Capital controls: a normative analysis
Preliminary and incomplete*

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Abstract

Countries' concerns with the value of their currency have been extensively studied and documented in the literature. Capital controls can be (and often are) used as a tool to manage exchange rate fluctuations. This paper investigates whether countries can benefit from using such tool. We develop a welfare based analysis of whether (or, in fact, how) countries should tax international borrowing. Our results suggest that restricting international capital flow with the use of these taxes can be beneficial for individual countries although it would limit cross-border pooling of risk. This is because while consumption risk-pooling is important, individual countries also care about domestic output fluctuations. Moreover, the results show that countries decide to restrict the international flow of capital exactly when this flow is crucial to ensure cross-border risk-sharing. Our findings thus point to important gains from international coordination in the use of capital controls.

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1 Introduction

Countries' concerns with the value of their currency have been extensively studied and documented in the literature. As detailed in Fry et al. (2000), the majority of central banks around the world actually include the exchange rate as one of their main policy objectives – and the rationale for this has been the topic of a large literature on monetary policy in open economies (Corsetti et al. (2010) and references there in). But apart from traditional monetary policy, capital controls can be (and often are) used as tool to manage exchange rate fluctuations (see survey by Edwards (1999), or more recently, Schmitt-Grohe and Uribe (2012)). The aim of this paper is to shed light on whether countries can in fact benefit from using such tool.

We present a welfare based analysis of whether (or, in fact, how) countries may wish to intervene in the international flow of capital. To do so we lay out a simple two-country model with incomplete financial markets. In the proposed model, controlling capital flows may be beneficial for two reasons.

Imperfect risk-sharing across countries introduces a natural role for intervention in the international flow of capital. While movements in international prices can automatically ensure cross-border risk-sharing in special circumstances (Cole and Obstfeld (1991)), this is not generally the case. As shown in Corsetti et al. (2008), when domestic demand is not too sensitive to changes in international relative prices (or the trade elasticity is low), movements in these prices are large and can create strong wealth effects that damage risk pooling among countries. Countries may also suffer from insufficient risk-sharing when shocks are persistent and the trade elasticity is large.

But apart from cross-border consumption risk-sharing, individual countries are also concerned with fluctuations in their own output – or the supply of labor by domestic households. Countries' incentives to strategically manage their terms of trade as to affect labor effort has been extensively studied in the monetary literature (e.g. Corsetti and Pesenti (2001), Tille (2001), Benigno and Benigno (2003), Sutherland (2006), De Paoli (2009)) [1].

[1] The literature has emphasized that, in light of a so-called "terms of trade externality", strategically

In our paper, it is the tug-of-war between these policy incentives that determines countries' desire to intervene in international capital flows. Our results suggest that restricting terms of trade movements with the use of capital controls can improve welfare when the trade elasticity is high. When this elasticity is low, policy should aim at enhancing flexibility in international relative prices. But such policy interventions, although optimal from the individual country point of view, critically limit cross-border pooling of risk. In fact, greater risk-sharing would call for opposite policies. Our findings thus point to important gains from international coordination in the use of capital controls.

The following is an illustration of the results. After a fall in productivity a subsidy to international borrowing can help domestic households share the burden of the shock with foreign households. This is particularly the case when domestic demand is too sensitive to changes in relative prices and the borrowing subsidy can enhance the, otherwise small, appreciation in domestic terms of trade. But individual countries actually find it optimal to tax, rather than subsidize, borrowing. With the goal of limiting fluctuations in domestic output and terms of trade, the country imposes restrictions on capital inflows that augment, rather than mitigate, the adverse effect of the shock on consumption.

Overall, our findings suggest that if capital controls are set in an uncoordinated fashion they can have damaging implications for global risk-sharing and welfare. Ultimately, if countries simultaneously and independently engage in such interventions in the international flow of capital, not only global but individual welfare would be adversely affected.

Other related literature:

Apart from the aforementioned works, our analysis is also related to that of Costinot et al. (2011) who study the role of capital controls in a two-country endowment model with growth. Although in their framework capital controls can be used to manipulate intertemporal prices, the lack of labor supply decisions removes the policy incentives driven by the terms of trade externality described above.

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managing the exchange rate may allow countries to reduce their labor effort without a corresponding fall in their consumption levels. This is the case when countries are able to switch consumption towards foreign goods via changes in relative prices.

Another important strand of the normative literature on capital control include the recent contributions by Benigno et al. (2010), Korinek (2011), Bianchi (2011) and Bianchi and Mendoza (2010). Differently from our work, these studies evaluate the role of capital control as a prudential tool – or a tool to reduce the probability of financial crisis.

2 The Model

The framework consists of two-country dynamic general equilibrium model featuring incomplete markets. The baseline framework is a version of Benigno (2009) that abstracts from nominal rigidities and allows for home bias in consumption. As shown in Corsetti et al. (2008) (or CDL, hereafter), introducing a non-unitary trade elasticity and consumption home-bias in incomplete markets models enables them to generate insufficient risk-sharing (and thus better match the empirical regularities documented in Backus and Smith (1993) and Kollmann (1995)). Finally, in order to introduce a tool with which countries can control the international flow of capital, we assume that policymakers set taxes/subsidies on international borrowing/lending.

2.1 Preferences

We consider two countries, H (Home) and F (Foreign). The world economy is populated with a continuum of agents of unit mass, where the population in the segment $[0, n)$ belongs to country H and the population in the segment $(n, 1]$ belongs to country F. The utility function of a consumer in country H is given by:

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s) - V(N_s) \right], \tag{1}$$

but in what follows we will assume the following isoelastic functional form

$$U(C_s) = \frac{C_t^{1-\rho}}{1-\rho} \text{ and } V(n_s^j) = \frac{(N_s)^{1+\eta}}{1+\eta}. \tag{2}$$

where ρ is the coefficient of risk aversion and η is the inverse of the elasticity of labor supply.

Households obtain utility from consumption $U(C^j)$ and supply labor N^j attaining disutility $V(N_s)$, and C is a C.E.S. (constant elasticity of substitution) aggregate of home and foreign goods, defined by

$$C = \left[v^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}. \tag{3}$$

The parameter $\theta > 0$ is the intratemporal elasticity of substitution between home and foreign-produced goods, C_H and C_F. As in Sutherland (2005), the parameter determining home consumers' preferences for foreign goods, $(1 - v)$, is a function of the relative size of the foreign economy, $(1 - n)$, and of the degree of openness, λ; more specifically, $(1 - v) = (1 - n)\lambda$.

Similar preferences are specified for the Foreign economy

$$C^* = \left[v^{*\frac{1}{\theta}} C_H^{*\frac{\theta-1}{\theta}} + (1-v^*)^{\frac{1}{\theta}} C_F^{*\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{4}$$

with $v^* = n\lambda$. That is, foreign consumers' preferences for home goods depend on the relative size of the home economy and the degree of openness. Note that the specification of v and v^* generates a home bias in consumption. This bias only disappears when $\lambda = 1$.

The consumption-based price indices that correspond to the above specifications of preferences are given by

$$P = \left[v P_H^{1-\theta} + (1-v) \left(P_F\right)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{5}$$

and

$$P^* = \left[v^* P_H^{*1-\theta} + (1-v^*) \left(P_F^*\right)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{6}$$

As Equations (5) and (6) illustrate, the home bias specification leads to deviations from purchasing power parity; that is, $P \neq SP^*$ For this reason, we define the real exchange rate as $Q \equiv SP^*/P$. We can also definite Home terms of trade as the relative price of imports from the Foreign economy versus the price of domestically produced goods: $ToT \equiv P_F/P_H$.

Consumers labor supply condition will imply:

$$w_t = \frac{V_y\left(N_t\right)}{U_c(C_t)} \tag{7}$$

$$= N_t^{\eta} C_t^{\rho}. \tag{8}$$

where w_t is the real wage.

2.2 Firms

We assume that there is a continuum of identical firms that take prices as given. Each individual firm produces an equal share of total output in each country. So the demand for domestic and foreign good is given by:

$$Y_t^H = \left[\frac{P_{H,t}}{P_t}\right]^{-\theta} \left[nvC_t + (1-n)v^* \left(\frac{1}{Q_t}\right)^{-\theta} C_t^*\right], \tag{9}$$

$$Y_t^F = \left[\frac{P_{F,t}}{P_t}\right]^{-\theta} \left[n\left(1-v\right)C_t + (1-n)(1-v^*) \left(\frac{1}{Q_t}\right)^{-\theta} C_t^*\right]. \tag{10}$$

We can derive the demand for an individual good produced in country H, and the demand for a good produced in country F:

$$Y_t = \left[\frac{P_{H,t}}{P_t}\right]^{-\theta} \left[vC_t + \frac{v^*(1-n)}{n} \left(\frac{1}{Q_t}\right)^{-\theta} C_t^*\right], \tag{11}$$

$$Y_t^* = \left[\frac{P_{F,t}}{P_t}\right]^{-\theta} \left[\frac{(1-v)\,n}{1-n} C_t + (1-v^*) \left(\frac{1}{Q_t}\right)^{-\theta} C_t^*\right]. \tag{12}$$

In the case of no-home bias (where $\lambda = 1$, as in Benigno (2009)), these reduce to:

$$Y_t = \left[\frac{P_{H,t}}{P_t}\right]^{-\theta} \left[nC_t + (1-n)C_t^*\right], \tag{13}$$

$$Y_t^* = \left[\frac{P_{F,t}}{P_t}\right]^{-\theta} \left[nC_t + (1-n)C_t^*\right]. \tag{14}$$

Given the following production function[2]

$$Y_t = \xi_t^{\frac{\eta}{\eta+1}} N_t.$$

where productivity shocks are denoted by ξ. Labor demand in the Home economy is, thus, given by

$$\frac{P_{H,t}}{P_t} = \xi_t^{-\frac{\eta}{\eta+1}} w_t,$$

and equating labor demand and labor supply, we obtain the following the labor leisure relationship

$$\frac{P_{H,t}}{P_t} C_t^{-\rho} = \xi_t^{-\eta} Y_t^{\eta}.$$

An analogous condition holds for the Foreign economy.

2.3 Asset Markets

We assume that households of both countries trade a real riskless bond paid in units of the Foreign consumption basket.[3] Moreover, we assume that households at Home face quadratic adjustment cost when changing their real asset position. As in Benigno (2009), the introduction of this cost enables us to pin down the steady state value of the foreign asset position. Moreover, we assume that Home (Foreign) policymakers can impose taxes on international borrowing and that they are rebated back to Home (Foreign) households in the form of transfers.

We can therefore write the household's budget constraint at Home as follows:

$$C_t + B_{F,t} \leq B_{F,t-1} \frac{Q_t}{Q_{t-1}} R_{t-1}^{*} (1 + \tau_{t-1}) + p_{H,t} Y_t + p_{H,t} Tr_t - \frac{\delta}{2} B_{F,t}^2, \tag{15}$$

[2]The production function has the power $\frac{\eta}{\eta+1}$ on productivity ξ in order to be consistent with a a Yeoman-farmer version of the model in Benigno (2009).

[3]The present framework does not include a portfolio problem for households. For recent contributions on optimal international portfolios in incomplete markets settings, see, for example, Devereux and Sutherland (2011) and Evans and Hnatkovska (2005).

where $B_{F,t}$ denotes foreign real bonds[4], Tr_t are transfers made in the form of domestic goods, $p_{H,t} \equiv P_{H,t}/P_t$ is the relative price of Home goods, R_t^* is a foreign real rate on foreign bond holdings and δ is a nonnegative parameter that measures the adjustment cost in terms of units of the consumption index. The variable τ_t is a tax on international bond holdings. Below we illustrate the role of this instrument:

- $B_{F,t} > 0$ and $\tau_t > 0$: Policy implies a subsidy on international lending or a subsidy on capital outflows
- $B_{F,t} > 0$ and $\tau_t < 0$: Policy implies a tax on international lending or a tax on capital outflows
- $B_{F,t} < 0$ and $\tau_t > 0$: Policy implies a tax on international borrowing or a tax on capital inflows
- $B_{F,t} < 0$ and $\tau_t < 0$: Policy implies a subsidy on international borrowing or a subsidy on capital inflows

Similarly to Equation (15), the budget constraint of Foreign households can be written as follows:

$$C_t^* + B_{F,t}^* \leq B_{F,t-1}^* R_{t-1}^* (1 + \tau_{t-1}^*) + p_{F,t}^* Y_t^* + p_{F,t}^* Tr_t^*. \tag{16}$$

where market clearing implies that $B_{F,t}^* = -B_{F,t}$. If, moreover, we assume that the adjustment costs faced by Home households are paid to Foreign households in the form of transfers, then the Home and Foreign economy-wide budget constraints can be written as:

$$C_t + B_{F,t} \leq B_{F,t-1} \frac{Q_t}{Q_{t-1}} R_{t-1}^* + p_{H,t} Y_t - \frac{\delta}{2} B_{F,t}^2 \tag{17}$$

and

$$C_t^* + B_{F,t}^* \leq B_{F,t-1}^* R_{t-1}^* + p_{F,t}^* Y_t^*. \tag{18}$$

Given the above specification, we can write the consumer's optimal intertemporal

[4] Alternatively, one can think of $B_{F,t}$ as the real value of a nominal bond paid in foreign currency. That is, $B_{F,t} \equiv S_t B_{F,t}^N / P_t$, where $B_{F,t}^N$ is a nominal bond paid in foreign currency.

choice as:

$$U_C(C_t)(1+\delta B_{F,t}) = R_t^*(1+\tau_t)\beta E_t\left[U_C(C_{t+1})\frac{Q_{t+1}}{Q_t}\right], \quad (19)$$

$$U_C(C_t^*) = R_t^*(1+\tau_t^*)\beta E_t\left[U_C\left(C_{t+1}^*\right)\right], \quad (20)$$

where (19) and (20) are the Home and Foreign Euler equations, respectively, both derived from the optimal choice of foreign bonds.

3 Welfare

In this section we illustrate how different features of the model affect global and national welfare. Such analysis allows us to understand the incentives driving the policy decisions presented in subsequent sections. Our conditional welfare measure is obtained using second-order perturbation methods – as described in Schmitt-Grohe and Uribe (2007) and Nam (2011).[5] National welfare is defined as the lifetime utility of each country (e.g. Home national welfare is given by 1). Global welfare is defined as the weighted average of these utilities, where the weights are given by country sizes. That is,

$$U_t^W = nU_t + (1-n)U_t^*.$$

So, every household in the world receives the same weight when computing global welfare.[6]

We first illustrate the implications of incomplete markets and the resulting inability of agents to fully share risk across-countries. As discussed in Cole and Obstfeld (1991), under certain conditions, movements in international relative prices can automatically ensure such cross-border risk-sharing regardless of countries' ability to trade financial assets. Other early works in the literature (e.g. Baxter and Crucini (1995)) have shown that the level of risk sharing in incomplete market models can be quite large. But, as

[5] All our numerical simulations use perturbation methods. We use a second-order approximation procedure to obtain theoretical moments. For impulse responses we use a first-order approximation of the model.

[6] Clearly, there is a variety of ways of specifying such weights (e.g. Negishi (1972)). Arguably, we will be considering the implications of using alternative weights – or at least extreme cases of it in which only the utility of one of the countries is weighted when setting policy – when comparing the optimal national versus optimal global policy.

extensively documented in the work by CDL, lack of risk sharing may be a significant feature of incomplete markets's models even when agents are allowed to trade bonds. The authors show that both the degree of substitutability between domestic and foreign goods, as well as the degree of home bias, are important determinants of risk sharing in such models.

When asset markets are complete, (i.e. when agents can trade, without any portfolio adjustment cost, a full set of contingent claims) adjusting for the real exchange rate, intertemporal marginal rates of substitution are equalized across borders. As a result, one can measure the lack of risk sharing based on the difference in such real exchange rate-adjusted intertemporal marginal rates of substitutions across countries (see, for example, Viani (2011) and CDL). So, we define the "risk-sharing gap" as

$$\frac{U_C\left(C_{t+1}\right)}{U_C\left(C_t\right)}\frac{Q_{t+1}}{Q_t}-\frac{U_C\left(C_{t+1}^*\right)}{U_C\left(C_t^*\right)}. \tag{21}$$

Figure 1 presents the standard deviation of this gap (named "Risk-sharing inefficiency" as in Viani (2011)) for different values of the trade elasticity, θ, and for different degrees of home bias, λ. The calibration used to produce Figure 1 is shown in Table 1. The exercise assumes no active tax policy (i.e. $\tau_t = \tau_t^* = 0$). Consistent with the results in CDL, the figure shows that such inefficiency is high as the trade elasticity deviates from unity. Intermediate levels of home bias also tend to deliver lower international risk-sharing.

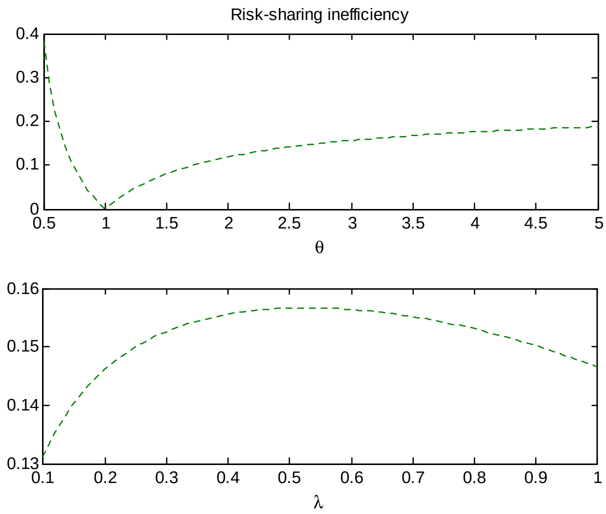

Figure 1: Standard deviation of risk-sharing gap (%), for different values of the trade elasticity, θ, and for different degrees of home bias, λ

Parameter	Value	Notes:
β	0.99	Specifying a quarterly model with 4% steady-state real interest rate
η	0.47	Following Rotemberg and Woodford (1997)
ρ	1	Log utility
λ	0.5; $[0.1, 1]$	Benchmark 0.5, but other values considered
n	0.5; $[0.1, 0.9]$	Symmetric country sizes, but other values considered
θ	3; $[0.5, 3]^*$	Following Obstfeld and Rogoff (1995)
		*Range allowing for complements and substitutes goods
δ	0.01	Following Benigno (2009)
$sdv(\varepsilon), sdv(\varepsilon^*)$	0.71%	Following Kehoe and Perri (2002)
$\kappa^{(\varepsilon)}$, $\kappa^{(\varepsilon^*)}$	0.95	Following Kehoe and Perri (2002)

Table 1: Parameter values used in the quantitative analysis[7]

Figure 2 presents another metric of the size of the inefficiencies created by incomplete markets by showing the level of global welfare (measured as a percentage of steady state consumption) for our benchmark model and for a version of the model in which asset markets are complete. Although the level of risk sharing shown in Figure 1 is still below the levels seen in the data (see Viani (2011)), welfare differences between complete and incomplete markets can be as large as 0.5%, a substantial difference when considering

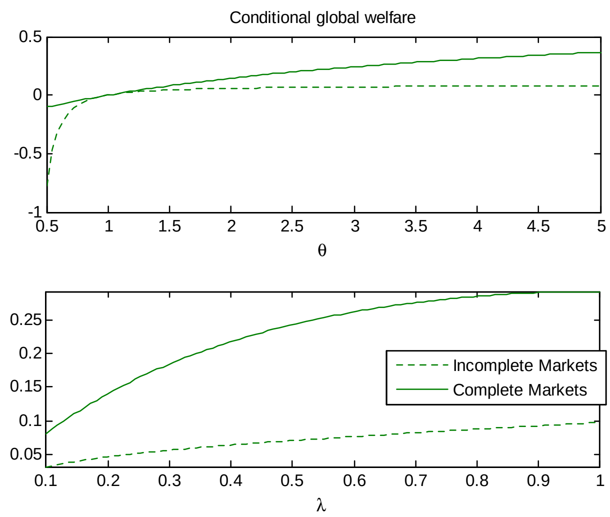

Figure 2: Conditional global welfare, measured as a percentage of steady state consumption, for different values of the trade elasticity, θ and for different degrees of home bias, λ.

that welfare costs of economic fluctuations in consumptions based models of our kind tend to be small (Lucas (1987)). Note that the Cole and Obstfeld (1991) result is replicated when $\theta = 1$.

In our model with endogenous labor supply, agents are not only concerned with cross-border consumption risk-sharing, but also with fluctuations in their own output. As discussed in the introduction and documented in the monetary literature, open economies are affected by a terms of trade externality. Individual countries strategically manage the terms of trade in order to reduce their labor effort without a corresponding fall in their consumption levels. This is particularly the case when the elasticity of substitution between goods is large and having a more appreciated exchange rate can divert consumption towards foreign goods.

Figure 3 illustrates that, although global welfare is always inferior when markets are incomplete, in an asymmetric world, welfare of individual countries may be larger under imperfect risk sharing. As shown in Figure 4, when the trade elasticity θ is large, bigger domestic purchasing power under incomplete markets (panel 3 of Figure 4), allows agent to produce less (panel 2 of Figure 4) without a proportional fall in consumption (panel 1 of Figure 4). When the share of imports in consumers baskets is sufficiently large, such equilibrium may deliver higher welfare (for the Home country, this is the case when $1 - n$

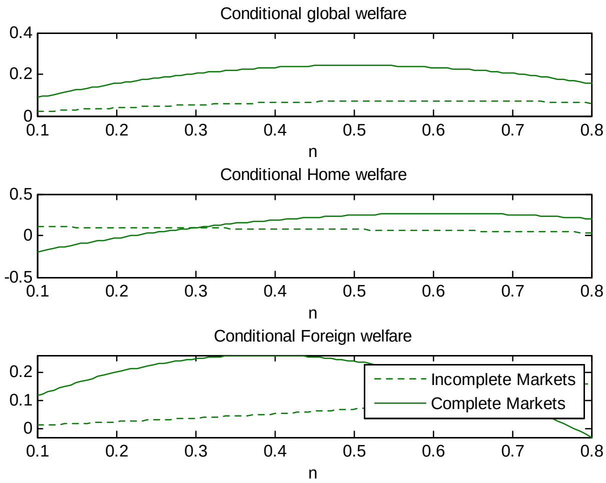

Figure 3: Conditional global and national welfare, measured in percentage deviations from steady state consumption, for different values of the trade elasticity, θ.

is large and for Foreign, this is the case when n is large – as shown in Figure 3).[8]

4 Optimal taxes under incomplete markets

We now analyze how policymakers would choose to tax international capital flows in light of the policy incentives described above. We consider different policy settings. First, we assume that the Home policymaker chooses taxes as to minimize domestic social losses, while the Foreign country does not have access to a tool to control capital flows. We then analyze the case in which taxes are determined by a global social planner who minimizes global social losses. Finally, we consider the case in which both countries decide how to set taxes on international bonds, arriving at a Nash equilibrium.

4.1 National optimal policy

In this section, we assume that only the Home policymaker has an active policy instrument. That is, while the Foreign policymaker keeps taxes constant ($\tau^* = 0$), the domestic

[8]For certain calibrations (e.g. no home bias), Home welfare can be under incomplete markets even when θ is low. In this case mean welfare is larger under incomplete markets as a more undervalued currency in this setting produces higher levels of consumption, without an equivalent increase in labor effort.

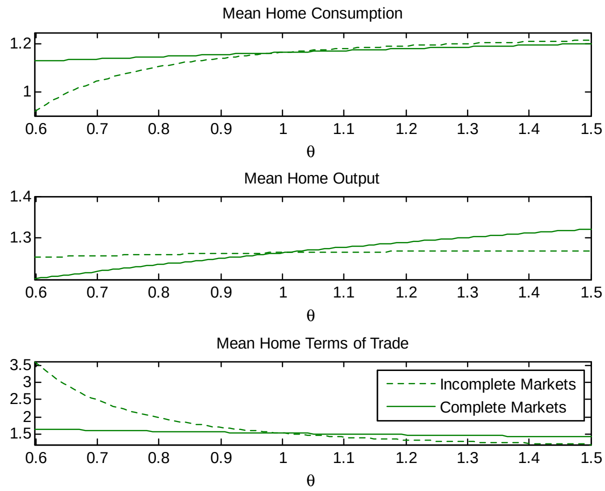

Figure 4: Unconditional mean of Home consumption, output and terms of trade for different values of the trade elasticity, θ.

policymaker decides on the evolutions of taxes, τ_t, that maximizes domestic welfare. The Ramsey policy problem and first order conditions are shown in Section 6.1 of the Appendix.

First, we analyze economic dynamics following a negative Home productivity shock under the assumption that $\theta = 3$, i.e. home and foreign goods are imperfect substitutes. As we can see in Figure 5, in response to the shock, home output and home consumption decrease while the terms of trade appreciate. Domestic households, in order to smooth consumption, would like to borrow from foreign agents. However, the domestic social planner increases taxes on international borrowing. Higher taxes effectively increase an interest rate paid on foreign bond holdings and discourage domestic households from borrowing. The result is an even stronger fall in consumption and a larger deviation from complete risk sharing. The policymaker's action reduces fluctuations in domestic labor supply – or lowers output volatility – at the expense of financial integration among countries.

Figure 6 considers the case in which domestic and foreign goods are complements. Under this specification, the strong appreciation in the terms of trade actually introduces a large positive wealth effect at home, (as described in CDL) which implies that domestic agents actually become net lenders to foreign households.The optimal policy implies a

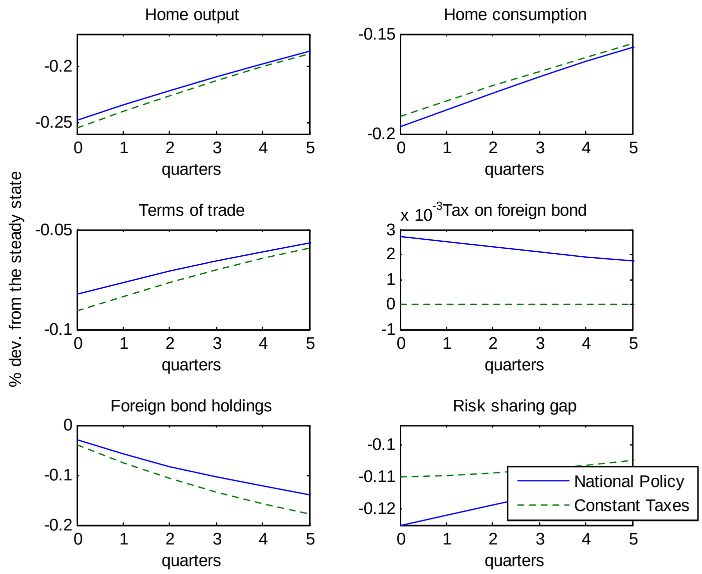

Figure 5: Optimal national policy following a negative Home productivity shock with $\theta = 3$: comparison with the case in which there is no active tax policy.

tax on capital outflow (as it reduces the effective returns to domestic lenders) that, again, limits international risk-sharing. The policy, however, allows for a smaller drop in consumption without a significant change in domestic labor supply.

4.2 Global optimal policy

We now consider the case in which a global policymaker sets the same instrument, τ_t, in order to maximize global welfare. Details of the optimal policy problem and first order conditions can be found in Section 6.3 in the Appendix.[9]

As shown in Figure 7, the optimal policy that maximizes global welfare has opposing tax prescriptions when compared to the policy designed to maximize national policy. After a negative shock to home productivity, when domestic and foreign goods are substitutes in the utility, the global social planner lowers taxes in order to promote international borrowing, increase capital flows and enhance cross border risk-sharing.[10] In fact, our

[9] Also, in order to simultaneously conduct some sensitivity analysis, we now consider the case of a symmetric country size, but introduce home bias in consumption. In particular, we set $n = 1 - n = \lambda = 0.5$. We find that the different size and home bias specification does not change the conclusions reached in the previous section.

[10] Taxes actually rise *permanently* as to minimize distortions in agents intertemporal decisions.

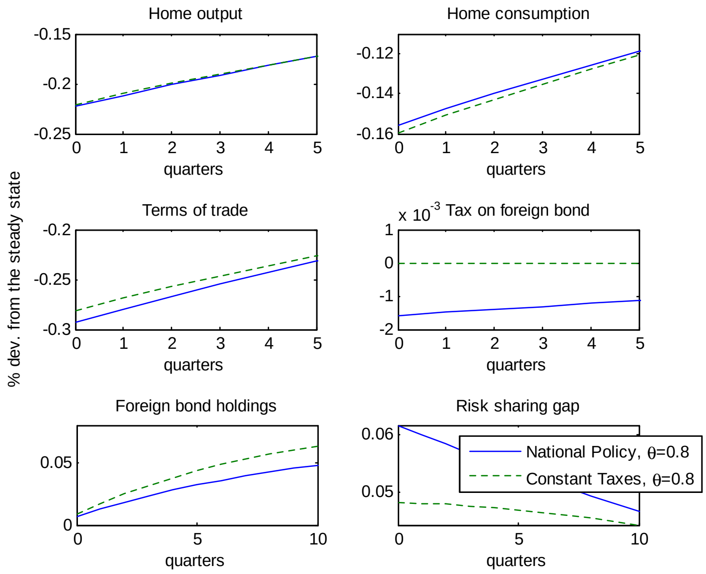

Figure 6: Optimal national policy with $\theta = 0.8$: comparison with the case in which there is no active tax policy.

measure of the risk-sharing gap (given by Expression 21) becomes non stationary under the global policy – which eliminates any fluctuations in the gap after the initial period.

But, as Figure 8 shows, while global policy increases global welfare and improves cross-border risk-sharing, it may reduce welfare of the Home economy.[11] As the effect of changes in the terms of trade on the composition of demand increase (or as θ moves away from unity), raising the strength of the terms of trade externality, Home welfare losses under the global optimal policy also increase.

4.3 Nash equilibrium

Finally, we consider a Nash equilibrium in which the Home policymaker chooses the optimal path for domestic borrowing taxes, τ_t, while the Foreign policymaker controls the evolution of τ_t^*. Again, the details of such policy problem and set of first order conditions can be found in Section 6.4 of the Appendix.

Figure 9 compares the Nash equilibrium (black line) with the case in which only one policymaker sets taxes actively (blue line), the case of a global central planner (red line),

[11] Note that for the global policy, the standard deviation of the risk sharing gap was calculated using simulated moments – given its non-stationary property.

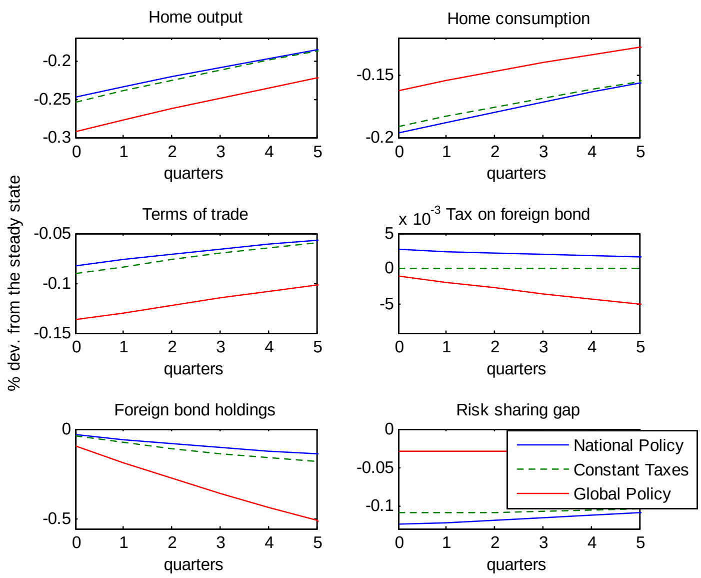

Figure 7: Optimal global and national policy following a negative Home productivity shock with $\theta = 3$: comparison with the case in which there is no active tax policy.

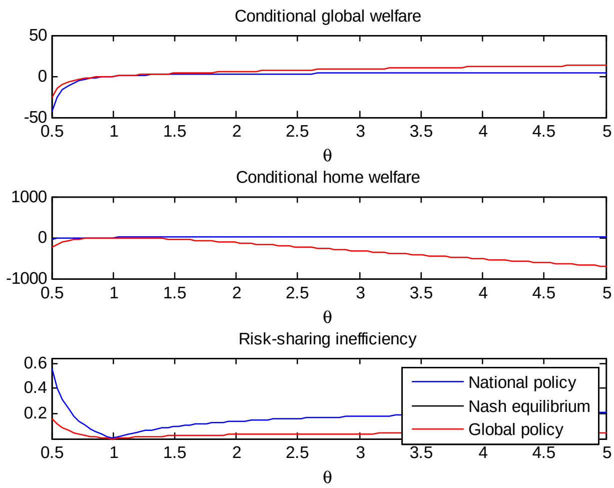

Figure 8: Risk-sharing, national and global welfare under the national policy and global policy.

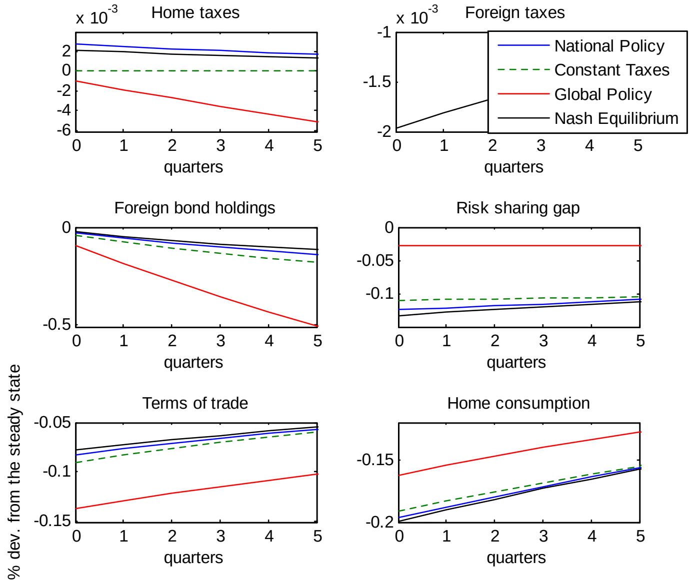

Figure 9: Optimal global and national policy following a negative Home productivity shock with $\theta = 3$: comparison with the Nash equilibrium.

and the case of constant taxes (green line). Note that when illustrating the global optimal policy we assume that there is only one policy instrument available to the global social planner. Adding τ_t^* as an additional tool would not change the economic dynamics since it would affect the same margin – namely the cross-border risk-sharing condition (or a combination of Equations 19 and 20) – and, with only one instrument, optimal global policy already implies zero volatility in the variable measuring deviations from full risk sharing after the initial period (see Figures 9).

Following a negative productivity shock, capital flows from Foreign to Home ($B_F < 0$). But instead of subsidizing such flow, Home taxes the capital inflow (as it reduces the domestic incentive to borrow). At the same time, the negative taxes in the Foreign country decrease returns to lenders, working as a tax on capital outflows from the Foreign country. Both policies, at home and abroad, contribute to reducing the flow of capital between countries. Domestic terms of trade are weaker under the Nash equilibrium in a period of low productivity at Home – consistent with lower cross-border risk-sharing.

Figure **??** already showed that the incentives of the Home economy to deviate from the socially optimal policy (i.e. the difference between Home welfare under the national

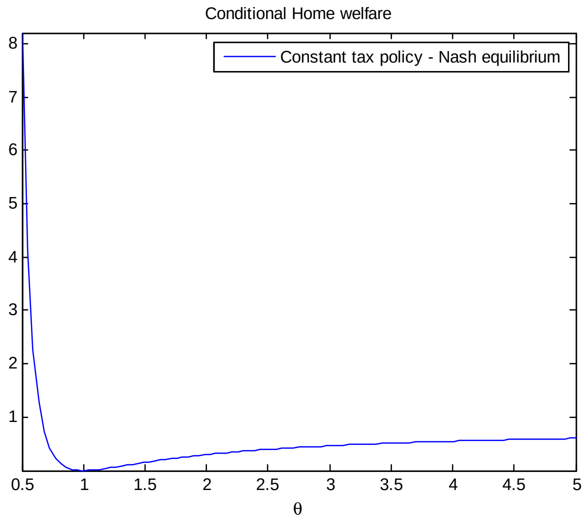

Figure 10: Difference in conditional Home welfare delivered under constant taxes and the Nash equilibrium.

policy and under the global policy) are the largest exactly when the losses from unilateral decision making (i.e. the difference between global welfare under the national policy and under the global policy) are the biggest. Moreover, if countries simultaneously and independently engage in such interventions in the international flow of capital, individual as well as global welfare would be adversely affected – as illustrated by the fact that Home welfare is smaller in the Nash equilibrium when compared with the constant tax policy (see Figure 10).

Our findings (see Figure 11) highlight that there is an important role for international coordination in how capital controls are set in different countries. The gains from international coordination, measured as the difference in conditional global welfare delivered under the global optimal policy and the Nash equilibrium may be close to 20%. This is because the incentives of individual countries and the global policy makers are completely orthogonal when it comes to interventions in capital flows.

5 Concluding remarks

In this paper, we analyze the effect of capital controls on domestic and world welfare. We show that countries incentive to limit cross-border flow of capital damages international

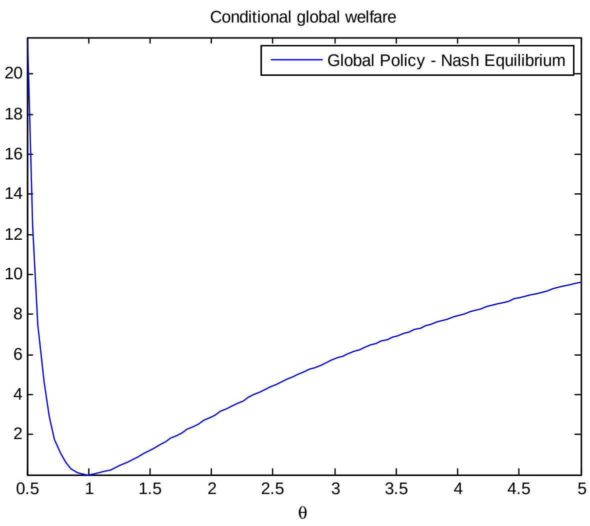

Figure 11: Gains from cooperation: difference in conditional global welfare delivered under the global optimal policy and the Nash equilibrium.

risk sharing. Such uncoordinated use of capital controls is beggar-thy-neighbor and, thus, there is a clear role for international coordination.

Our proposed model is stylized. This allows us to keep the welfare and policy analysis parsimonious and clear. Nevertheless, to quantify the real gains from international coordination, a richer model may be required. Early works in the literature have shown that the level of risk sharing in incomplete market models (where agents can trade bonds) can be quite large. As shown in CDL, frameworks like ours may need to feature near-permanent shocks and possibly a distribution sector (that introduces significant deviations from the law of one price) in order to generate an insufficient level of risk-sharing that matches the data. A fruitful avenue for this research may be to enrich the model in these directions and move from a qualitative to a quantitative analysis of the effects of capital controls.

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6 Appendix: Optimal policy problem

6.1 Derivation of first order conditions: National optimal policy, two-country model

Period utility function

$$W = \ln C_t - A_t^{-\eta} \frac{Y_t^{\eta+1}}{\eta+1} \tag{22}$$

Structural equations:

1. Home demand equation

$$Y_t P_{H,t}^{\theta} = \nu C_t + \frac{(1-n)\nu^*}{n} C_t^* Q_t^{\theta} \tag{23}$$

2. Foreign demand equation

$$Y_t^* P_{F,t}^{\theta} = \frac{n(1-\nu)}{1-n} C_t + (1-\nu^*) C_t^* Q_t^{\theta} \tag{24}$$

3. Home labor supply

$$P_{H,t} C_t^{-\rho} = \left(\frac{Y_t}{A_t}\right)^{\eta} \tag{25}$$

4. Foreign labor supply

$$\frac{P_{F,t}}{Q_t} C_t^{*-\rho} = \left(\frac{Y_t^*}{A_t^*}\right)^{\eta} \tag{26}$$

5. Relative prices (1)

$$P_{H,t}^{\theta-1} = \nu + (1-\nu)\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\theta} \tag{27}$$

6. Relative prices (2)

$$\left(\frac{P_{H,t}}{Q_t}\right)^{\theta-1} = \nu^* + (1-\nu^*)\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\theta} \tag{28}$$

7. Euler equation (1)

$$R_t^* = \frac{1}{\beta} E_t\left(\frac{{C_{t+1}^*}^{\rho}}{C_t^{*\rho}}\right) \tag{29}$$

8. Euler equation (2)

$$R_t^*(1+\tau_t) = \frac{1}{\beta} E_t\left(\frac{{C_{t+1}}^{\rho} Q_t}{{C_t}^{\rho} Q_{t+1}}\right)(1+\delta_f Bf_{h,t}) \tag{30}$$

9. Budget constraint

$$P_{H,t}Y_t + Bf_{h,t-1}R_{t-1}^*\frac{Q_t}{Q_{t-1}} = C_t + Bf_{h,t} + \frac{1}{2}\delta_f Bf_{h,t}^2 \tag{31}$$

First order conditions:

- wrt Y_t

$$-A_t^{-\eta}Y_t^{\eta} + P_{H,t}^{\theta}\gamma_{1,t} - \eta\gamma_{3,t}A_t^{-\eta}Y_t^{\eta-1} + \gamma_{9,t}P_{H,t} = 0$$

- wrt Y_t^*

$$P_{F,t}^{\theta}\gamma_{2,t} - \eta\gamma_{4,t}A_t^{*-\eta}Y_t^{*\eta-1} = 0$$

- wrt $P_{H,t}$

$$\theta P_{H,t}^{\theta-1} Y_t \gamma_{1,t} + C_t^{-\rho}\gamma_{3,t} + \gamma_{5,t}(\theta-1)P_{H,t}^{\theta-2} - (1-\nu)(\theta-1)P_{F,t}^{1-\theta}P_{H,t}^{\theta-2}\gamma_{5,t}$$
$$+ (\theta-1)P_{H,t}^{\theta-2}Q_t^{1-\theta}\gamma_{6,t} - (1-\nu^*)(\theta-1)P_{F,t}^{1-\theta}P_{H,t}^{\theta-2}\gamma_{6,t} + \gamma_9 Y_t$$
$$= 0$$

- wrt $P_{F,t}$

$$\gamma_{2,t}Y_t^*\theta P_{F,t}^{\theta-1} + \frac{C_t^{*-\rho}}{Q_t}\gamma_{4,t} - (1-\nu)P_{H,t}^{\theta-1}(1-\theta)P_{F,t}^{-\theta}\gamma_{5,t} - (1-\nu^*)P_{H,t}^{\theta-1}(1-\theta)P_{F,t}^{-\theta}\gamma_{6,t} = 0$$

- wrt Q_t

$$-\theta\frac{(1-n)\nu^*}{n}C_t^*Q_t^{\theta-1}\gamma_{1,t}$$
$$-\theta(1-\nu^*)C_t^*Q_t^{\theta-1}\gamma_{2,t} - P_{F,t}C_t^{*-\rho}Q_t^{-2}\gamma_{4,t}$$
$$+\gamma_{6,t}P_{H,t}^{\theta-1}(1-\theta)Q_t^{-\theta}$$
$$-E_t\left(\frac{1}{\beta}\frac{{C_{t+1}}^{\rho}}{{C_t}^{\rho}Q_{t+1}}\right)(1+\delta_f Bf_{h,t})\gamma_{8,t}$$
$$+\frac{1}{\beta^2}\frac{{C_t}^{\rho}Q_{t-1}}{{C_{t-1}}^{\rho}Q_t^2}(1+\delta_f Bf_{h,t-1})\gamma_{8,t-1} + \gamma_{9,t}Bf_{h,t-1}R_{t-1}^*\frac{1}{Q_{t-1}} - \beta E_t(\gamma_{9,t+1}Q_{t+1})Bf_{h,t}R_t^*\frac{1}{Q_t^2}$$
$$= 0$$

- wrt C_t

$$\frac{1}{C_t} - \nu\gamma_{1,t} - \frac{n(1-\nu)}{1-n}\gamma_{2,t}$$
$$-\rho\gamma_{3,t}P_{H,t}C_t^{-\rho-1}$$
$$+\rho E_t\left(\frac{1}{\beta}\frac{{C_{t+1}}^{\rho}Q_t}{{C_t}^{\rho+1}Q_{t+1}}\right)(1+\delta_f Bf_{h,t})\gamma_{8,t}$$
$$-\rho\frac{1}{\beta^2}\frac{{C_t}^{\rho-1}Q_{t-1}}{{C_{t-1}}^{\rho}Q_t}(1+\delta_f Bf_{h,t-1})\gamma_{8,t-1} - \gamma_{9,t}$$
$$= 0$$

- C_t^*

$$
\begin{aligned}
&-(1-n)\frac{\nu^*}{n}Q_t^\theta\gamma_{1,t}-(1-\nu^*)Q_t^\theta\gamma_{2,t}-\rho\gamma_{4,t}\frac{P_{F,t}}{Q_t}C_t^{*-\rho-1}\\
&+\rho\frac{1}{\beta}E_t\left(C_{t+1}^*\right)^\rho C_t^{*-\rho-1}\gamma_{7,t}-\frac{1}{\beta^2}\rho C_t^{*\rho-1}{C_{t-1}^*}^{-\rho}\gamma_{7,t-1}\\
&=0
\end{aligned}
$$

- wrt R_t^*

$$(1+\tau_t)\gamma_{8,t}+\gamma_{7,t}+\beta Bf_{h,t}E_t\left(\frac{Q_{t+1}}{Q_t}\gamma_{9,t+1}\right)=0$$

- wrt $Bf_{h,t}$

$$-\gamma_{8,t}E_t\left(\frac{1}{\beta}\frac{{C_{t+1}}^\rho Q_t}{{C_t}^\rho Q_{t+1}}\right)\delta_f+\beta E_t\left(\gamma_{9,t+1}R_t^*\frac{Q_{t+1}}{Q_t}\right)-\gamma_{9,t}(1+\delta_f Bf_{h,t})=0$$

- wrt τ_t

$$\gamma_{8,t}R_t^*=0$$

6.2 Derivation of first order conditions: Small open economy optimal policy

Period utility function

$$W_{soe}=\left(\ln C_t-A_t^{-\eta}\frac{Y_t^{\eta+1}}{\eta+1}\right) \tag{32}$$

Structural equations:

1. Home demand equation

$$Y_tP_{H,t}^\theta=\nu C_t+\frac{(1-n)\nu^*}{n}C_t^*Q_t^\theta \tag{33}$$

2. Home labor supply

$$P_{H,t}C_t^{-\rho}=\left(\frac{Y_t}{A_t}\right)^\eta \tag{34}$$

3. Relative prices

$$Q_t = \left(\frac{1-(1-\lambda)P_{H,t}^{1-\theta}}{\lambda} \right)^{\frac{1}{1-\theta}} \tag{35}$$

4. Euler equation

$$R_t^*(1+\tau_t) = \frac{1}{\beta} E_t \left(\frac{{C_{t+1}}^{\rho} Q_t}{{C_t}^{\rho} Q_{t+1}} \right) (1+\delta_f Bf_{h,t}) \tag{36}$$

5. Budget constraint

$$P_{H,t}Y_t + Bf_{h,t-1}R_{t-1}^* \frac{Q_t}{Q_{t-1}} = C_t + Bf_{h,t} + \frac{1}{2}\delta_f Bf_{h,t}^2 \tag{37}$$

First order conditions:

- wrt Y_t

$$-A_t^{-\eta}Y_t^{\eta} + P_{H,t}^{\theta}\gamma_{1,t} - \eta\gamma_{2,t}A_t^{-\eta}Y_t^{\eta-1} + \gamma_{5,t}P_{H,t} = 0$$

- wrt $P_{H,t}$

$$\theta P_{H,t}^{\theta-1}Y_t\gamma_{1,t} + C_t^{-\rho}\gamma_{2,t} + \left(\frac{1-(1-\lambda)P_{H,t}^{1-\theta}}{\lambda} \right)^{\frac{1}{1-\theta}-1} P_{H,t}^{-\theta}(\frac{1}{\lambda}-1)\gamma_{3,t} + \gamma_{5,t}Y_t = 0$$

- wrt Q_t

$$\begin{aligned}
&-\theta\lambda C_t^* Q_t^{\theta-1}\gamma_{1,t} + \gamma_{3,t} - \frac{1}{\beta}E_t \left(\frac{{C_{t+1}}^{\rho}}{{C_t}^{\rho}Q_{t+1}} \right)(1+\delta_f Bf_{h,t})\gamma_{4,t} \\
&+ \frac{1}{\beta^2}\frac{{C_t}^{\rho}Q_{t-1}}{{C_{t-1}}^{\rho}Q_t^2}(1+\delta_f Bf_{h,t-1})\gamma_{4,t-1} + \gamma_{5,t}Bf_{h,t-1}R_{t-1}^*\frac{1}{Q_{t-1}} - \beta E_t \left(\gamma_{5,t+1}Bf_{h,t}R_t^*\frac{Q_{t+1}}{Q_t^2} \right) \\
&= 0
\end{aligned}$$

- wrt C_t

$$\frac{1}{C_t} - (1-\lambda)\gamma_{1,t} - \rho\gamma_{2,t}P_{H,t}C_t^{-\rho-1} + \rho\frac{1}{\beta}E_t\left(\frac{{C_{t+1}}^{\rho}Q_t}{{C_t}^{\rho+1}Q_{t+1}}\right)(1+\delta_f Bf_{h,t})\gamma_{4,t}$$
$$-\rho\frac{1}{\beta^2}\frac{{C_t}^{\rho-1}Q_{t-1}}{{C_{t-1}}^{\rho}Q_t}(1+\delta_f Bf_{h,t-1})\gamma_{4,t-1} - \gamma_{5,t}$$
$$= 0$$

- wrt $Bf_{h,t}$

$$-\gamma_{4,t}\frac{1}{\beta}E_t\left(\frac{{C_{t+1}}^{\rho}Q_t}{{C_t}^{\rho}Q_{t+1}}\right)\delta_f + \beta E_t\left(\gamma_{5,t+1}R_t^*\frac{Q_{t+1}}{Q_t}\right) - \gamma_{5,t}(1+\delta_f Bf_{h,t}) = 0$$

- wrt τ_t

$$\gamma_{4,t}R_t^* = 0$$

6.3 Derivation of first order conditions: Global optimal policy, two-country model

Period utility function

$$W_g = n\left(\ln C_t - A_t^{-\eta}\frac{Y_t^{\eta+1}}{\eta+1}\right) + (1-n)\left(\ln C_t^* - A_t^{*-\eta}\frac{Y_t^{*\eta+1}}{\eta+1}\right) \tag{38}$$

Structural equations:

1. Home demand equation

$$Y_t P_{H,t}^{\theta} = \nu C_t + \frac{(1-n)\nu^*}{n}C_t^* Q_t^{\theta} \tag{39}$$

2. Foreign demand equation

$$Y_t^* P_{F,t}^{\theta} = \frac{n(1-\nu)}{1-n}C_t + (1-\nu^*)C_t^* Q_t^{\theta} \tag{40}$$

3. Home labor supply

$$P_{H,t}C_t^{-\rho} = \left(\frac{Y_t}{A_t}\right)^{\eta} \tag{41}$$

4. Foreign labor supply

$$\frac{P_{F,t}}{Q_t}C_t^{*-\rho} = \left(\frac{Y_t^*}{A_t^*}\right)^{\eta} \tag{42}$$

5. Relative prices (1)

$$P_{H,t}^{\theta-1} = \nu + (1-\nu)\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\theta} \tag{43}$$

6. Relative prices (2)

$$\left(\frac{P_{H,t}}{Q_t}\right)^{\theta-1} = \nu^* + (1-\nu^*)\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\theta} \tag{44}$$

7. Euler equation (1)

$$R_t^* = \frac{1}{\beta}E_t\left(\frac{{C_{t+1}^*}^{\rho}}{{C_t^*}^{\rho}}\right) \tag{45}$$

8. Euler equation (2)

$$R_t^*(1+\tau_t) = \frac{1}{\beta}E_t\left(\frac{{C_{t+1}}^{\rho}Q_t}{{C_t}^{\rho}Q_{t+1}}\right)(1+\delta_f Bf_{h,t}) \tag{46}$$

9. Budget constraint

$$P_{H,t}Y_t + Bf_{h,t-1}R_{t-1}^*\frac{Q_t}{Q_{t-1}} = C_t + Bf_{h,t} + \frac{1}{2}\delta_f Bf_{h,t}^2 \tag{47}$$

First order conditions (global policy):

- wrt Y_t

$$-nA_t^{-\eta}Y_t^{\eta} + P_{H,t}^{\theta}\gamma_{1,t} - \eta\gamma_{3,t}A_t^{-\eta}Y_t^{\eta-1} + \gamma_{9,t}P_{H,t} = 0$$

- wrt Y_t^*

$$-(1-n)A_t^{*-\eta}Y_t^{*\eta} + P_{F,t}^{\theta}\gamma_{2,t} - \eta\gamma_{4,t}A_t^{*-\eta}Y_t^{*\eta-1} = 0$$

- wrt $P_{H,t}$

$$\begin{aligned}
&\theta P_{H,t}^{\theta-1}Y_t\gamma_{1,t} + C_t^{-\rho}\gamma_{3,t} + \gamma_{5,t}(\theta-1)P_{H,t}^{\theta-2} - (1-\nu)(\theta-1)P_{F,t}^{1-\theta}P_{H,t}^{\theta-2}\gamma_{5,t} \\
&+ (\theta-1)P_{H,t}^{\theta-2}Q_t^{1-\theta}\gamma_{6,t} - (1-\nu^*)(\theta-1)P_{F,t}^{1-\theta}P_{H,t}^{\theta-2}\gamma_{6,t} + \gamma_9 Y_t \\
&= 0
\end{aligned}$$

- wrt $P_{F,t}$

$$\gamma_{2,t}Y_t^*\theta P_{F,t}^{\theta-1} + \frac{C_t^{*-\rho}}{Q_t}\gamma_{4,t} - (1-\nu)P_{H,t}^{\theta-1}(1-\theta)P_{F,t}^{-\theta}\gamma_{5,t} - (1-\nu^*)P_{H,t}^{\theta-1}(1-\theta)P_{F,t}^{-\theta}\gamma_{6,t} = 0$$

- wrt Q_t

$$\begin{aligned}
&-\theta\frac{(1-n)\nu^*}{n}C_t^*Q_t^{\theta-1}\gamma_{1,t} \\
&-\theta(1-\nu^*)C_t^*Q_t^{\theta-1}\gamma_{2,t} - P_{F,t}C_t^{*-\rho}Q_t^{-2}\gamma_{4,t} + \gamma_{6,t}P_{H,t}^{\theta-1}(1-\theta)Q_t^{-\theta} \\
&-\frac{1}{\beta}E_t\left(\frac{{C_{t+1}}^{\rho}}{{C_t}^{\rho}Q_{t+1}}\right)(1+\delta_f Bf_{h,t})\gamma_{8,t} \\
&+\frac{1}{\beta^2}\frac{{C_t}^{\rho}Q_{t-1}}{{C_{t-1}}^{\rho}Q_t^2}(1+\delta_f Bf_{h,t-1})\gamma_{8,t-1} + \gamma_{9,t}Bf_{h,t-1}R_{t-1}^*\frac{1}{Q_{t-1}} - \beta E_t\left(\gamma_{9,t+1}Bf_{h,t}R_t^*\frac{Q_{t+1}}{Q_t^2}\right) \\
&= 0
\end{aligned}$$

- wrt C_t

$$\begin{aligned}&\frac{n}{C_t} - \nu\gamma_{1,t} - \frac{n(1-\nu)}{1-n}\gamma_{2,t} - \rho\gamma_{3,t}P_{H,t}C_t^{-\rho-1}\\&+\rho\frac{1}{\beta}E_t\left(\frac{{C_{t+1}}^{\rho}Q_t}{{C_t}^{\rho+1}Q_{t+1}}\right)(1+\delta_f Bf_{h,t})\gamma_{8,t}\\&-\rho\frac{1}{\beta^2}\frac{{C_t}^{\rho-1}Q_{t-1}}{{C_{t-1}}^{\rho}Q_t}(1+\delta_f Bf_{h,t-1})\gamma_{8,t-1} - \gamma_{9,t}\\&=0\end{aligned}$$

- C_t^*

$$\begin{aligned}&\frac{1-n}{C_t^*} - (1-n)\frac{\nu^*}{n}Q_t^{\theta}\gamma_{1,t} - (1-\nu^*)Q_t^{\theta}\gamma_{2,t} - \rho\gamma_{4,t}\frac{P_{F,t}}{Q_t}C_t^{*-\rho-1}\\&+\rho\frac{1}{\beta}E_t\left(C_{t+1}^*\right)^{\rho}C_t^{*-\rho-1}\gamma_{7,t} - \frac{1}{\beta^2}\rho C_t^{*\rho-1}{C_{t-1}^*}^{-\rho}\gamma_{7,t-1}\\&=0\end{aligned}$$

- wrt R_t^*

$$(1+\tau_t)\gamma_{8,t} + \gamma_{7,t} + \beta Bf_{h,t}E_t\left(\frac{Q_{t+1}}{Q_t}\gamma_{9,t+1}\right) = 0$$

- wrt $Bf_{h,t}$

$$-\gamma_{8,t}\frac{1}{\beta}E_t\left(\frac{{C_{t+1}}^{\rho}Q_t}{{C_t}^{\rho}Q_{t+1}}\right)\delta_f + \beta E_t\left(\gamma_{9,t+1}R_t^*\frac{Q_{t+1}}{Q_t}\right) - \gamma_{9,t}(1+\delta_f Bf_{h,t}) = 0$$

- wrt τ_t

$$\gamma_{8,t}R_t^* = 0$$

6.4 Nash equilibrium in a two-country world

Structural equations:

1. Home demand equation

$$Y_t P_{H,t}^{\theta} = \nu C_t + \frac{(1-n)\nu^*}{n}C_t^* Q_t^{\theta} \tag{48}$$

2. Foreign demand equation

$$Y_t^* P_{F,t}^{\theta} = \frac{n(1-\nu)}{1-n} C_t + (1-\nu^*) C_t^* Q_t^{\theta} \tag{49}$$

3. Home labor supply

$$P_{H,t} C_t^{-\rho} = \left(\frac{Y_t}{A_t}\right)^{\eta} \tag{50}$$

4. Foreign labor supply

$$\frac{P_{F,t}}{Q_t} C_t^{*-\rho} = \left(\frac{Y_t^*}{A_t^*}\right)^{\eta} \tag{51}$$

5. Relative prices (1)

$$P_{H,t}^{\theta-1} = \nu + (1-\nu)\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\theta} \tag{52}$$

6. Relative prices (2)

$$\left(\frac{P_{H,t}}{Q_t}\right)^{\theta-1} = \nu^* + (1-\nu^*)\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\theta} \tag{53}$$

7. Euler equation (1)

$$R_t^*(1+\tau_t^*) = E_t\left(\frac{1}{\beta}\frac{{C_{t+1}^*}^{\rho}}{C_t^{*\rho}}\right) \tag{54}$$

8. Euler equation (2)

$$R_t^*(1+\tau_t) = \frac{1}{\beta} E_t\left(\frac{{C_{t+1}}^{\rho} Q_t}{{C_t}^{\rho} Q_{t+1}}\right)(1+\delta_f Bf_{h,t}) \tag{55}$$

9. Budget constraint

$$P_{H,t} Y_t + Bf_{h,t-1} R_{t-1}^* \frac{Q_t}{Q_{t-1}} = C_t + Bf_{h,t} + \frac{1}{2}\delta_f Bf_{h,t}^2 \tag{56}$$

Home first order conditions (almost the same as national policy):

Period utility function

$$W = \ln C_t - A_t^{-\eta}\frac{Y_t^{\eta+1}}{\eta+1} \tag{57}$$

First order conditions:

- wrt Y_t

$$-A_t^{-\eta}Y_t^{\eta} + P_{H,t}^{\theta}\gamma_{1,t} - \eta\gamma_{3,t}A_t^{-\eta}Y_t^{\eta-1} + \gamma_{9,t}P_{H,t} = 0$$

- wrt Y_t^*

$$P_{F,t}^{\theta}\gamma_{2,t} - \eta\gamma_{4,t}A_t^{*-\eta}Y_t^{*\eta-1} = 0$$

- wrt $P_{H,t}$

$$\theta P_{H,t}^{\theta-1}Y_t\gamma_{1,t} + C_t^{-\rho}\gamma_{3,t} + \gamma_{5,t}(\theta-1)P_{H,t}^{\theta-2} - (1-\nu)(\theta-1)P_{F,t}^{1-\theta}P_{H,t}^{\theta-2}\gamma_{5,t}$$
$$+(\theta-1)P_{H,t}^{\theta-2}Q_t^{1-\theta}\gamma_{6,t} - (1-\nu^*)(\theta-1)P_{F,t}^{1-\theta}P_{H,t}^{\theta-2}\gamma_{6,t} + \gamma_9 Y_t = 0$$

- wrt $P_{F,t}$

$$\gamma_{2,t}Y_t^*\theta P_{F,t}^{\theta-1} + \frac{C_t^{*-\rho}}{Q_t}\gamma_{4,t} - (1-\nu)P_{H,t}^{\theta-1}(1-\theta)P_{F,t}^{-\theta}\gamma_{5,t} - (1-\nu^*)P_{H,t}^{\theta-1}(1-\theta)P_{F,t}^{-\theta}\gamma_{6,t} = 0$$

- wrt Q_t

$$\begin{aligned}
&-\theta\frac{(1-n)\nu^*}{n}C_t^*Q_t^{\theta-1}\gamma_{1,t}\\
&-\theta(1-\nu^*)C_t^*Q_t^{\theta-1}\gamma_{2,t} - P_{F,t}C_t^{*-\rho}Q_t^{-2}\gamma_{4,t}\\
&+\gamma_{6,t}P_{H,t}^{\theta-1}(1-\theta)Q_t^{-\theta}\\
&-\frac{1}{\beta}E_t\left(\frac{{C_{t+1}}^{\rho}}{{C_t}^{\rho}Q_{t+1}}\right)(1+\delta_f Bf_{h,t})\gamma_{8,t}\\
&+\frac{1}{\beta^2}\frac{{C_t}^{\rho}Q_{t-1}}{{C_{t-1}}^{\rho}Q_t^2}(1+\delta_f Bf_{h,t-1})\gamma_{8,t-1} + \gamma_{9,t}Bf_{h,t-1}R_{t-1}^*\frac{1}{Q_{t-1}} - \beta E_t\left(\gamma_{9,t+1}Bf_{h,t}R_t^*\frac{Q_{t+1}}{Q_t^2}\right)\\
&=0
\end{aligned}$$

- wrt C_t

$$
\begin{aligned}
&\frac{1}{C_t} - \nu\gamma_{1,t} - \frac{n(1-\nu)}{1-n}\gamma_{2,t} \\
&- \rho\gamma_{3,t}P_{H,t}C_t^{-\rho-1} \\
&+ \rho\frac{1}{\beta}E_t\left(\frac{{C_{t+1}}^{\rho}Q_t}{{C_t}^{\rho+1}Q_{t+1}}\right)(1+\delta_f Bf_{h,t})\gamma_{8,t} \\
&- \rho\frac{1}{\beta^2}\frac{{C_t}^{\rho-1}Q_{t-1}}{{C_{t-1}}^{\rho}Q_t}(1+\delta_f Bf_{h,t-1})\gamma_{8,t-1} - \gamma_{9,t} \\
&= 0
\end{aligned}
$$

- C_t^*

$$
\begin{aligned}
&-(1-n)\frac{\nu^*}{n}Q_t^{\theta}\gamma_{1,t} - (1-\nu^*)Q_t^{\theta}\gamma_{2,t} - \rho\gamma_{4,t}\frac{P_{F,t}}{Q_t}C_t^{*-\rho-1} \\
&+ \rho\frac{1}{\beta}RE_t({C_{t+1}^*}^{\rho})C_t^{*-\rho-1}\gamma_{7,t} - \frac{1}{\beta^2}\rho C_t^{*\rho-1}{C_{t-1}^*}^{-\rho}\gamma_{7,t-1} \\
&= 0
\end{aligned}
$$

- wrt R_t^*

$$
(1+\tau_t)\gamma_{8,t} + (1+\tau_t^*)\gamma_{7,t} + \beta E_t\left(Bf_{h,t}\frac{Q_{t+1}}{Q_t}\gamma_{9,t+1}\right) = 0
$$

- wrt $Bf_{h,t}$

$$
-\gamma_{8,t}\frac{1}{\beta}E_t\left(\frac{{C_{t+1}}^{\rho}Q_t}{{C_t}^{\rho}Q_{t+1}}\right)\delta_f + \beta E_t\left(\gamma_{9,t+1}R_t^*\frac{Q_{t+1}}{Q_t}\right) - \gamma_{9,t}(1+\delta_f Bf_{h,t}) = 0
$$

- wrt τ_t

$$
\gamma_{8,t}R_t^* = 0
$$

First order conditions (foreign policy)

Period utility function

$$
W = \ln C_t^* - A_t^{*-\eta}\frac{Y_t^{*\eta+1}}{\eta+1} \tag{58}
$$

First order conditions:

- wrt Y_t

$$P_{H,t}^{\theta}\gamma_{1,t}^{*} - \eta\gamma_{3,t}^{*}A_t^{-\eta}Y_t^{\eta-1} + \gamma_{9,t}^{*}P_{H,t} = 0$$

- wrt Y_t^*

$$-A_t^{*-\eta}Y_t^{*\eta} + P_{F,t}^{\theta}\gamma_{2,t}^{*} - \eta\gamma_{4,t}^{*}A_t^{*-\eta}Y_t^{*\eta-1} = 0$$

- wrt $P_{H,t}$

$$\begin{aligned}
&\theta P_{H,t}^{\theta-1}Y_t\gamma_{1,t}^{*} + C_t^{-\rho}\gamma_{3,t}^{*} + \gamma_{5,t}^{*}(\theta-1)P_{H,t}^{\theta-2} - (1-\nu)(\theta-1)P_{F,t}^{1-\theta}P_{H,t}^{\theta-2}\gamma_{5,t}^{*} \\
&+ (\theta-1)P_{H,t}^{\theta-2}Q_t^{1-\theta}\gamma_{6,t}^{*} - (1-\nu^*)(\theta-1)P_{F,t}^{1-\theta}P_{H,t}^{\theta-2}\gamma_{6,t}^{*} + \gamma_{9,t}^{*}Y_t \\
&= 0
\end{aligned}$$

- wrt $P_{F,t}$

$$\gamma_{2,t}^{*}Y_t^{*}\theta P_{F,t}^{\theta-1} + \frac{C_t^{*-\rho}}{Q_t}\gamma_{4,t}^{*} - (1-\nu)P_{H,t}^{\theta-1}(1-\theta)P_{F,t}^{-\theta}\gamma_{5,t}^{*} - (1-\nu^*)P_{H,t}^{\theta-1}(1-\theta)P_{F,t}^{-\theta}\gamma_{6,t}^{*} = 0$$

- wrt Q_t

$$\begin{aligned}
&-\theta\frac{(1-n)\nu^*}{n}C_t^{*}Q_t^{\theta-1}\gamma_{1,t}^{*} \\
&-\theta(1-\nu^*)C_t^{*}Q_t^{\theta-1}\gamma_{2,t}^{*} - P_{F,t}C_t^{*-\rho}Q_t^{-2}\gamma_{4,t}^{*} \\
&+\gamma_{6,t}^{*}P_{H,t}^{\theta-1}(1-\theta)Q_t^{-\theta} \\
&-\frac{1}{\beta}E_t\left(\frac{{C_{t+1}}^{\rho}}{{C_t}^{\rho}Q_{t+1}}\right)(1+\delta_f Bf_{h,t})\gamma_{8,t}^{*} \\
&+\frac{1}{\beta^2}\frac{{C_t}^{\rho}Q_{t-1}}{{C_{t-1}}^{\rho}Q_t^2}(1+\delta_f Bf_{h,t-1})\gamma_{8,t-1}^{*} + \gamma_{9,t}^{*}Bf_{h,t-1}R_{t-1}^{*}\frac{1}{Q_{t-1}} - \beta E_t\left(\gamma_{9,t+1}^{*}Bf_{h,t}R_t^{*}\frac{Q_{t+1}}{Q_t^2}\right) \\
&= 0
\end{aligned}$$

- wrt C_t

$$
\begin{aligned}
&-\nu\gamma^*_{1,t} - \frac{n(1-\nu)}{1-n}\gamma^*_{2,t} \\
&-\rho\gamma^*_{3,t}P_{H,t}C_t^{-\rho-1} \\
&+\rho\frac{1}{\beta}E_t\left(\frac{{C_{t+1}}^{\rho}Q_t}{{C_t}^{\rho+1}Q_{t+1}}\right)(1+\delta_f Bf_{h,t})\gamma^*_{8,t} \\
&-\rho\frac{1}{\beta^2}\frac{{C_t}^{\rho-1}Q_{t-1}}{{C_{t-1}}^{\rho}Q_t}(1+\delta_f Bf_{h,t-1})\gamma^*_{8,t-1} - \gamma^*_{9,t} \\
&= 0
\end{aligned}
$$

- C_t^*

$$
\begin{aligned}
&\frac{1}{C_t^*} - (1-n)\frac{\nu^*}{n}Q_t^\theta\gamma^*_{1,t} - (1-\nu^*)Q_t^\theta\gamma^*_{2,t} - \rho\gamma^*_{4,t}\frac{P_{F,t}}{Q_t}C_t^{*-\rho-1} \\
&+\rho\frac{1}{\beta}E_t({C^*_{t+1}}^{\rho})C_t^{*-\rho-1}\gamma^*_{7,t} - \frac{1}{\beta^2}\rho {C_t^*}^{\rho-1}{C^*_{t-1}}^{-\rho}\gamma^*_{7,t-1} \\
&= 0
\end{aligned}
$$

- wrt R_t^*

$$
(1+\tau_t)\gamma^*_{8,t} + (1+\tau_t^*)\gamma^*_{7,t} + \beta Bf_{h,t}E_t\left(\frac{Q_{t+1}}{Q_t}\gamma^*_{9,t+1}\right) = 0
$$

- wrt $Bf_{h,t}$

$$
-\gamma^*_{8,t}\frac{1}{\beta}E_t\left(\frac{{C_{t+1}}^{\rho}Q_t}{{C_t}^{\rho}Q_{t+1}}\right)\delta_f + \beta E_t\left(\gamma^*_{9,t+1}R_t^*\frac{Q_{t+1}}{Q_t}\right) - \gamma^*_{9,t}(1+\delta_f Bf_{h,t}) = 0
$$

- wrt τ_t^*

$$
\gamma^*_{7,t}R_t^* = 0
$$