectors)

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Granular and unique joint firm-level information on:

- Frontier innovation (R&D) and non-frontier innovation (diffusion)
- Identification of crisis-induced adjustment: realized vs. pre-crisis plan
- Reasons for adjustment
- Crisis-induced drop in production /business activity
- Detailed further firm characteristics (general; financing and frictions)
- Firms' expectations (firm-specific, macroeconomic)

Estimation results: crisis exposure and innovation cuts

	Probability to decrease: R&D				Probability to decrease: diffusion			
	1	2	3	4	5	6	7	8
Crisis-induced production/ activity drop (0-1)	0.116*** (0.018)	0.092*** (0.019)			0.085*** (0.018)	0.071*** (0.018)		
Crisis-induced production/ activity drop (pct.)			0.002*** (0.000)	0.001*** (0.000)			0.001*** (0.000)	0.001*** 0.000)
Covariates	No	Yes	No	Yes	No	Yes	No	Yes
Observations	1317	1309	1186	1178	1295	1287	1163	1155

Probability to decrease: R&D and technological diffusion. Marginal effects after Heckmann probit. Exclusion criteria is having planned investment in respectively R&D or TD. Report on investments decisions of the firms is collected in the 2021, July-September. Information on recession impact and expectations about next 6 months are collected in June-July 2020.

Estimation results: elasticities

	R&D investment cuts (pct.)				Diffusion investment cuts (pct.)			
	1	2	3	4	5	6	7	8
Crisis-induced production/ activity drop (0-1)	15.745*** (5.310)	15.256*** (4.994)			11.852** (5.146)	10.639** (5.025)		
Crisis-induced production/ activity drop (pct.)			0.271*** (0.078)	0.275*** (0.077)			0.309*** (0.073)	0.338*** (0.076)
Covariates	No	Yes	No	Yes	No	Yes	No	Yes
Observations	166	166	157	157	153	153	146	146

Elasticities: R&D and diffusion.

Adjustment patterns of investment in technology

	(1) Planned R&D mean	(2) Did not plan R&D mean
No change, R&D	0.693	0.991
Decreased, R&D	0.245	.
Increased, R&D	0.062	0.009
Observations	2629	2182

Adjustment shares (R&D)

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Observations	2629	2182

Adjustment shares (R&D)

	(1) Planned TD mean	(2) Did not plan TD mean
No change, TD	0.763	0.990
Decreased, TD	0.191	.
Increased, TD	0.046	0.010
Observations	2934	1846

Adjustment shares (TD)

Core innovators

Reasons for non-adjustment

Conditional on having plans to invest in R&D or TA. Source: BOP-F, Waves 6-8; trimmed data; own calculations.

- 46% did not experience a sufficiently strong change in own economic conditions which would have necessitated adjustment.
- Sufficient financial resources (despite changed situation at the firm-level) prevented further adjustment (33%).
- For a small fraction of firms adjustment was not feasible ("sticky" investment).
- Episode as a lower bound for response during a crisis; importance of fiscal and monetary support.

Categories

Plans and adjustment in amounts

	p10	p50	p90	mean	count
R&D investment: '000 planned	5	50	1200	1952	2629
Decrease R&D, '000 euro	-700	-30	-5	-750	644
Increase R&D, '000 euro	5	33	338	179	162
Change in R&D, '000 euro	-50	0	0	-173	2629

Investment in R&D, '000 euro

	p10	p50	p90	mean	count
TA investment: '000 planned	5	40	1000	2049	2932
Decrease TD, '000 euro	-650	-30	-4	-954	559
Increase TD, '000 euro	5	20	225	144	135
Change in TD, '000 euro	-25	0	0	-175	2932

Investment in TD, '000 euro

R&D (dec.)

TD (dec.)

- Economically substantial downward adjustment
- Increases negligible both in shares ($\sim 5\%$) and magnitude
- Average adjustment in R&D -9% (aggregate decline: -6.3% BERD vs. 4% pre-crisis growth)

Downward adjustment relative to plans

	p10	p50	p75	p90	mean	count
R&D investment: '000 planned	5	50	200	1200	1952	2629
Decrease R&D, % planned amounts	25	67	93	100	65	644

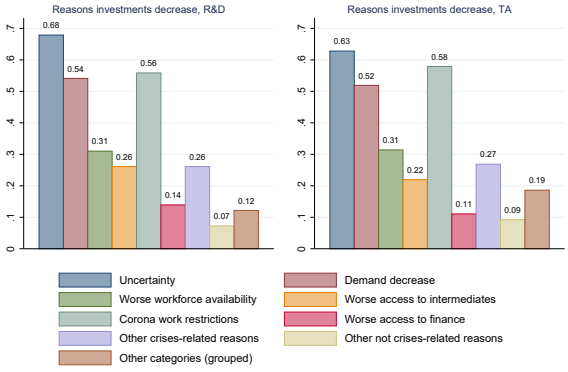
Plans and downward adjustment in R&D, conditional on having plans, by innovator type

	p10	p50	p75	p90	mean	count
TD investment: '000 euro planned	5	40	200	1000	2049	2932
Decrease TD, % planned amounts	25	71	100	100	69	559

Plans and downward adjustment in TA, conditional on having plans, by innovator type

Adj. (dec.)

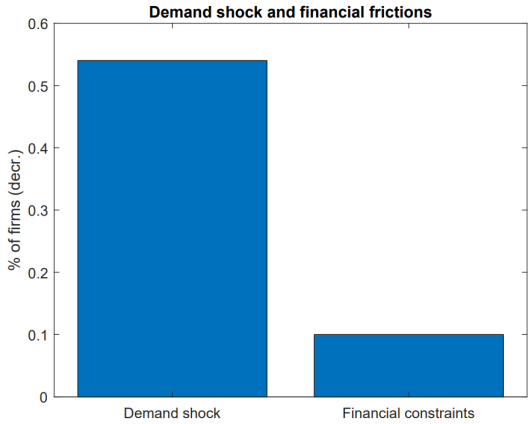
Reasons behind technology-enhancing investment cuts



- R&D and diffusion driven by similar shocks (Comin and Gertler (2006)) mechanism
- Key shocks: uncertainty and <u>demand</u>
- Supply-chain disruptions $\rightarrow$ innovation investment$\downarrow$ (Fornaro and Wolf (2023))
- COVID policy restrictions contributed *negatively* $\nrightarrow$ "innovating out of pandemic"
- Non-binding financial frictions (policy support; non-financial shock)

Demand shock and financial frictions

- 2008/09: procyclical slowdown in TFP growth $\rightarrow$ relative role of financial shock + frictions and the role of weak demand?
- We show: demand shocks can slow innovation investment and thus long-term aggregate supply even without financial frictions.
- Amplification in the absence of large-scale policy support.
- Amplification under simultaneous demand shock and binding financial constraints (higher share of decreasers; model).



Demand-supply

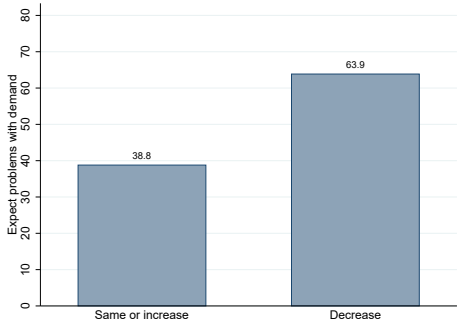
The role of expectations: demand and financial constraints

	Probability to decrease investment			
	R&D		Diffusion	
Expect demand problems	0.101***	0.075***	0.076***	0.058***
	(0.019)	(0.020)	(0.019)	(0.020)
Expect financing problems	0.059**	0.056**	0.052*	0.060**
	(0.027)	(0.027)	(0.027)	(0.027)
Expect problems due to covid restrictions	-0.006	0.007	0.020	0.028
	(0.027)	(0.027)	(0.020)	(0.019)
Covariates	No	Yes	No	Yes
N observations	1300	1293	1278	1271

Decreased investment in R&D, effect of crisis-induced production drop and expectations

Appendix

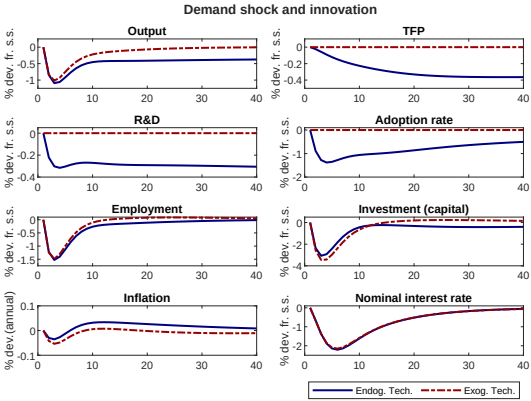
Expecations about demand



Model

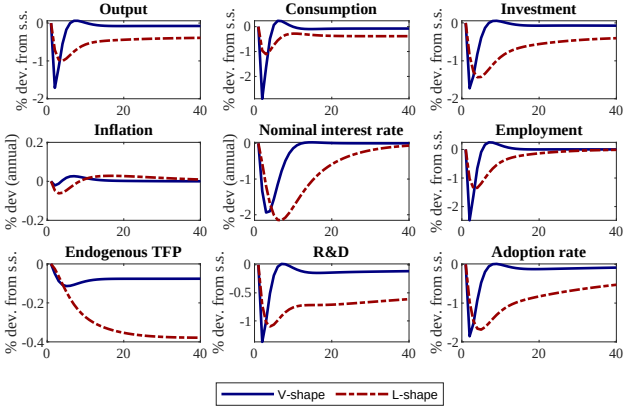
- New Keynesian DSGE model with endogenous investment in innovation and technology growth
- Endogenous TFP growth: horizontal innovation through expanding varieties in intermediate goods (Romer (1990))
- Two-stage technology growth process (Comin and Gertler (2006, AER)):
  1. R&D sector: technological frontier
  2. Endogenous diffusion: costly technology adoption
- Medium-scale DSGE model setup (Christiano et al. (2005); Smets and Wouters (2007))
  - Calvo price and wage rigidities
  - Investment adjustment costs
  - Monetary policy rule

Demand shock, innovation and aggregate supply



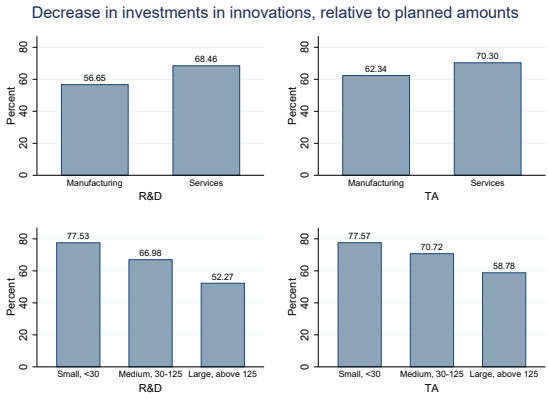
Macroeconomic dynamics under a contractionary demand shock

Short-lived vs. persistent crises



Magnitude of slowdown in technology growth (V-shape vs. L-shape)

Adjustment by firm size and sector



Downward adjustment by sectors and firm size

Notes: Conditional on having plans to invest in R&D or TA; source: FDSZ der Deutschen Bundesbank, BOP-F, Waves 6-8; own calculations.

- Relatively similar relative decrease in services vs. manufacturing
  - Role of relative output drop
  - Larger plans in M for R&D, difference less pronounced in TD
- Downward adjustment more pronounced in small firms vs. large firms
  - Role of financial constraints
  - Larger firms with larger plans

Sect. adj.

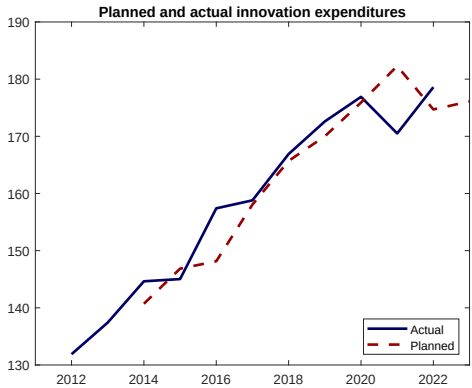
Distr.

Joint adjustment patterns in R&D and technology diffusion

	(1) Increased TD	(2) Decreased TD	(3) No change TD
Increased R&D	1.56	1.19	3.25
Decreased R&D	1.19	14.68	9.52
No change R&D	3.93	2.10	60.98

Source: BOP-F, Waves 6-8; trimmed data; own calculations; conditional on having plans to invest in both R&D and T&A.

No systemic downward revisions outside crisis episode



Pre-crisis trends in planned (red line) vs. actually realized innovation expenditures (blue line) in Germany; source: Mannheim Innovation Panel (ZEW); units: bn. euros.

Additional results

- Data: Innovation investment and TFP trajectory in subsequent periods.
- Quantitative analysis:
  - Effect of policy support and macroeconomic stabilization policies.
  - The role of pressimistic expectations and their revisions.

Conclusion

- **Micro-level evidence shows that firms cut investment in technology in a recession**
  - Fall in both frontier and non-frontier innovation investment → slowdown in technology growth
  - Economically substantial cuts (65%/70% of pre-crisis plans)
- Firm-level evidence of spillovers **from short-run fluctuations and demand shifts to innovation and at least medium aggregate supply**
- **Implications for macroeconomic modeling and policy**
  - Stabilization policies and persistent effects of recessions
  - Strict division of cycle and trend and related concepts: potential output and gap measures

Additional slides

Questionnaire: Reasons for non-adjustment

***Question**: You stated that your firm did not adjust its plans regarding expenditure on R&D or other innovation activities in 2020. Which of the following reasons were the most important?*

- We would have reduced investment in innovation, but were not able to make adjustments.
- We would have increased investment in innovation, but were not able to make adjustments.
- Overall, the situation for my firm did not change significantly in 2020.
- We had sufficient financial resources.
- Other reasons.

Non-adjustment

Questionnaire: Reasons for change

***Question**: Which of the following changes linked to the coronavirus pandemic induced an adjustment of your plans regarding expenditure for R&D activities and other innovation activities (excluding R&D) in 2020?*

Firms select separately for R&D and other innovation activities.

- More uncertain economic outlook
- Lower/ higher customer demand for existing products and services
- Worse/ better access to intermediate inputs
- Worse/ better availability of suitable specialist staff
- Worse/ better access to financing sources
- Closures or work restrictions due to the coronavirus pandemic (hygiene rules, lockdown etc.)
- Other reasons linked to the coronavirus pandemic
- Reasons not linked to the coronavirus pandemic

Questionnaire: attachment to R&D

Question: *The previous questions referred to research and development (R&D) specifically in 2020. What is the situation more generally, does your firm invest in research and development (R&D)?*

- Yes, continuously with a specific R&D budget.
- Yes, continuously without a specific R&D budget.
- Yes, occasionally.
- No.

Questionnaire: change in business activity

Question: *How has your production/business activity changed as a result of the coronavirus pandemic?*

1 = decreased, 2= stayed the same, 3= increased

Question:
Your production/business activity has decreased as a result of the coronavirus pandemic. How large was the decrease in your production/ business activity as a result of the coronavirus pandemic in the month of May compared with a "normal" situation, e.g. in May 2019?

Estimation

Adjustment patterns of investment in technology: shares (core innovators)

	Planned R&D		No R&D planned	
	core	non-core	core	non-core
	(1)	(2)	(3)	(4)
No change, R&D	0.664	0.729	0.946	0.994
Increased, R&D	0.077	0.043	.	0.006
Decreased, R&D	0.259	0.228	.	.
Observations	1455	1171	148	2028

Table: Adjustment (shares) in R&D

	Planned TA		No TA planned	
	core	non-core	core	non-core
	(1)	(2)	(3)	(4)
No change, TA	0.732	0.787	0.985	1.000
Increased, TA	0.054	0.040	.	.
Decreased, TA	0.214	0.173	.	.
Observations	1296	1634	259	1582

Table: Adjustment (shares) in TA

Source: BOP-F, Waves 6-8; trimmed data; own calculations.

Main

Firms by investment behavior in R&D - weighted

	(1) Invest in RD continuously mean	(2) Do not invest in RD continuously mean
Invest continuously with budget	0.224	
Invest continuously w/o budget	0.776	
Invest occasionally		0.319
Do not invest typically		0.681
Observations	1817	3671

Source: BOP-F, Waves 6-8; trimmed data; own calculations.

Firms by investment behavior in R&D

	(1) Invest in R&D continuously mean	(2) Invest in R&D occasionally mean
Invest continuously with budget	0.286	
Invest continuously w/o budget	0.714	
Invest occasionally		0.358
Do not invest typically		0.642
Observations	1818	3672

Source: BOP-F, Waves 6-8; trimmed data; own calculations.

Adjustment in amounts: R&D (decomposition)

	(1) All					(2) Core innovators				
	p10	p50	p90	mean	count	p10	p50	p90	mean	count
R&D investments: '000 planned	5	50	1200	1952	2629	10	100	3000	3083	1455
Decrease R&D, '000 euro	-700	-30	-5	-750	644	-1000	-50	-7	-966	377
Increase R&D, '000 euro	5	33	338	179	162	5	50	499	174	112
Change in R&D, '000 euro	-50	0	0	-173	2629	-100	0	0	-237	1455

Investment in R&D, conditional on having plans, by innovator type, '000 euro

Amounts

Adjustment in amounts: diffusion (decomposition)

	(1) All					(2) Core innovators				
	p10	p50	p90	mean	count	p10	p50	p90	mean	count
TA investments: '000 planned	5	40	1000	2049	2932	10	80	2000	2581	1295
Decrease TA, '000 euro	-650	-30	-4	-954	559	-1000	-50	-5	-1687	276
Increase TA, '000 euro	5	20	225	144	135	5	50	390	199	70
Change in TA, '000 euro	-25	0	0	-175	2932	-50	0	0	-349	1295

Investment in TD, conditional on having plans, by innovator type, '000 euro

Amounts

Downward adjustment relative to plans (decomposition)

	(1) All						(2) Core innovators					
	p10	p50	p75	p90	mean	count	p10	p50	p75	p90	mean	count
R&D investments: '000 planned	5	50	200	1200	1952	2629	10	100	500	3000	3083	1455
Decrease R&D, % planned amounts	25	67	93	100	65	644	20	56	80	100	57	377

Table: **Plans and downward adjustment in R&D, conditional on having plans, by innovator type**

	(1) All						(2) Core innovators					
	p10	p50	p75	p90	mean	count	p10	p50	p75	p90	mean	count
TA investments: '000 euro planned	5	40	200	1000	2049	2932	10	80	300	2000	2581	1295
Decrease TA, % planned amounts	25	71	100	100	69	559	20	67	90	100	63	276

Table: **Plans and downward adjustment in TA, conditional on having plans, by innovator type**

Rel.adjustment

Firms by investment behavior in R&D

	(1) Invest in R&D continuously mean	(2) Invest in R&D occasionally mean
Invest continuously with budget	0.286	
Invest continuously w/o budget	0.714	
Invest occasionally		0.358
Do not invest typically		0.642
Observations	1818	3672

Table: Change of Plans to invest in R&D, BOP-F

Trimmed data; source: Forschungsdaten- und Servicezentrum (FDSZ) der Deutschen Bundesbank, BOP-F, Waves 6-8; trimmed data; own calculations.

Change in business activity in the data

Average production drop:

- **Decrease:** 57% of firms, average decrease: 38%.
- **Approx. unchanged:** 33% of firms.
- **Increase:** 10% of firms, average decrease: 17%.

Change of Plans to invest

	(1) Planned RD only	(2) Planned TA only	(3) Planned RD and TA	(4) Didnt plan
	mean	mean	mean	mean
No change, RD	0.737	0.986	0.681	0.993
No change, TA	0.984	0.799	0.749	0.992
No change, TA and RD	0.728	0.791	0.620	0.986
Increased, RD	0.079	0.014	0.061	0.007
Increased, TA	0.016	0.039	0.049	0.008
Decreased, RD	0.184	.	0.258	.
Decreased, TA	.	0.162	0.202	.
Observations	380	700	2164	1463

Table: Change of Plans to invest, BOP-F

Source: Forschungsdaten- und Servicezentrum (FDSZ) der Deutschen Bundesbank, BOP-F, Waves 6-8, own calculations; trimmed data.

Reasons for increase

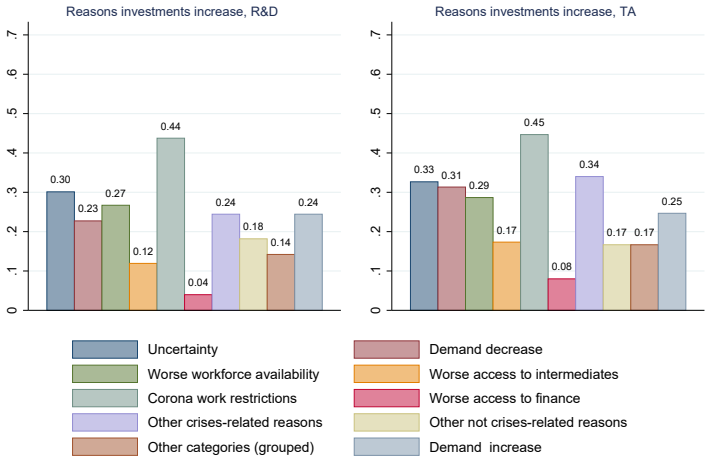
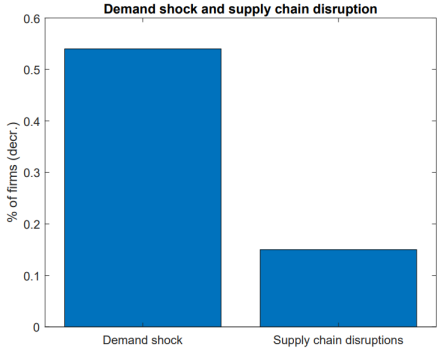


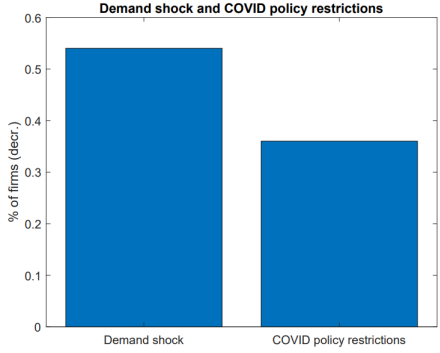
Figure: Reasons or firms increasing investments in R&D and TD

Demand shock and supply chain disruptions



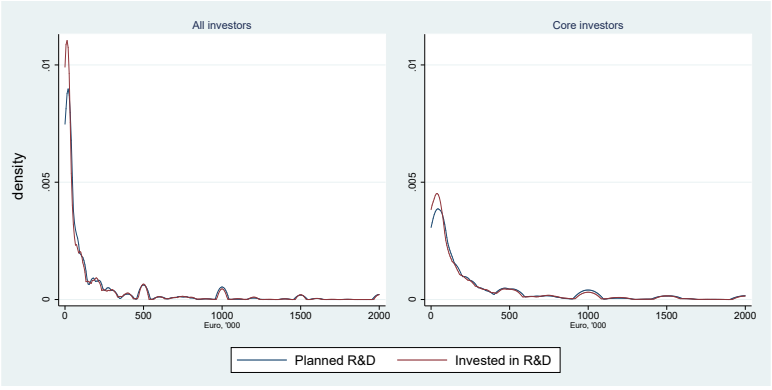
Demand-finance

Demand shock and COVID policy restrictions



Demand-finance

R&D: DISTRIBUTION OF PLANS AND REALIZATION

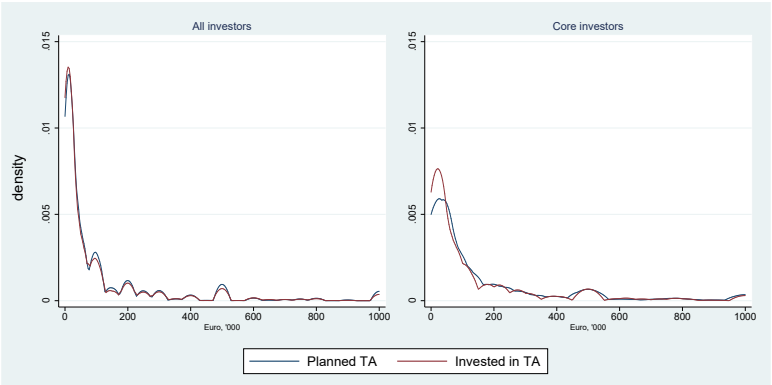


Notes: Conditional on having plans to invest in R&D

Source: Forschungsdaten- und Servicezentrum (FDSZ) der Deutschen Bundesbank, BOP-F, Waves 6-8, own calculations.

Sectors, size

Diffusion : distribution of plans and realization



Notes: Conditional on having plans to invest in TD.

Source: Forschungsdaten- und Servicezentrum (FDSZ) der Deutschen Bundesbank, BOP-F, Waves 6-8, own calculations.

R&D sector: technological frontier

- Growth through expanding varieties
- Innovators invest in R&D to invent new intermediate goods
- Law of motion of technological frontier: $Z_{t+1} = \phi Z_t + \varphi_t X_t$
- Innovator i's production function: $V_t^i = \varphi_t X_t^i = \frac{\chi Z_t}{Z_t^\zeta X_t^{1-\zeta}} X_t^i$
- Positive spillover from aggregate innovation stock Z_t, externality from aggregate R&D efforts ($\frac{1}{Z_t^\zeta X_t^{1-\zeta}}$, where $0 < \zeta < 1$)
- Aggregate R&D: $X_t = \int_i X_t^i di$

Entrepreneurs' problem

Innovator i chooses R&D investment X_t^i to maximize:

$$\max_{\{X_{t+j}^i\}_{j=0}^{\infty}} \mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \left[\Lambda_{t,t+1+j} J_{t+1+j} \varphi_{t+j} X_{t+j}^i - (1 + f^x) X_{t+j}^i \right] \right\}$$

Optimality condition for R&D:

$$\mathbb{E}_t \left(\Lambda_{t,t+1} J_{t+1} \varphi_t \right) = \Delta f^x$$

Aggregate new technologies:

$$V_t = \int_i V_t^i di = \chi Z_t^{1-\zeta} X_t^{\zeta}$$

Technology adoption sector

- Adopters buy right to use unadopted technology from innovators at competitive price J_t
- Technologies are rendered usable in production using equipment E_t^i
- Probability of successful adoption ($\kappa_\lambda > 0$, $0 < \eta < 1$, $0 < \rho_\lambda < 1$)

$$\lambda_t\left(E_t^i\right) = \kappa_\lambda \left(\frac{X_t}{A_t}\right)^\eta \left(E_t^i\right)^{\rho_\lambda}$$

- Successfully adopted technology is sold at price H_t $H_t = \Pi_t + \phi \mathbb{E}_t\left(\Lambda_{t,t+1} H_{t+1}\right)$

Adopters' problem

- Adopters weigh adoption costs against the expected gains from technology adoption:

$$J_t = \max_{E_t^i} -Q_t^a E_t^i + \phi \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left[\lambda_t H_{t+1} + (1 - \lambda_t) J_{t+1} \right] \right\}$$

- Optimality condition for adoption:

$$\rho_\lambda \kappa_\lambda \phi \left(\frac{X_t}{A_t} \right)^\eta \mathbb{E}_t \left[\Lambda_{t,t+1} \left(H_{t+1} - J_{t+1} \right) \right] = Q_t^a E_t^{1-\rho_\lambda}$$

- Law of motion for adopted technologies:

$$A_{t+1} = \phi A_t + \phi \left[\lambda_t \left(Z_t - A_t \right) \right]$$

Intermediate goods production

- Intermediate goods output: $Y_t^m = \left[\int_0^{A_t} \left(Y_t^{im}\right)^{\frac{\vartheta-1}{\vartheta}} di\right]^{\frac{\vartheta}{\vartheta-1}}$
- Price of intermediate good composite: $P_t^m = \left[\int_0^{A_t} \left(P_t^i\right)^{1-\vartheta} di\right]^{\frac{1}{1-\vartheta}}$
- Intermediate good production function: $Y_t^{im} = \theta_t \left(K_t^i\right)^{\alpha} \left(L_t^i\right)^{1-\alpha}$
- Cost minimization:

$$\alpha \frac{\vartheta - 1}{\vartheta} \frac{P_t^m}{P_t} \frac{Y_t^m}{K_t} = R_t^k$$

$$(1-\alpha) \frac{\vartheta - 1}{\vartheta} P_t^m \frac{Y_t^m}{L_t} = W_t$$

- Aggregation:

$$Y_t = \theta_t A_t^{\frac{1}{\vartheta-1}} K_t^{\alpha} L_t^{1-\alpha}$$

Final good production

- Final good composite: $Y_t = \left[\int_0^1 Y_t^{i\,\frac{\mu-1}{\mu}} di\right]^{\frac{\mu}{\mu-1}}$
- Price index of final good: $P_t = \left[\int_0^1 P_t^{i\,1-\mu} di\right]^{\frac{1}{1-\mu}}$
- Final goods producer i's output:

$$Y_t^i = \left(\frac{P_t^i}{P_t}\right)^{-\mu} Y_t$$

- Price indexation: $P_t^i = P_{t-1}^i \pi_{t-1}^{\iota_p} \bar{\pi}^{1-\iota_p}$
- Final good producer's problem (s.t. equ. 25)

$$\max_{P_t^*} \mathbb{E}_t \sum_{j=0}^{\infty} \xi_p^j \Lambda_{t,t+j} \left(\frac{P_t^* \prod_{k=1}^{j} \pi_{t+k-1}^{\iota_p} \bar{\pi}^{1-\iota_p}}{P_{t+j}} - \frac{P_{t+j}^m}{P_{t+j}} \right) Y_{t+j}^i$$

Capital producers: investment

- Capital producers turn final output into capital which they sell to households at price Q_t

$$\mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \Lambda_{t,t+1+j} \left[Q_{t+j} I_{t+j} - (1 + f^I) I_{+j} \right] \right\}$$

- Marginal costs of generating investment goods equals their price:

$$Q_t = 1 + f_i \left(\frac{I_t}{I_{t-1}} \right) + \frac{I_t}{I_{t-1}} f_i' \left(\frac{I_t}{I_{t-1}} \right) - \mathbb{E}_t \left[\Lambda_{t+1} \left(\frac{I_t}{I_{t-1}} \right)^2 f_i' \left(\frac{I_t}{I_{t-1}} \right) \right]$$

- Law of motion of capital:

$$K_{t+1} = (1 - \delta) K_t + I_t$$

Employment agencies

- Continuum of households $i \in [0,1]$ monopolistically supply specialized labor L_t^i
- Large number of competitive employment agencies:

$$L_t = \left[\int_0^1 L_t^{i\,\frac{\omega-1}{\omega}} di \right]^{\frac{\omega}{\omega-1}}$$

- Labor demand for type i:

$$L_t^i = \left(\frac{W_t^i}{W_t} \right)^{-\omega} L_t$$

- Wages:

$$W_t = \left[\int_0^1 W_t^{i\,1-\omega} di \right]^{\frac{1}{1-\omega}}$$

Households

- Household i maximizes utility

$$\mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[log\left(C_{t+j} - hC_{t+j-1}\right) - \frac{\psi}{1+\nu} L_{i,t+j}^{1+\nu} \right] \right\}$$

subject to the budget constraint

$$\frac{W_t^i}{P_t} L_t^i + R_t \frac{B_t}{P_t} + \left(R_t^k + (1-\delta) Q_t \right) K_t + \Pi_t = C_t + \frac{B_{t+1}}{P_t} + Q_t K_{t+1}$$

- Optimal wage set subject to labor demand:

$$\max_{W_t^*} \mathbb{E}_t \sum_{j=0}^{\infty} \left\{ (\xi_w \beta)^j \left[\frac{U_{c,t+j}}{P_{t+j}} L_{t+j}^i W_t^* \prod_{k=1}^{j} (1+g)\, \pi_{t+k-1}^{\iota_w} \bar{\pi}^{1-\iota_w} - \frac{\psi}{1+\nu} \left(L_{t+j}^i \right)^{1+\nu} \right] \right\}$$

- Wage indexation: $W_t^i = W_{t-1}^i (1+g)\, \pi_{t-1}^{\iota_w} \bar{\pi}^{1-\iota_w}$

Monetary policy and aggregation

Monetary authority sets policy rate according to:

$$R_t = \left(\left(\frac{\pi_t}{\pi^*} \right)^{\gamma_\pi} \left(\frac{y_t}{y_t^{pot}} \right)^{\gamma_y} R_n \right)^{1-\rho_r} (R_{t-1})^{\rho_r} \, r_t^m$$

Aggregation

$$Y_t = C_t + f^I I_t + f^X X_t + f^A I_t^A$$

Parameterization

Parameter	Description	Value
α	Capital share	0.33
β	Discount factor	0.999
h	Habit persistence	0.50
ν	Inverse Frisch elasticity	0.50
δ	Capital depreciation	0.025
f_k''	Capital adjustment costs	5.5
$\bar{L}$	Steady state employment	1
θ_p	Calvo prices	0.93
θ_w	Calvo wages	0.9
ι_p	Price indexation	0.5
ι_w	Wage indexation	0.5
μ	Elasticity of substitution (final goods)	6
ω	Elasticity of substitution (labor)	6
γ_π	Inflation weight	1.5
γ_y	Output weight	1
ρ_r	Persistence (policy rule)	0.8
π^*	Inflation target (quarterly)	0.005
ϑ	Elasticity of substitution (intermediates)	2.493
ζ	R&D elasticity	0.304
ρ^λ	Adoption elasticity	0.925
$\bar{\lambda}$	Steady state adoption rate	0.05
η	R&D-adoption spillover	0.294
$1-\phi$	Obsolescence rate	0.025
$f_{R\&D}''$	Adjustment costs R&D	6
$f_{\bar{\iota}a}''$	Adjustment costs adoption	6
$100 * (\bar{g}^{\frac{1}{\vartheta-1}})$	Technology growth (steady state)	0.5