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Derivatives and Credit Contagion in Interconnected Networks

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Outline

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Motivation

- Credit defaults clustered around times of economic stress
  - dependency on macro-economic factors
  - credit contagion

- Contagion dynamics radically changed in the last decade through Credit Default Swaps (CDS)

- Yet role of CDS in contagion only looked at in the context of pricing individual products:

  Hull and White (2001); Haworth and Reisinger (2007); Haworth, Reisinger and Shaw (2008); Frey and Backhaus (2008); Brigo and Chourdakis (2009); Frey and Backhaus (2010); Errais, Giesecke and Goldberg (2010)

Motivation – Cont.

- Little research on influence of CDS on contagion dynamics, at systemic level, though recent crisis has clearly highlighted their significance (Lehman – AIG)
- Network analyses of contagion have not included CDS: e.g. Frey and Backhaus (2003), Giesecke and Weber (2004), Neu and RK (2004), Hatchett and RK (2006/2009), Cont et al. (2009), Gai, Haldane and Kapadia (2011)
- Recent studies
  - CDS as indicators of contagion: Jorion (2007)
  - Effect of CDS on systemic stability (concentration of CDS markets): ECB (2009)
  - Stability analysis of a network of reconstructed CDS exposures of major US banks: Markose et al. (2010)

Mechanics of CDS

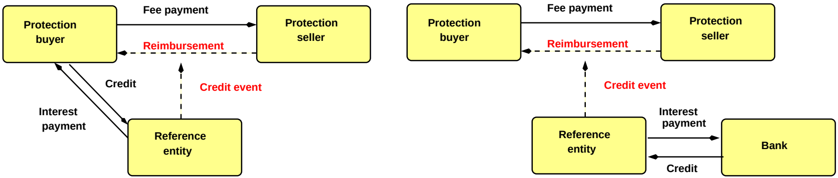

Fig 1. Mechanics of CDS contracts used for hedging and speculation.

CDS

- are used to manage credit risk (hedging), and for speculation
- are zero-sum games
- create additional ‘three-particle’ contagion channels

Method — Take-Home Message

- Introduce CDS into existing model of credit contagion

  P Neu & RK, Physica A (2004), JPL Hatchett & RK, J Phys A (2006), Quant. Fin. (2009)

- CDS cannot completely eliminate risk, and amplify contagion in times of stress, in particular if used to expand loan books.

Outline

Interacting Companies

- Interacting Companies Model

  P Neu & RK, Physica A (2004), JPL Hatchett & RK, J Phys A (2006), Quant. Fin. (2009)

- Two state model:

  company up and running ($n_i = 0$), or defaulted ($n_i = 1$)

- Probabilities of default and mutual impacts of defaults (exposures) heterogeneous across the set of companies (“frozen disorder”); connectivity functionally defined

  $\Longleftrightarrow$ lattice gas model defined on random graph

- Losses determined randomly (recovery process) when a company defaults

Contagion Dynamics I: Firms Only

- Companies need “orders” (support, cash inflow) to maintain wealth and avoid default
- W_{it} wealth position of firm i at time t,

$$W_{it} = \vartheta_i - L_{it} - \eta_{it} = \vartheta_i - \sum_{j \in F} J_{ij}\, n_{jt} - \eta_{it}$$

  - ϑ_i initial wealth
  - J_{ij} impact of a default of j on wealth of i
  - Noise η_{it} idiosyncratic & economy-wide (minimal Basel II)

$$\eta_{it} = \sigma_i \left(\sqrt{\rho_i}\, \eta_{0,t} + \sqrt{1-\rho_i}\, \xi_{it} \right)$$

- Company i defaults, if the total wealth falls below zero

$$n_{it+1} = n_{it} + (1 - n_{it}) \Theta(-W_{it})$$

- No recovery within ‘risk horizon’ T: $n_i = 1$ is absorbing state. Time unit: 1 month; $T = 12 \Leftrightarrow 1$ year. $\Rightarrow$ no equilibrium dyn.

Contagion Dynamics II: banks, insurers, & CDS

- Banks and insurers engage in several types α of interaction among each other, and with firms
  - direct exposures (d),
  - unhedged loans (u),
  - hedged loans (hb),
  - protection selling for hedged loans (hs),
  - speculative buying/selling (sb/ss) of CDS
- Wealth dynamics as for firms:

$$W_{it} = \vartheta_i - \sum_{\alpha} L_{it}^{\alpha} - \eta_{it}$$

$$n_{it+1} = n_{it} + (1 - n_{it})\Theta(-W_{it})$$

Loss Types

- Direct exposures: material impact of default (as for firms)

$$L_{i,t}^{(d)} = \sum_j J_{ij}^{(d)} \, n_{j,t}$$

- Unhedged loans: losses through defaults, income from interest payments

$$L_{i,t}^{(u)} = \sum_{j \in F,B} J_{ij}^{(u)} \left[n_{j,t} - \sum_{\tau=1}^{t} \varepsilon_{ij,\tau} \right]$$

- Hedged loans: losses through (coincident) defaults & fees, income from interest

$$L_{i,t}^{(hb)} = \sum_{j \in F,B} \sum_{k \in B,I} J_{ij}^{k} \sum_{\tau=1}^{t} \left[(n_{j,\tau} - n_{j,\tau-1}) n_{k,\tau} + f_{ij,\tau}^{k} - \varepsilon_{ij\tau} \right]$$

- Protection selling: Losses through credit events, fee income

$$L_{i,t}^{(hs)} = \sum_{j \in F,B} \sum_{k \in B} J_{ij}^{k} \sum_{\tau=1}^{t} \left[(n_{j,\tau} - n_{j,\tau-1})(1 - n_{i,\tau}) - f_{ij,\tau}^{k} \right]$$

- Speculative protection buying: income from credit events, fee-payments

$$L_{i,t}^{(sb)} = - \sum_{j \in F,B} \sum_{k \in B,I} K_{ij}^{k} \sum_{\tau=1}^{t} \left[(n_{j,\tau} - n_{j,\tau-1})(1 - n_{k,\tau}) - f_{ij,\tau}^{k} \right]$$

- Speculative protection selling: losses from credit events, fee income

$$L_{i,t}^{(ss)} = \sum_{j \in F,B} \sum_{k \in B} K_{ij}^{k} \sum_{\tau=1}^{t} \left[(n_{j,t} - n_{j,\tau-1})(1 - n_{i\tau}) - f_{ij,\tau}^{k} \right]$$

Loss Types

- Hedged loans: losses through (coincident) defaults & fees, income from interest

$$L_{i,t}^{(hb)} = \sum_{j \in F,B} \sum_{k \in B,I} J_{ij}^{k} \sum_{\tau=1}^{t} \left[(n_{j,\tau} - n_{j,\tau-1}) n_{k,\tau} + f_{ij,\tau}^{k} - \varepsilon_{ij\tau} \right]$$

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Note: losses incurred **only, if protection sellers have defaulted at (or prior to) time of default** of reference entities

Analysis for a Stochastic Setting

- Heterogeneous initial wealths ϑ_i and interactions/exposures J_{ij}, J_{ij}^k, K_{ij}^k on Erdös-Renyi random graphps

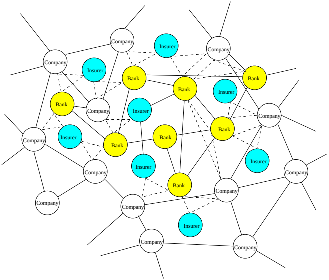

Fig 2. Network of financial dependencies. Full lines: direct exposures and unhedged loans. Triangles: CDS contracts.

- Assume: large sectors, large number of interactions (loans, CDS contracts) $\Rightarrow$ only low-order statistics of exposure sizes needed.
- $\Rightarrow$ Contagion dynamics at system level in terms of fractions $m_{s,t}$ of defaulted nodes in various sectors $s \in \{F, B, I\}$.

Outline

Losses and Defaults in Banking Sector

- Concentrate on
  - End of year losses per bank in banking sector

$$L = \frac{1}{N_B} \sum_{i \in B} L_{i,T}$$

  - End of year fraction of defaulted banks in banking sector

$$m = \frac{1}{N_B} \sum_{i \in B} n_{i,T}$$

- Their distributions $P(L)$ and $P(m)$ are ‘driven by’ the macro-economic noise $\eta_{0,t}$,
  - assumed to be slowly varying Gaussian
  - simplification: keep constant within year $\eta_{0,t} = \eta_o$

Unhedged Lending

- Starting point: no CDS

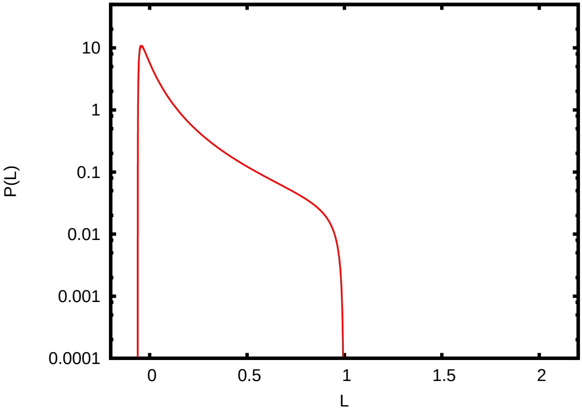

Fig 3. Unhedged lending: baseline scenario.

Unhedged Lending

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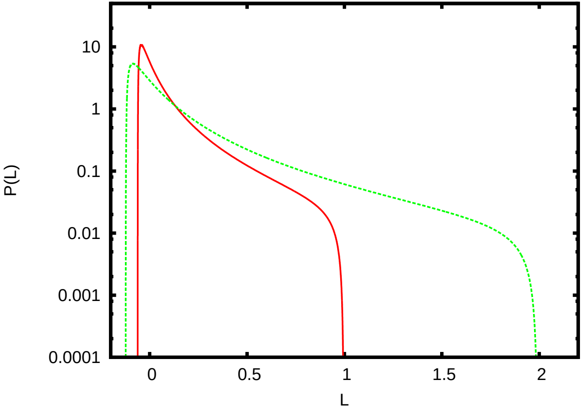

Fig 3. Unhedged lending: baseline scenario.
Effect of doubling loan books with firms

Unhedged Lending

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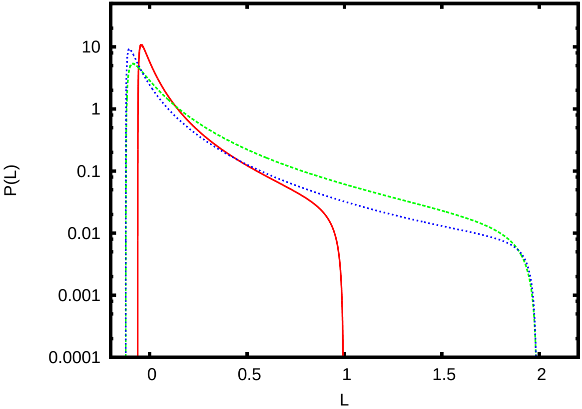


Fig 3. Unhedged lending: baseline scenario.
Effect of doubling loan books with firms , doubling, but half-half firm & inter bank

Three Scenarios with CDS

- Scenario 1: B & F, only hedging
- Scenario 2: B, F & I, only hedging
- Scenario 3: B, F & I, hedging and speculation

Hedging Exposures — Losses

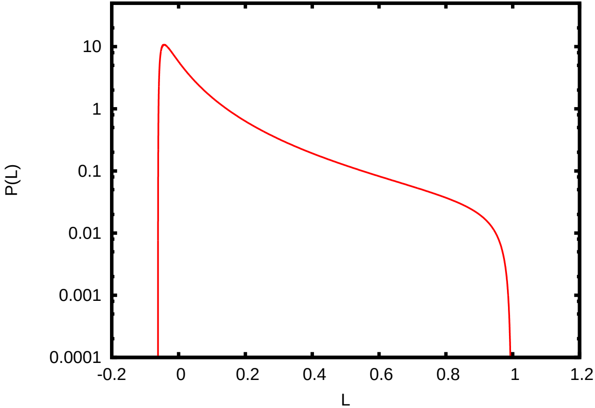


Fig 4. Scenario 1: the effect of CDS, hedging exposures **within banking sector**

unhedged base-line scenario .

Hedging Exposures — Losses

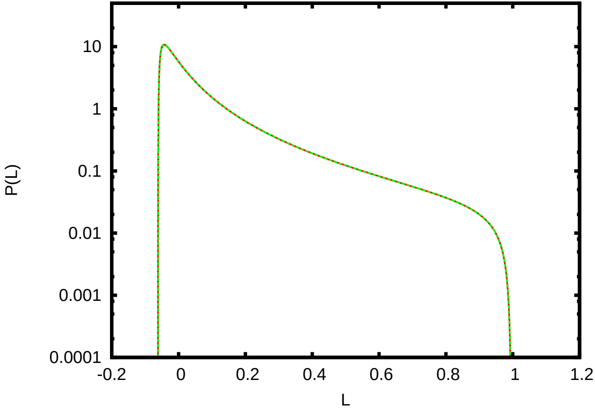
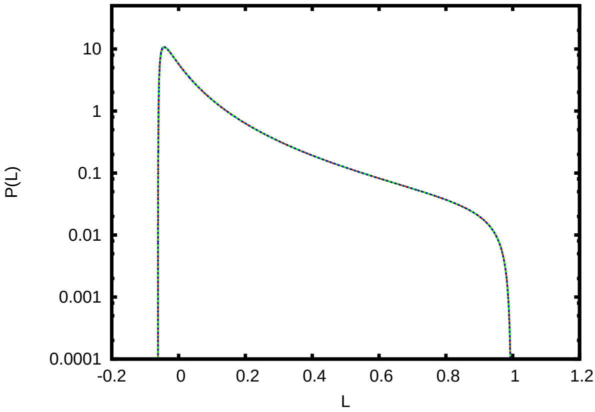


Fig 4. Scenario 1: the effect of CDS, hedging exposures **within banking sector**

unhedged base-line scenario, 1/3 hedged .

Hedging Exposures — Losses



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Hedging Exposures — Losses

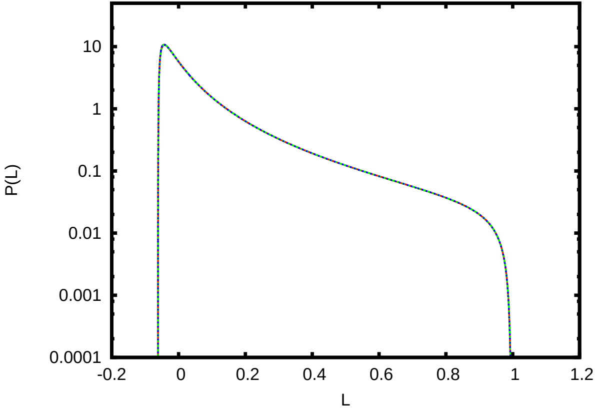


Fig 4. Scenario 1: the effect of CDS, hedging exposures **within banking sector**

unhedged base-line scenario, 1/3 hedged, 2/3 hedged ⇔ **CDS are zero-sum game**.

Hedging Exposures — Default Rates

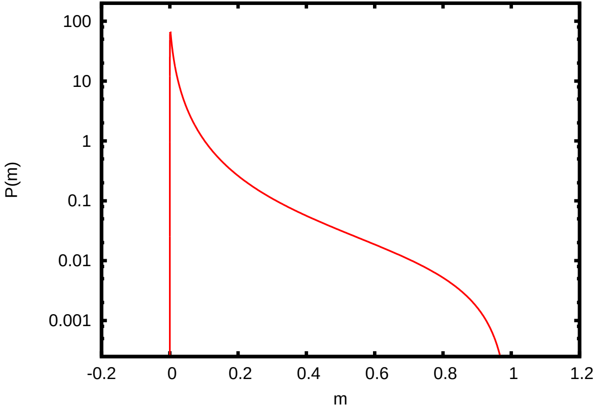
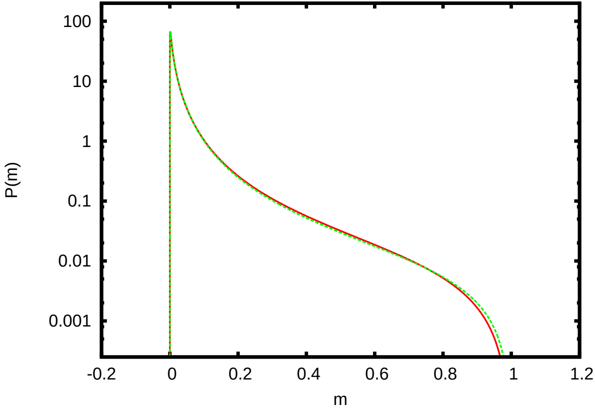


Fig 5. Scenario 1: the effect of CDS, hedging exposures **within banking sector**

unhedged base-line scenario .

Hedging Exposures — Default Rates



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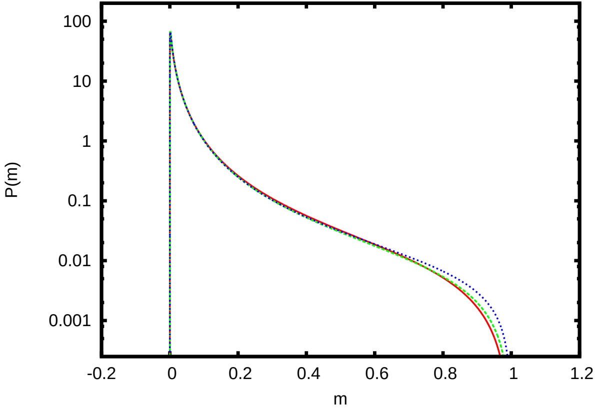
unhedged base-line scenario, 1/3 hedged .

Hedging Exposures — Default Rates



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Hedging Exposures — Default Rates

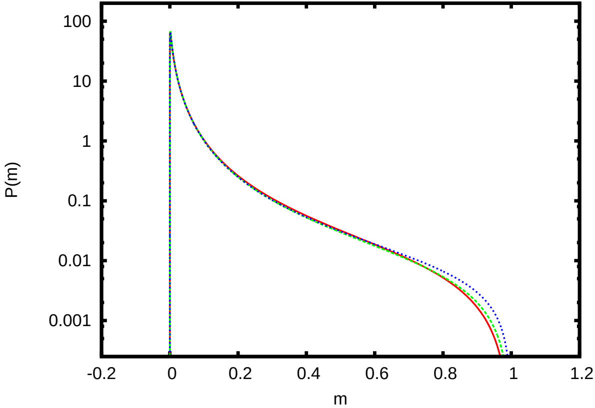


Fig 5. Scenario 1: the effect of CDS, hedging exposures **within banking sector**

unhedged base-line scenario, 1/3 hedged, 2/3 hedged $\Rightarrow$ **more defaults, despite unchanged loss distribution.**

Hedging Exposures — Effect of Hedging Ratio

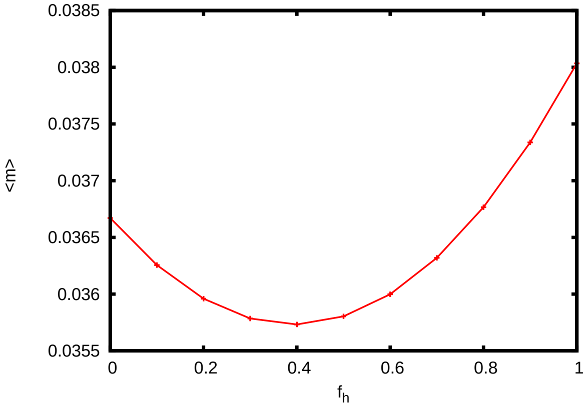


Fig 6. Scenario 1: the effect of hedging ratio on **average default rates**.

Hedging Exposures — Effect of Hedging Ratio

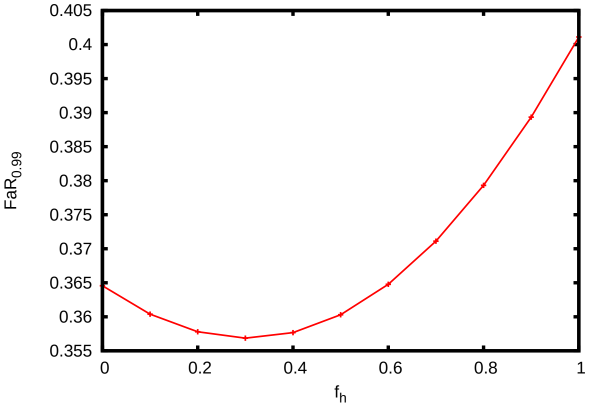


Fig 7. Scenario 1: the effect of hedging ratio on **Fraction at Risk** at 99% confidence level.

Hedging with Insurers

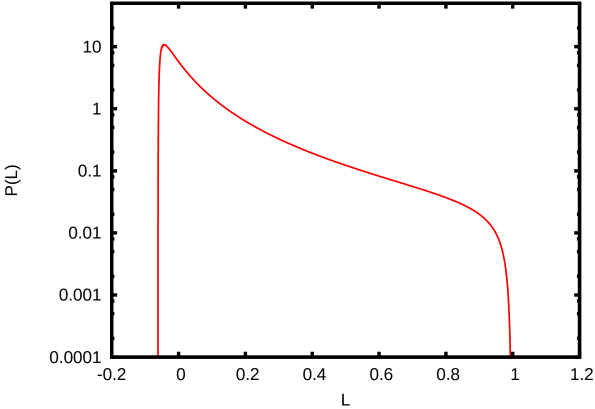


Fig 8. Scenario 2: Unhedged lending: baseline scenario (losses in banking sector).

Hedging with Insurers

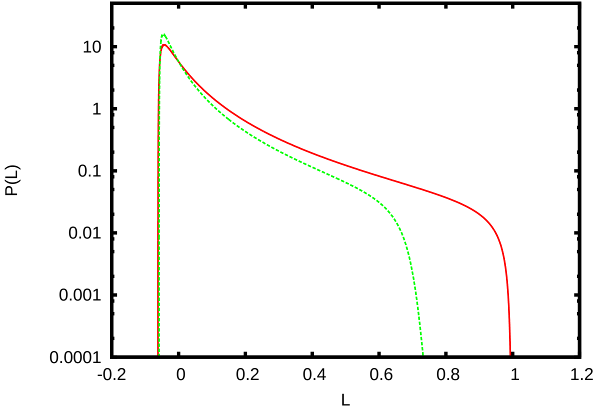
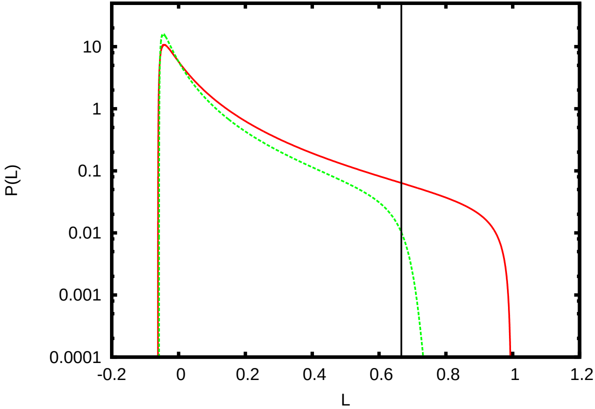


Fig 8. Scenario 2: Unhedged lending: baseline scenario (losses in banking sector). Effect of hedging one third of exposures with insurers

Hedging with Insurers



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Effect of hedging one third of exposures with insurers , and naively expected maximum loss.

Hedging with Insurers

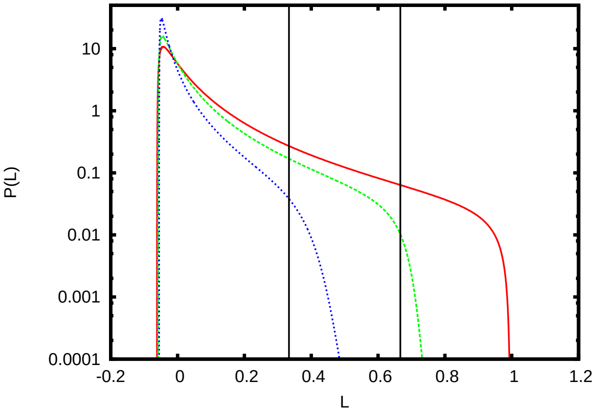


Fig 8. Scenario 2: Unhedged lending: baseline scenario (losses in banking sector).
Effect of hedging one third of exposures with insurers , and naively expected maximum loss.
Effect of hedging two thirds of exposures with insurers

Hedging Increased Exposures with Insurers

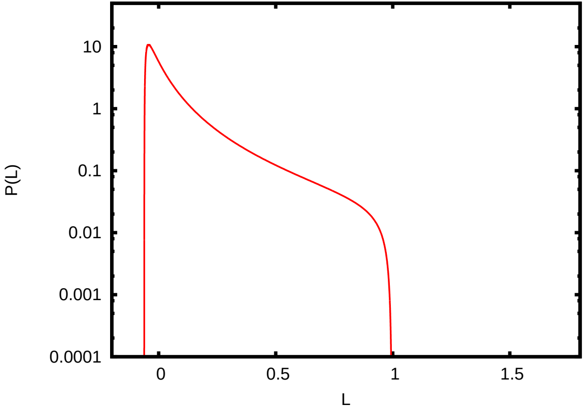


Fig 9. Scenario 2: Unhedged lending: baseline scenario (losses in banking sector).

Hedging Increased Exposures with Insurers

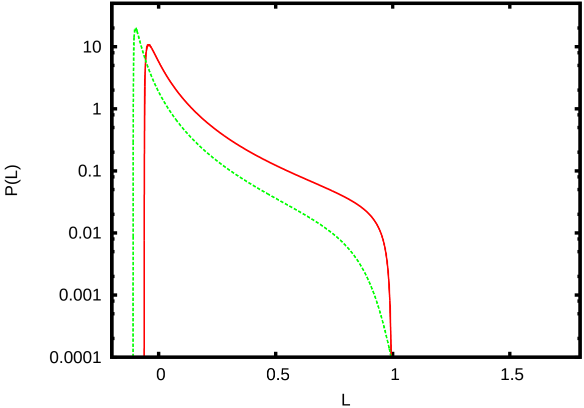
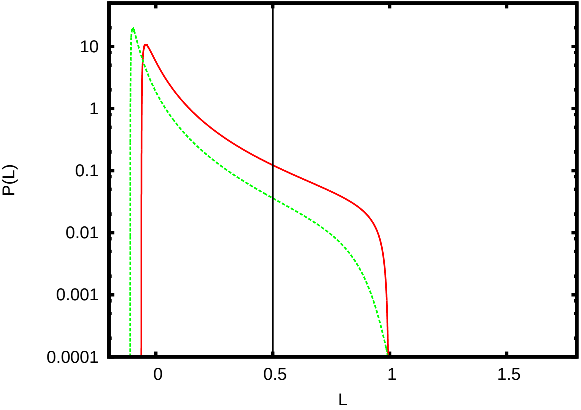


Fig 9. Scenario 2: Unhedged lending: baseline scenario (losses in banking sector). Effect of doubling the size of loan books, hedging half of original exposures with banks, the remainder with with insurers

Hedging Increased Exposures with Insurers



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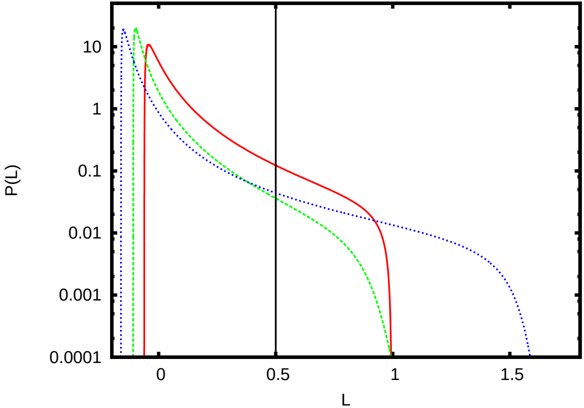

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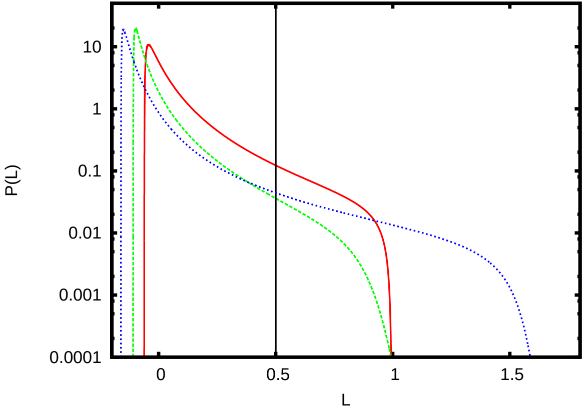

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Note: **incentives and dangers** of this strategy!

Adding Speculative CDS

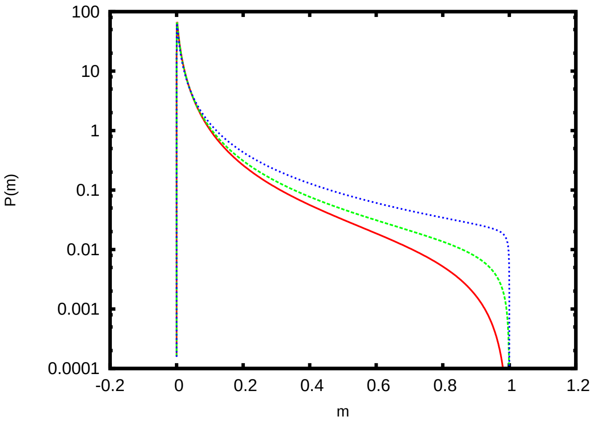

Fig 10. Scenario 3: Distribution of the fraction of defaulted banks; the base-line scenario compared with situations where speculative CDS of a volume matching the base-line exposure, or twice the volume of the base-line exposure are taken out *inside* the banking sector. **Note:** loss distributions are unaffected!

Outline

Summary I

- Looked at stylized model of networks of firms, banks and insurers
  - Limit of large number of loans/CDS contracts allows to exploit LLN, CLT to obtain macroscopic dynamics.
  - Only low order statistics of interaction effects required.
- CDS are **zero-sum games**.
  - They **do not change loss distributions at system level.**
  - They do not protect against the risk of increased losses, when expanding loan books.
- CDS create **additional contagion channels** which **destabilize the system in times of economic stress**.

Summary II

- Areas for improvement
  - more realistic networks of dependencies, wealth- and exposure distributions
  - modifications for small banking sector (e.g. UK)
  - include economic impact of defaulting banks on network of firms
  - CDS fees correlated defaults . . .
- **Main findings** expected to be qualitatively **insensitive against modification of detail**
  - zero-sum nature of CDS
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  - conclusions based on comparison of scenarios with/without CDS

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THANK YOU!

Literature

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