IMF Working Paper

Financial Crises in DSGE Models:
A Prototype Model

Jaromir Benes, Michael Kumhof, and Douglas Laxton

IMF Working Paper

Research Department

**Financial Crises in DSGE Models:
A Prototype Model**

Prepared by Jaromir Benes, Michael Kumhof, and Douglas Laxton

Authorized for distribution by Douglas Laxton

April 2014

Abstract

This paper presents the theoretical structure of MAPMOD, a new IMF model designed
to study vulnerabilities associated with excessive credit expansions, and to support
macroprudential policy analysis. In MAPMOD, bank loans create purchasing power
that facilitates adjustments in the real economy. But excessively large and risky loans
can impair balance sheets and sow the seeds of a financial crisis. Banks respond to
losses through higher spreads and rapid credit cutbacks, with adverse effects for the real
economy. These features allow the model to capture the basic facts of financial cycles.
A companion paper studies the simulation properties of MAPMOD.

Contents   Page

I. INTRODUCTION

This paper discusses the theoretical structure of MAPMOD, a new model that has been developed at the IMF to support macrofinancial and macroprudential policy analysis. A companion paper (Benes, Kumhof, and Laxton, 2014) presents the model's simulation properties. MAPMOD has been designed specifically to study vulnerabilities associated with excessive credit expansions and asset price bubbles, and the consequences of different macroprudential policies that attempt to guard against or cope with such vulnerabilities.

As has been emphasized in a number of recent theoretical and empirical studies by the world's leading policy institutions (see for example Macroeconomic Assessment Group, 2010), the critical macroprudential policy tradeoff is between reducing the risks of very costly financial crises and minimizing the costs of macroprudential policies during normal times. It is therefore crucial to design analytical frameworks that clearly articulate the role of the financial sector and of macroprudential policies. We argue that such new analytical frameworks require a major revamp of the conventional linear dynamic stochastic general equilibrium (DSGE) models that, in the period before the financial crisis of 2007/8, had been designed for conventional monetary policy analysis (Borio, 2012). Some progress has been made, but much work remains to be done. In our view, an area that requires particular attention is the special role played by banks, most importantly the role of bank balance sheets.[1] This includes, as we will discuss in much more detail in the next section, the role of bank equity in absorbing lending losses, the role of bank loans in creating new purchasing power to finance consumption and investment (both real and financial), and the role of bank deposits as the economy's principal medium of exchange, with all of these subject to balance sheet risks that generate highly nonlinear feedback between bank balance sheets, borrower balance sheets and the real economy during financial crises.

MAPMOD does feature banks and bank balance sheets that play these fundamental roles, and the globally nonlinear version of the model allows to capture the basic stylized facts of both the pre-crisis and crisis phases of financial cycles. Claessens, Ayhan, and Terrones (2011) and Borio (2012) suggest that the financial cycle can be described parsimoniously in terms of credit and property prices, with rapid growth in these variables providing an important early warning indicator of potential future financial crises, and with very painful

---

[1]Many of the recent DSGE models with financial frictions abstract from bank balance sheets altogether by modeling all lending as direct, others (the majority) feature trivial bank balance sheets that require no net worth because all lending risk is diversifiable, and the remaining small group of models that does feature bank equity models banks not as lenders subject to lending risk, but as investors subject to price risk.

recessions and slow recoveries being associated with large contractions in credit and house price busts. However, Claessens, Ayhan, and Terrones (2011) also find that not all credit expansions are followed by financial crises. Furthermore, it can be very difficult to distinguish fundamentally sound ("good") and excessive ("bad") credit expansions and asset price bubbles in real time. MAPMOD is designed to study the critical differences between good and bad credit expansions. Moreover, it also allow us to study alternative macroprudential policies, including not only their role in dealing with the immediate aftermath of a crisis, but also their role in preventing a crisis from occurring in the first place, for example by making it unattractive for banks to let credit grow too fast or too far.

The remainder of this paper is organized as follows. Section II provides an overview of our modeling philosophy and the key design features of MAPMOD. Section III discusses the three most important aspects of these design features in greater depth. Section IV describes some important aspects of the notation that is used to describe the model. Section V introduces the first building blocks of the model, including the notions of aggregate credit risk and of capital adequacy regulation. Section VI provides a complete description of the model. Section VII discusses a number of extensions. Section VIII concludes. Furthermore, Appendix A shows derivations of some key equations, and Appendix B provides a glossary of variables.

II. MODELING PHILOSOPHY AND KEY DESIGN FEATURES OF MAPMOD

During financial crises we observe major deviations in the behavior of agents and in macroeconomic variables from what prevails during normal times. Specifically, the economic mechanisms become inherently nonlinear when subjected to large distress events, a point emphasized by many authors, such as Milne (2009), and furthermore there can be vicious interactions between asset prices, bank lending conditions and the real economy that magnify such effects (Borio, 2012). Bank balance sheets play a critical role in such interactions. Conventional linearized DSGE models are not very useful for evaluating macroprudential policy tradeoffs under such conditions, first because by construction they do not capture the effects of nonlinearities, and second because they ignore the special role played by banks in contributing to vulnerabilities and nonlinearities. Banks, especially if left unregulated, can fundamentally change the economic propagation mechanism, through their response to standard demand and supply shocks that emanate from outside the banking system. But in addition, banks can themselves become an important source

of shocks, for example by setting lending terms that reflect overly optimistic expectations concerning growth prospects, borrower riskiness or asset prices.

Given the existence of fundamental uncertainty about the nature and persistence of the underlying shocks, and therefore about the sustainability of existing lending practices, models can provide an important framework for assessing alternative policies, and ensuring that these policies are reasonably robust to such uncertainty, and based on the existing state of knowledge. This paper presents a prototype model that has been specifically designed to support macrofinancial and macroprudential policy analysis. As such, it assigns a central role to banks, and it incorporates important endogenous and nonlinear feedback mechanisms between bank balance sheets, borrower balance sheets and the real economy. The role of banks in this model differs in several fundamental ways from the way in which banks are conceived in existing DSGE models. Below we provide a partial list of these differences. Section III discusses the three major differences in much more detail.

First, banks maintain a stock of net worth that enables them to absorb loan losses. They do so for a number of reasons: because of Basel-style minimum capital adequacy regulation, because acquiring additional net worth directly from the equity markets is subject to frictions, and because bank lending is subject not only to diversifiable borrower-specific idiosyncratic risk, but also to non-diversifiable aggregate risk that makes lending inherently and endogenously risky.

Second, banks' determination of the price and quantity of loans, and their maintenance of capital buffers above minimum requirements, arise as an optimal equilibrium phenomenon resulting from the interactions between loan contracts, endogenous loan losses and regulation.

Third, in the process of making new loans, commercial banks create matching liabilities (bank deposits) for their borrowers, thereby expanding their balance sheets. In so doing, banks are limited only by their perceptions of profitability and by the risk absorption capacity of their capital.[2] The main implication of the credit creation process is that bank loans give borrowers new purchasing power that *did not previously exist.* This is highly beneficial during periods of strong economic fundamentals, when it is essential that banks provide the purchasing power that the economy needs to allow consumption, investment and real wages to grow in line with the economy's potential. Macroprudential policies that

---

[2]Unlike in the loanable funds model, the decisions by bankers to make loans are not constrained by an available supply of pre-existing saving, or by central bank reserves, but rather by expectations of return and risk, and their interactions with prudential regulation insofar as it affects the return and risk of lending.

attempt to prevent a credit expansion under such circumstances can therefore be costly. But the very same flexibility can at times result in an excessively large and risky loan book, especially when risk becomes underpriced. As emphasized by Borio (2013), this can make the balance sheets of both banks and their borrowers very vulnerable to shocks, thereby sowing the seeds of a financial crisis that may happen many years later.

Fourth, banks respond to financial shocks, and the resulting balance sheet dislocations, through a combination of higher spreads and non-price credit rationing. With bank credit rapidly shrinking during financial distress, households and firms are cut off from one of their principal sources of financing exactly when they need it the most.

Fifth, during severe financial crises vicious and highly nonlinear feedback effects between borrower balance sheets, banks balance sheets, and the real economy characterize the economy.

There are two such nonlinear feedback effects. First, lending losses can lead to a serious erosion in banks' capital adequacy taking them close to, or even below, their regulatory capital minimum, where penalties start to apply. This triggers a very rapid contraction in lending to immediately move banks out of that danger zone. It is accompanied by higher lending spreads, as banks attempt to replenish their equity buffers so as to move more durably away from that danger zone, on the basis of higher equity rather than reduced lending. But, second, lending losses are also a reflection of the fact that the balance sheets of banks' borrowers have become far more vulnerable, due to declines in the value of their assets, with the resulting steep increases in loan-to-value ratios providing another reason for reduced lending volumes and higher lending spreads. The resulting nonlinear responses of lending volumes and lending spreads act as a strong amplifying mechanism that further exacerbates the balance sheet problems of bank borrowers, and therefore of banks themselves. The adjustment to such shocks can therefore be very protracted, and very costly. Due to the nature of their operations and regulatory environment, banks impose tighter financial conditions precisely at the time when the real economy would benefit from more countercyclical lending (Borio (2012)). This then of course directly leads to an important policy implication, namely the need for macroprudential policy to encourage banks to adopt a more countercyclical stance.[3]

We emphasize that the painful contractions that accompany financial cycles in MAPMOD, and in the real world, are not only a reflection of deteriorating fundamentals themselves,

---

[3]Boissay, Collard and Smets (2013) have emphasized the important role of nonlinearities and liquidity in generating financial cycles in a DSGE model.

such as large downward revisions of expected future economic growth rates, as would be the case in conventional monetary business cycle models. Nor are they only a function of deteriorating bank-specific fundamentals, such as reductions in bank borrowers' credit-worthiness, although this does of course play an important role. Rather, the severity of financial crises stems to a very large extent from the fact that, following downward shocks to economic fundamentals, banks endogenously become more vulnerable. The reason is that the leverage of banks and their borrowers, which will have grown in response to previous exaggerated expectations concerning future growth and future lending risk, can quickly expose banks to nonlinear contractionary effects when expectations are revised downwards and when, consequently, lending losses occur.

In modern financial systems banks are not constrained on the margin by a pre-existing level of deposits, but have the ability to expand both sides of their balance sheets simultaneously, by making loans and generating demand deposits. This means that their lending, and provision of purchasing power to the economy, can expand and shrink at a much faster rate than traditional models would suggest. But, given uncertainty about the underlying creditworthiness of borrowers, this flexibility can be a double-edged sword. If banks correctly anticipate stronger fundamentals, the decision to make loans and generate purchasing power can result in a good credit expansion that helps to facilitate adjustment in the real economy. However, if these decisions are not in line with fundamentals, they can create vulnerabilities and a potential financial cycle, with an associated real contraction that can be very severe. MAPMOD is designed with precisely these features at its core.

Finally, it is helpful to contrast some basic aspects of macroprudential policy analysis with those of traditional monetary policy analysis. We see five main differences, all of which are reflected in the design of MAPMOD. First, monetary policy works over regular business cycles, while macroprudential policy deals with macrofinancial cycles, which are typically longer and much more asymmetric. Second, the main focus of monetary policy analysis is on producing the most likely future projections of key macroeconomic variables, accompanied by an assessment of risks. Macroprudential policy is concerned with not-so-likely yet plausible stress scenarios and an assessment of the economy's tipping points. Third, flow variables and prices dominate monetary policy analysis, while balance sheets, stock-flow relationships, and aggregate risk dominate macroprudential policy analysis. Fourth, monetary policy in normal times can be thought of as a linear-quadratic optimal control problem. Macroprudential policy needs to be addressed as a highly nonlinear robust control problem; in other words, as policy designed to avert catastrophic scenarios. Fifth, monetary policy is, in regular times, characterized by fairly stable tradeoffs that can,

at least to some extent, be empirically quantified. Estimation and other empirical methods are a useful and important input into desigiating and parameterizing monetary policy tools. The essence of macroprudential policy lies in global nonlinearities arising when the economy is subjected to large distress. Macroprudential policymakers therefore face far more uncertainty, and this cannot be resolved by empirical methods in a straightforward manner. Judgment and simulation-based validation play irreplaceable roles.

III. DETAILED DISCUSSION OF THREE KEY DESIGN FEATURES

Section II provided an overview of the key features involved in the design of MAPMOD, insofar as they differ from all or most other DSGE models in the current literature. The discussion in Section II is largely identical to that presented in the non-technical companion paper. Because this paper, starting in Section IV, deals with a number of deeper technical issues involved in building appropriate macrofinancial frameworks, this section provides a more in-depth discussion of three key features of MAPMOD.

A. Bank Credit Creation

Macrofinancial vulnerabilities arise as a result of large and risky balance sheet exposures. Put very simply, we can measure these vulnerabilities as the product of two factors, the size of gross balance sheets and the degree of overall risk. Modeling both of these factors correctly is therefore critical for macroprudential policy analysis. We discuss the nature and sources of macrofinancial risk in the next subsection. In this subsection we describe how gross positions are created, how their size is linked to the rest of the macroeconomic environment, and more specifically, what role different bank assets and bank liabilities play in the broader macroeconomy. The traditional and so far exclusive view in macro models sees banks as intermediaries channeling loanable funds from depositors/savers to borrowers/investors. In this environment, bank deposits are savings, i.e. real resources set aside through, for example, additional work effort or reduced consumption, and subsequently lent out to borrowers in the form of bank loans. However, this view, in a very fundamental way, fails to capture the essence of how assets and liabilities are created (and destroyed). We therefore argue that models based on this view necessarily underestimate both the *size* of bank balance sheets and (more importantly) the *flexibility* with which banks can inflate or deflate their balance sheets, and thereby expand or contract credit to the economy. By imposing constraints that do not exist in the real world, namely that the quantity of bank

deposits, and hence also of bank loans, is limited by the amount of real savings available in the economy, such models almost inevitably produce misleading policy advice. Disyatat and Borio (2011) make a very similar point.

What, therefore, is the correct model of bank lending? Most importantly, banks do not lend real savings that they receive *beforehand* from depositors. Rather, upon the approval of a loan application, a bank creates *new* journal entries consisting of a new asset (the loan granted) and a new deposit (the loan disbursed). The new deposit is not a *real* resource withdrawn from some other use, such as foregone consumption. Instead it is new *financial* purchasing power created along with the loan. New bank deposits are therefore not created by a process of saving but by a process of financing, a key distinction also emphasized by Disyatat and Borio (2011). The only way in which aggregate deposits can grow is therefore through the extension of additional bank loans, and the only way in which aggregate deposits can shrink is through the repayment of some existing loans.[4]

Why is a correct model of the process of bank credit creation so important for a correct model of the overall financial system, and therefore also of macroprudential policy? Most importantly, the correct model implies that bank credit represents financing, which means that its expansion is not limited by the prior availability of deposits or saving. Saving needs to be accumulated through a process of either producing additional resources, or foregoing consumption of existing resources, and the speed of such a process is subject to clear physical limitations. The same is not true for financing, which can be created through offsetting financial gross positions on bank balance sheets that require essentially no resources. And banks are in the business of financing; they are issuers of the economy's medium of exchange through lending, and not the intermediary of the economy's real resources. Real resources are bought and sold *outside* the banking sector using the medium of exchange. Bank balance sheets are therefore very elastic, as banks have the ability to expand them on demand. The saver-investor (or depositor-borrower) distinction is therefore, in a theoretical model, not needed to motivate the existence of a banking sector. A much better assumption is that of representative agents who, in a world where bank deposits are the predominant medium of exchange, would simultaneously demand bank loans and bank deposits because the former are the means of creating the latter, and because bank deposits enable them to carry out their spending plans. We do not deny the existence of savers and investors in the real world, or their contribution to the determination of mac-

---

[4]Note also that when a deposit is transferred from one bank to another, the total consolidated amount of bank deposits held by the non-financial sector does not change. The transaction is settled through interbank positions and positions with the central bank.

roeconomic variables like real interest rates. We simply maintain that banks do not engage in a direct middleman role of conveying real loanable funds from savers to investors. Non-bank financial institutions come closer to this role, but even they do not intermediate real resources, they intermediate purchasing power that has to first be created by banks. Intermediaries of real resources, of the kind envisaged by the loanable funds model, simply do not exist in the real world. If they did, banking would be much safer by design, because such agents would have much greater difficulty in rapidly expanding or contracting their balance sheets.

From a modeling point of view, the above implies that there are two critical questions that need to be addressed to correctly portray the functioning of a banking sector and its balance sheets. First, what is the economic function of bank deposits for households and firms? Second, what are the true limits on the expansion of bank balance sheets in the real world?

To answer the first question, bank assets and liabilities need to be associated in the model with demand for *financing*, not with an act of *saving*. Saving is a change in real *net* worth. By contrast, financing and bank balance sheets are concepts intrinsically related to *gross* financial positions (assets and liabilities). Consequently, the amount of aggregate saving cannot be a factor that directly limits the size of bank balance sheets. The modeling of gross positions requires either the assumption of a representative household, as in MAP-MOD, or of depositors and borrowers who can trade both financial and nonfinancial assets among themselves (see Jakab and Kumhof, 2014, for details). And finally, the demand for the medium of exchange that is created through financing needs to be associated with the ability of households or firms to carry out their spending plans.

As for the second question, in the real world there are only two relevant constraints on the size of bank balance sheets. The first is the demand by non-banks for the economy's medium of exchange, which determines whether the demand for loans expands, thereby adding to the medium exchange if the loans are approved, or whether loans are repaid, thereby reducing the quantity of the medium of exchange. The second constraint is the expected implications of new lending for banks' profitability, solvency and capitalization, which among other factors are affected by prudential regulation. For this constraint to exist, banks must be able to make profits or losses, with the latter potentially putting their solvency at risk. This in turn requires that banks be exposed to some risks that are not diversifiable. We turn to the existence of nondiversifiable, or aggregate, risk in the next subsection.

B. Aggregate Risk

The notion of bank balance sheet exposures to nondiversifiable risks is at the very heart of macroprudential policy analysis. Some portion of the risks on the balance sheets of both financial and non-financial agents cannot be diversified even in large and granular portfolios. This is because in the real world markets are incomplete, and hedging, insurance, or state-contingent contracts are simply not available for a majority of states of the macroeconomy. While the existence of nondiversified balance sheet risk is a well-acknowledged fact, it is difficult to formalize in a model, especially in general equilibrium. This seems also to be one of the reasons behind the fact that many macroeconomic models with macrofinancial extensions only focus on the implications of risk that is diversifiable by means of optimal state-contingent contracts, such as the classical model of the financial accelerator of Bernanke, Gertler, and Gilchrist (1999), or the more recent model of inefficient credit booms of Lorenzoni (2008).

The exclusion of aggregate risk removes the most interesting and most fundamental element from models meant for macroprudential policy analysis, and is at odds with observed reality. Furthermore, not only is it critical to preserve nondiversifiable risk in the model, but also to let it be determined endogenously by the real economy, jointly with the optimal leverage ratio, individual lending and borrowing behavior, optimal spreads, and the probability of default.

We include aggregate risk in MAPMOD. In the model, unexpected lending losses, due to non-diversifiable aggregate risk, are absorbed by bank net worth, and the risk absorption capacity of bank net worth is the most important factor in the expansion of bank balance sheets. With large enough portfolio losses, banks may experience significant capital shortfalls. Since such shortfalls are costly, banks tend to maintain regulatory capital buffers in excess of the regulatory minimum, while the size of the buffers varies with macroeconomic and financial conditions.

C. Global Nonlinearities

The goal of macroprudential policies is to strengthen the resilience of the overall economy to large balance sheet crises, while accepting that such policies can have smaller but ongoing costs during normal times. Crises exhibit significant deviations in the behavior of agents and of macroeconomic variables from what prevails during normal times. In

other words, the economic mechanisms become inherently nonlinear when subjected to large distress, and the nonlinearities are greatly magnified by various types of macro-financial feedback. Macroprudential policy models therefore need to be very carefully designed to have the ability to depict such real-world distress nonlinearities. Otherwise, they cannot possibly be useful in correctly evaluating the underlying policy trade-offs. Local behavior around the long-run equilibrium of an economy tells us very little about what is going to happen in times when vulnerabilities do materialize. We therefore argue that attempts to conduct macroprudential policy on the basis of models solved by local approximation methods, and estimated on regular business cycle data, are likely to result in major biases, misleading advice, and potentially very painful policy errors. By the same token, attempts to address macroprudential policy as an optimal control problem (let alone a linear-quadratic one) will move most of policymakers' attention to fine-tuning over the regular business cycle dominated by local dynamics, while missing out the broad picture of the possible build-up in macrofinancial risk and its globally nonlinear implications. Such an approach will give a false impression of full control and safety. We prefer to think of macroprudential policy as a robust control problem of minimizing the impact of very bad scenarios at some reasonable cost paid during normal times. This is also the view adopted by the BIS in several recent studies of macroprudential policies, such as Macro-economic Assessment Group (2010). In agreement with the robust control approach is the now commonly accepted principle of pro-active macroprudential policy design. That is, create safety buffers during the macrofinancial upswings, and stand by to release them quickly in case things go wrong. This will be reflected in some of the macroprudential policy designs that we study in this paper.

IV. NOTATION

In this short section, we explain some notational conventions we follow throughout the derivation of the model:

- All interest rates are are expressed as *gross* rates of interest.

- Time subscripts are used so as to comply with the underlying information sets. The realization of a variable indexed by t is observed (and becomes part of everyone's information set) at time t. A choice variable indexed by t is decided upon at time t. This means, for instance, that (state non-contingent) lending rates accruing between

t and $t+1$ are denoted by $R_{L,t}$, while return on bank equity during the same period of time is denoted by $R_{E,t+1}$ since the uncertainty therein is only resolved at time $t+1$.

- Variables not internalized (taken as given) by the respective agent are marked with a bar, as e.g. in the habit reference level in consumption, $\bar{C}_{t-1}$.

- There is, in general, no necessary connection between a variable denoted by an upper-case letter and another one denoted by the respective lower-case letter. Because of the large number of variables and intermediate results, we take the liberty to use lower-case and upper-case letters independently.

- Steady-state levels of variables are denoted by the respective letter with no time subscript; e.g. R is the steady-state level of R_t.

To greatly economize on notation, we also use the following simplified way to model monopolistic competition with price and wage adjustment costs in labor and goods markets, respectively. Instead of continua of differentiated households or retailers, we assume the existence of a single representative agent in either market who faces a downward sloping demand curve. The locus of the demand curve, determined by the aggregate price and quantity, is not internalized by the optimizing agent but set equal to its individual counterpart in symmetric equilibrium. It is worth noting that the results are identical to those from a model where Dixit-Stiglitz CES indexes would enter the production and utility functions.

Finally, we use several types of adjustments costs to produce realistic dynamic properties of the model. We are only interested in the first-order dynamic effects that the adjustment costs have on the optimal choice of the model agents. We therefore make simplifications by dropping higher-order terms relating to adjustment costs.

V. Aggregate Credit Risk and Capital Regulation

s

In this section, we introduce several building blocks that are used to introduce the notion of aggregate credit risk and capital regulation in our model:

- The asymptotic single risk factor framework.

- Loan portfolio value theory.

- Capital regulation as an incentive-based mechanism.

- Imperfections in external capital flows.

Since these building blocks are often unfamiliar to the existing macroeconomics litera-
ture and modeling practice, we recommend that the readers familiarize themselves with
the theoretical arguments developed in this section before proceeding to the full model
described in Sections VI and VII.

Most importantly, the four building blocks give rise to an endogenous lending spread con-
sisting of two components, related loosely speaking to individual credit risk and the risk
of a regulatory capital shortfall. The lending spread plays a critical role in the feedback
mechanisms between banks and the real economy, and is a major source of nonlinear-
ities in the model. Since the lending spread responds to the leverage of both banks and
households, including foreign debt leverage, and affects household consumption, it has
the power to close a small open economy model in the sense that it uniquely determines
the foreign debt to GDP ratio in the long run. No additional mechanism, such as one of
those discussed by Schmitt-Grohé and Uribe (2003), is needed to close the model.

A. Asymptotic Single Risk Factor Model

The theoretical foundations of bank credit risk in our model derive from the asymptotic
single risk factor (ASRF) framework. The ASRF model can be viewed as the conceptual
framework underlying the internal risk based approach defined in Basel II, as illustrated
by Gordy (2000, 2003). We first explain how credit risk and defaults arise in a simple ver-
sion of the ASRF model, and describe the probabilistic assumptions about individual de-
faults. These results will be used subsequently to calculate default ratios within an entire
portfolio of bank loans, and to make a connection between defaults and macroeconomic
variables.

The main idea of the ASRF model is that the percentage of defaults (or, in general, the total
loss) in a large homogenous portfolio of state non-contingent exposures is ex-ante un-
certain, but this uncertainty can be analytically described if it exists as a result of a single

common systematic risk factor—in other words, if the occurrence of each individual default is driven by a combination of some idiosyncratic factors (that can be fully diversified away in a large enough portfolio) and a single aggregate factor common to all exposures (that cannot be diversified). The aggregate, or systematic, risk factor can be thought of as a variable, or a scalar combination of a number of variables, describing the state of the general macroeconomy. Its distribution is assumed to be known or can be, for instance, inferred from an empirical model. How limiting it to assume a single aggregate factor (albeit a composite index of many macroeconomic variables)? Tarashev and Zhu (2007) show that the possible errors arising from imposing this assumption are much *smaller* than other sources of errors, mainly those associated with parameter uncertainty.

We use the following simple variant of the ASRF model. Each bank holds a portfolio of one-period loans extended to a large number, n, of non-financial borrowers indexed by $i = 1, \ldots, n$. We denote the representative portfolio by L_t, and the individual exposures within the portfolio by L_t^i, where $L_t = \sum_{i=1}^n L_t^i$. At time t, the bank and borrower i agree on the amount, L_t^i, and a non-contingent nominal gross rate of interest, $R_{L,t}^i$.[5] At the time of repayment, $t+1$, some obligors may default on their loans.[6] The default is an event occurring under specific circumstances, which are obviously rather complex in the real world, and can be only captured on a very stylized level in formal models.

Along with taking a bank loan, each borrower also holds a certain amount of tradable assets. In our model, it is claims to physical capital employed in production.[7] The value of borrower i's holdings of physical capital at time t is $P_{K,t} K_t^i$. At the beginning of time $t+1$, the return on capital (including capital gains), $R_{K,t+1}$, is observed, and hence the value of the capital changes to $R_{K,t+1} P_{K,t} K_t^i$. The borrower defaults on the loan if the ex-post value of her capital falls below a stochastic threshold based on the amount owed,

$$
R_{K,t+1} P_{K,t} K_t^i < \frac{R_{L,t}^i L_t^i}{\kappa \exp u_{t+1}^i}, \tag{1}
$$

where κ is a constant (whose role is explained in more depth when we parameterize the model), and u_{t+1}^i is a random variable that captures the effect of all possible idiosyncratic factors underlying an individual default. The idiosyncratic factor is assumed to have a known distribution (identical for all i's) and be independent of any aggregate quantities.

---

[5]Non-contingent means that the lending rate is a fixed number determined at time t and does not change in response to aggregate or individual outcomes observed at time $t+1$.

[6]Since the duration of the loans is one period only, we do not make a distinction between default, non-performance, or delinquency, and use the terms interchangeably in the paper.

[7]We use the term "physical" capital to make a clear distinction from bank capital introduced later.

Taking the logarithm of both sides of the inequality (1), we re-write the default condition in more compact form leading to a simple discrete-time version of Merton (1974). Concentrating all random quantities unknown at the time of extending the loan on the left-hand side, and the quantities known at time t on the right-hand side, we get

$$r_{t+1}^i < \hat{r}_t^i, \tag{2}$$

where

$$\overbrace{r_{t+1}^i}^{\text{individual}} = \overbrace{r_{t+1}}^{\text{aggregate}} + \overbrace{u_{t+1}^i}^{\text{idiosyncratic}}, \tag{3}$$

is an overall individual risk factor, a random quantity consisting of the idiosyncratic risk factor u_{t+1}^i (whose realizations differ for different individual borrowers) and an aggregate—or systematic— risk factor, the log of the aggregate return on physical capital, $r_{t+1} = \log R_{K,t+1}$ (whose realization is common to all individuals), The right-hand side of (2),

$$\hat{r}_t^i = \log R_{L,t}^i L_t^i - \log \kappa P_{K,t} K_t^i,$$

is an individual default cut-off point determined entirely at time t and observed by both the borrower and the bank. Note that the default cut-off point involves an individual loan-to-value ratio, $L_t^i / P_{K,t} K_t^i$.

To formally describe the state of borrower i and her loan at time $t+1$, we introduce a Bernoulli variable H_{t+1}^i, that takes the value 0 if the loan performs, and 1 if the borrower defaults,

$$H_{t+1}^i = \begin{cases} 0 & \text{(loan } i \text{ performs)} & \text{if } r_{t+1}^i \geq \hat{r}_t^i, \\ 1 & \text{(default on loan } i\text{)} & \text{if } r_{t+1}^i < \hat{r}_t^i. \end{cases}$$

Furthermore, we denote the proportion of defaults in an entire bank loan portfolio, called the portfolio default ratio, by H_{t+1} (dropping the superscript i). The ratio is given by

$$H_{t+1} = \sum_{i=1}^n \frac{L_t^i}{L_t} H_{t+1}^i \in [0, 1].$$

The focal point of the ASRF theory is the notion of unconditional and conditional probabilities of default. To derive these, we first need to make probabilistic assumptions about the underlying risk factors. In our model, both of the risk factors, the aggregate one and the

idiosyncratic one, are distributed normally,

$$r_{t+1} \sim N\left(\mathbb{E}_t\left[\log R_{K,t+1}\right], \varsigma\sqrt{\varrho}\right), \tag{4a}$$

$$u_{t+1}^i \sim N\left(0, \varsigma\sqrt{1-\varrho}\right), \tag{4b}$$

and independent of each other. The overall individual risk factor is thus distributed as

$$r_{t+1}^i \sim N\left(\mathbb{E}_t\left[\log R_{K,t+1}\right], \varsigma\right), \tag{5}$$

where both $\varsigma > 0$ and $\varrho > 0$. The mean of the aggregate risk factor, $\mathbb{E}_t[r_{t+1}]$, is determined by the aggregate behavior of the model economy: $\mathbb{E}_t\left[\log R_{K,t+1}\right]$ denotes the model-consistent expectation of the log of the return on physical capital conditional upon time t information. The standard deviations of the two risk factors, $\varsigma\sqrt{\varrho}$ and $\varsigma\sqrt{1-\varrho}$, respectively, are treated parametrically.

The key implication of this probabilistic structure is that each pair of individual risk factors, r_{t+1}^i and r_{t+1}^j ($\forall i \neq j$), is cross-correlated owing to the presence of the aggregate factor, and the coefficient of correlation is ϱ. As will become clear later, the non-zero cross-correlation of individual risk factors results in an inability of banks to fully diversify away all risk: the aggregate factor renders some portion of the risk on bank balance sheets non-diversifiable.

The *unconditional*[8] probability of individual default is the expectation of H_{t+1}^i. Since H_{t+1}^i is a Bernoulli variable, its expectation is simply given by the probability of $H_{t+1}^i = 1$. From our definition of H_{t+1}^i, this probability identically equals the probability of the overall individual risk factor falling below the default cut-off point, $r_{t+1}^i < \hat{r}_t^i$. With the distribution of r_{t+1}^i known and given by (5), we can write

$$\mathbb{E}_t\left[H_{t+1}^i\right] = \Pr\left(H_{t+1}^i = 1\right) = \Pr\left(r_{t+1}^i < \hat{r}_t^i\right) = \Phi\left(\frac{\hat{r}_t^i - \mathbb{E}_t[r_{t+1}]}{\varsigma}\right), \tag{6}$$

where $\Phi(\cdot)$ denotes a standardized normal c.d.f. For ease of notation, we define

$$p_t^i = \Phi\left(\frac{\hat{r}_t^i - \mathbb{E}_t[r_{t+1}]}{\varsigma}\right), \quad p_t^{i\prime} = \Phi'\left(\frac{\hat{r}_t^i - \mathbb{E}_t[r_{t+1}]}{\varsigma}\right), \tag{7}$$

to represent the respective functions on the right-hand side.

---

[8]In the ASRF framework, *unconditional* refers to the fact that the aggregate risk factor is unknown.

The *conditional* probability of individual default is the probability given a particular realization of the aggregate risk factor, r_{t+1}. It can be easily calculated from equation (3) and assumptions (4a) and (4b):

$$\mathbb{E}\left[H_{t+1}^i\big|r_{t+1}\right] = \Pr\left(r_{t+1}^i < \hat{r}_t^i\big|r_{t+1}\right) = \Phi\left(\frac{\hat{r}_t^i - r_{t+1}}{\varsigma\sqrt{1-\varrho}}\right) = \Phi\left(\frac{\Phi^{-1}(p_t^i) - \Phi^{-1}(q_{t+1})\sqrt{\varrho}}{\sqrt{1-\varrho}}\right), \quad (8)$$

where q_{t+1} in the last expression on the RHS is the percentile of the given realisation of the aggregate risk factor, r_{t+1},

$$q_{t+1} = \Phi\left(\frac{r_{t+1} - \mathbb{E}_t[r_{t+1}]}{\varsigma\sqrt{\varrho}}\right).$$

The details of deriving the conditional probability in (8) are provided in Appendix A.1.

B. Loan Portfolio Value Theory

We now turn to the loan portfolio as a whole, and following Vasicek (2002), we show how to calculate a portfolio default ratio. The asymptotic results that we state here are valid only under two regularity conditions, usually referred to as *portfolio homogeneity*:

1. (*Perfect granularity*) The portfolio must consist of a large number of individual exposures none of which is dominating. Formally, the results are valid for a limit case where

$$\sum_{i=1}^{n}\left(\frac{L_t^i}{L_t}\right)^2 \to 0.$$

2. (*Equal probability of default*) All exposures in the portfolio must have the same unconditional probabilities of default,

$$p_t^i = p_t^j = p_t, \quad \forall i,j = 1,\ldots,n.$$

As will become clear in the next sections, both conditions are satisfied in our model. First, the number of individuals taking bank loans is (infinitely) large by assumption. Second, the individuals are all identical ex-ante in symmetric equilibrium, thus making the same choices. The amounts borrowed are therefore equal, and the loan portfolio is not dominated by any of them.

We first observe that the law of large numbers implies the following:

- The unconditional expectation of the portfolio default ratio coincides with the unconditional probability of individual default,

$$\mathbb{E}_t \left[H_{t+1} \right] = \mathbb{E}_t \left[H_{t+1}^i \right] = p_t. \tag{9}$$

- The actual ex-post portfolio default ratio, H_{t+1}, i.e. the percentage of defaults in a homogenous portfolio for a given observation of the aggregate risk factor, r_{t+1}, equals the conditional probability of individual default

$$H_{t+1} | r_{t+1} = \mathbb{E}_t \left[H_{t+1}^i | r_{t+1} \right], \tag{10}$$

where the latter is given by (8).

These two results are based on the fact that all idiosyncratic risk can be, in theory, diversified fully away in a homogenous portfolio. With the aggregate risk factor fixed and known, the proportion of defaulted loans in such a portfolio is a deterministic number equal to the conditional probability of individual default given by (8). We will use this result to evaluate the actual ex-post losses facing banks in our model.

We are now ready to derive the ex-ante distribution function for the portfolio default ratio. Using equation (10) together with the probabilistic assumption about the aggregate risk factor, (4a), it can be shown the the c.d.f. for H_{t+1}, denoted by $\Pi(\cdot)$, is a function of only the unconditional probability of individual default, p_t (equal across the portfolio), and the pairwise cross-correlation of risk factors, ϱ, and is given by

$$\Pi_t(x) = \Pr\left(H_{t+1} < x\right) = \Phi\left(\frac{\Phi^{-1}(x)\sqrt{1-\varrho} - \Phi^{-1}(p_t)}{\sqrt{\varrho}}\right). \tag{11}$$

Note that we index the c.d.f. by t because of its dependence on the time-varying (endogenously determined) probability of default, p_t. Finally, by differentiating the right-hand side of (11), we can calculate the corresponding p.d.f.

$$\Pi'_t(x) = \sqrt{\frac{1-\varrho}{\varrho}} \; \Phi'\left(\frac{\Phi^{-1}(x)\sqrt{1-\varrho} - \Phi^{-1}(p_t)}{\sqrt{\varrho}}\right) \frac{1}{\Phi'\left(\Phi^{-1}(x)\right)}. \tag{12}$$

Further details of deriving these c.d.f. and p.d.f. are provided in Appendix A.2.

This completes the description of all the probabilistic characteristics of individual defaults and default ratios that are needed to derive the optimal behavior of banks and households in our model: (i) the unconditional probability of individual default and its relation to ag-

gregate variables determined by the rest of the model; (ii) the ex-post actual default ratio in a representative portfolio; and (iii) the distribution function for the ex-ante default ratio.

C. Bank Balance Sheets and Capital Regulation

We now outline the structure of bank balance sheets, and introduce capital regulation as an incentive-based mechanism, as proposed by Milne (2002). Bank loans and bank deposits are both one-period instruments. At the time of extending new loans and creating new deposits, the representative bank has the following balance sheet:

<div align="center">

Bank balance sheet

L_t Loans	Deposits d_t
	Bank capital E_t

</div>

The loan portfolio consists of a large number of loans to individual borrowers (members of the representative households), $L_t = \sum_{i=1}^{n} L_t^i$. The deposits include both deposits with residents (local households), D_t, and non-residents (rest of the world), F_t,

$$d_t = D_t + F_t,$$

We abstract from issues relating to the currency of denomination here, and treat these two types of deposits as perfect substitutes. The details of how we treat possible currency mismatches are postponed until later in this section.

At the beginning of time $t+1$ (before new lending and new deposit creation take place), the return on assets and the cost of liabilities are realized. We denote the ex-post realized values of the loans, deposits, and bank capital by LL_{t+1}, dd_{t+1}, and EE_{t+1}, respectively. The ex-post realized balance sheet looks as follows:

<div align="center">

Ex-post realized balance sheet

$LL_{t+1} = R_{L,t} L_t (1 - \lambda H_{t+1})$	$dd_{t+1} = R_t d_t$
	$EE_{t+1} = LL_{t+1} - dd_{t+1}$

</div>

The terms in the balance sheet are as follows:

- $R_{L,t}$ is the aggregate average gross rate of interest on new lending, defined by $R_{L,t} = \sum_{i=1}^{n} R_{L,t}^i L_t^i / L_t$.

- H_{t+1} is the portfolio default ratio defined in subsection V.A, $H_{t+1} = \sum_{i=1}^{n} H_{t+1}^{i} L_{t}^{i} / L_{t}$. The portfolio default ratio is the only source of uncertainty on the ex-post balance sheet.

- λ is the loss given default, i.e. the percentage of the amount owed on a defaulted loan that the bank is not able to recover. The loss given default is treated as a fixed parameter in our model.

- R_{t} is the deposit rate, and also the policy rate in our model, subject to no uncertainty.

- EE_{t+1} is the ex-post bank capital (equity) calculated residually as the difference between assets and non-equity liabilities. It is this ex-post equity that enters capital requirements defined below.

When choosing the size of its balance sheet, banks are limited by minimum capital adequacy ratios. The requirements apply to the *ex-post* value of equity such that bank capital must end up above a certain fraction φ of the ex-post realized value of assets. Banks with a shortfall in regulatory capital are liable to a penalty given by a fraction v of their ex-ante book value of assets:

$$\text{if } EE_{t+1} < \varphi LL_{t+1} \Rightarrow \text{penalty } vL_{t}.$$

The shortfall will occur whenever the portfolio default ratio, H_{t+1}, exceeds a critical, or cut-off, level, $\hat{H}_{t}$. The cut-off default ratio, known at time t with certainty, follows directly from the ex-post balance sheet; it is defined as the ratio for which

$$R_{L,t} L_{t} \left(1 - \lambda \hat{H}_{t}\right) - R_{t} d_{t} = \varphi R_{L,t} L_{t} \left(1 - \lambda \hat{H}_{t}\right).$$

By concentrating $\hat{H}_{t}$ on the left-hand side, we obtain

$$\hat{H}_{t} = \frac{1}{\lambda} \left[1 - \frac{R_{t} d_{t}}{(1 - \varphi) R_{L,t} L_{t}} \right] = \frac{1}{\lambda} \left[1 - \frac{R_{t}}{(1 - \varphi) R_{L,t}} \left(1 - \frac{E_{t}}{L_{t}} \right) \right], \tag{13}$$

where we substitute for d_{t} from the balance sheet identity in the last term. Because we know the probabilistic distribution of the default ratio, see equation (11), we can evaluate the probability of a regulatory capital shortfall for a given balance sheet. The probability is completely determined by quantities known at time t, and is given by $1 - \Pi_{t}\left(\hat{H}_{t}\right)$, i.e. one minus the value of the c.d.f. at the cut-off point. The expected value of the regulatory penalty associated with time t lending is given by $vL_{t} \cdot \left[1 - \Pi_{t}\left(\hat{H}_{t}\right)\right]$. This term will be used in the bank optimization problem to account for the ex-ante effect of capital adequacy regulation.

Finally, we define the actual ex-post return on bank equity, including the possible regula-
tory penalty, and a possible exogenous shock to the return (which can be interpreted e.g.
as a loss incurred on assets not included in the model balance sheets and unrelated to do-
mestic economic activity, such as valuation losses on foreign assets). We have

$$R_{E,t} = \frac{EE_t - G_t \cdot vL_{t-1}}{E_{t-1}} + \varepsilon_{E,t}, \tag{14}$$

where G_t is a Bernoulli variable describing the occurrence of a shortfall in regulatory capi-
tal experienced by the representative bank,

$$G_t = \begin{cases} 0 & \text{(complies with regulation)} & \text{if } EE_t \geq \varphi\, LL_t, \\ 1 & \text{(short of capital)} & \text{if } EE_t < \varphi\, LL_t, \end{cases}$$

and $\varepsilon_{E,t}$ is an exogenous component in the return on bank capital.

D. Imperfections in Equity Flows

External flows of bank capital (i.e. issuance of new equity in the market, or changes in div-
idend policies) are rather slow in their response to macrofinancial and macroeconomic
developments, both in times of balance sheet expansions and during banking crises. The
existing literature provides a wealth of economic reasons that give rise to such capital flow
imperfections, including informational costs, time delays in recapitalization, or strate-
gic behavior of banks; see Estrella (2004); Peura and Jokivuolle (2004); Peura and Keppo
(2006); Van den Heuvel (2006), and others. Importantly, the imperfections have three cru-
cial implications from the point of view of the scope of our paper:

1. Relative to assets and non-equity liabilities, book equity of banks (which is the rele-
   vant quantity for bank lending, as opposed to market capitalization) is fairly inelas-
   tic. Expansions and contractions in bank balance sheets are marked by large changes
   in plain, risk-unadjusted leverage ratios. In other words, the balance sheet cycles are
   not supported by inflows and outflows of capital, but rather by changes in the un-
   derlying risk, perceived or fundamental. This is an empirical finding documented by
   Adrian, Colla, and Shin (2013).

2. With bank recapitalization choices limited to internal sources (retained earnings)
   most of the time, capital requirements exerts much stronger influence over bank
   lending decisions and risk management. Were banks able to recapitalize themselves

from the market immediately and costlessly under a wide range of circumstances, capital regulation would see very little effect on their behavior.

3. With aggregate, non-diversifiable risk on their balance sheet, and with limited recapitalization options, banks choose to maintain regulatory capital buffers: they hold capital in excess of the minimum required by the regulator. The buffers vary over time as they respond to cycles in financial risk and other factors. This fact is documented in a number of empirical studies including, for instance, Jokipii and Milne (2008) or Milne and Wood (2009).

We introduce two layers of external capital flow imperfections. The first one is embedded in the very definition of our capital requirements. As the minimum capital adequacy policy applies to the ex-post balance sheet at the beginning of time $t + 1$, banks are effectively prevented from using any sources, external or internal, to replenish their capital at the very moment of calculating the regulatory ratio (before they start new lending).

The second layer of imperfections is added directly to the optimization problem of the representative bank. The bank faces an adjustment cost designed to express the idea that any deviations in ex-ante capital, either increases or decreases, from the level determined by retained earnings are costly. The cost is quite likely to be asymmetric in the real world. We nevertheless resort to a simple quadratic function in the basic specification of the model. The cost is expressed as a percentage of today's capital, E_t, itself,

$$E_t \cdot \tfrac{1}{2}\xi_E \left\{\log E_t - \log\left[(\bar{R}_{E,t} - \tau_E)\bar{E}_{t-1}\right]\right\}^2.$$
(15)

The terms of this expression have the following meaning:

- $R_{E,t}$ is the gross rate of return on last period's equity, E_{t-1}, defined by equation (14).

- $\tau_E \in (0,1)$ is a technical constant to guarantee a well behaved steady state where bank capital neither explodes nor implodes over time. The economic interpretation of the constant is a fixed dividend policy pursued by the bank.

- $\xi_E \in [0,\infty)$ is a cost parameter. In the extreme case with $\xi_E \to \infty$, banks can only recapitalize themselves from retained earnings.

- The bars over variables $(\bar{E}_t, \bar{R}_{E,t})$ mean that the respective quantities are taken as given and not internalized by the bank.

VI. BASIC SPECIFICATION OF THE MODEL

In this section, we present the basic specification of our model, building extensively on the macrofinancial concepts introduced in Section V. In the basic specification, we concentrate on the core mechanisms critical for understanding the banking-macro nexus. The basic specification is further extended and modified along several dimensions in the next section, where a number of practical, real-world extensions are introduced.

The model depicts a small open economy with a financial sector consisting of a representative competitive bank. The bank engages in extending loans and creating the corresponding deposits. It can therefore freely inflate or deflate its balance sheet. The only true limitation for the size of the bank balance sheet is, just as in the real world, the risk-bearing capacity of bank capital (equity). This mechanism is often termed a bank capital channel; see e.g. Van den Heuvel (2002). Two assumptions are essential for the channel to function. First, bank capital is subject to regulation in the form of capital requirements. Second, imperfections in equity markets and other possible costs associated with *external* capital flows prevent banks from acquiring fresh capital instantaneously and costlessly when they need it. Banks are therefore rendered more reliant on retained earnings and other sources of *internal* capital flows.

The economy is populated by a representative household consisting of a large number of individuals (household members). The household as a whole makes decisions on consumption, investment, labor, and deposits. The individual members choose bank loans (acting as individual borrowers) and claims on physical capital. The individual borrowers are identical ex-ante, but have different stochastic thresholds determining their defaults ex-post. Defaults are driven by both idiosyncratic and aggregate risk factors as detailed in Section V. The corresponding bank deposits created when loans are granted are a source of liquidity, or purchasing power, for the household to finance its outlays on consumption and investment, and its trade in claims on physical capital.

The real side of the economy is relatively standard in our model. The supply side combines domestic input factors, capital and labor, with intermediate imports to produce local goods. Some of the local goods are demanded for consumption or investment by the household, some are further processed in exporting industries. The market for local consumption and investment goods operates under monopolistic competition. The final prices, as well as nominal wages, are sticky. The country is a price-taker in international trade, with its terms of trade exogenous and driven by random persistent shocks.

Finally, access to international finance is restricted to banks in the basic specification. Households cannot issue any international debt or equity instruments, and their international transactions are cleared through bank-issued liabilities (deposits).

A. Banks

We split the bank decision-making process into two stages. In the first stage, the bank chooses the overall size of its balance sheet subject to capital requirements, taking the credit risk of individual exposures on the loan book as given. In the second stage, the bank designs its individual lending policy consistent with its first-stage decisions.

1. Optimal Size of Bank Balance Sheet

The bank chooses the size of its balance sheet, i.e. a homogenous loan portfolio, L_t, the volume of deposits, d_t, and bank capital, E_t, to maximize the ex-post shareholder value net of initial equity investment. This objective is designed in the same way as e.g. Van den Heuvel (2008). In the basic specification of the model, banks are fully owned by domestic households, and the cash flows are therefore evaluated using the representative household's shadow value of wealth. We, however, use a smaller discount parameter $\hat{\beta}$ in place of β for reasons explained later.

$$\max_{\{L_t, d_t, E_t\}} \mathbb{E}_t \left[\frac{\hat{\beta} \Lambda_{t+1}}{\Lambda_t} \left(\overbrace{R_{L,t} L_t (1 - \lambda H_{t+1}) - R_t d_t}^{\text{Gross earnings}} - \overbrace{v L_t \left[1 - \Pi_t \left(\hat{H}_t \right) \right]}^{\text{Regulatory penalty}} \right) \right]$$
$$- \underbrace{E_t}_{\text{Initial investment}} - \underbrace{E_t \cdot \tfrac{1}{2} \xi_E \Omega_{E,t}{}^2}_{\text{Adjustment cost}}, \quad (16)$$

subject to the ex-ante balance sheet identity, $L_t = d_t + E_t$. The terms are as follows:

- Λ_t is the representative household's shadow value of wealth, introduced in subsection VI.B.

- $R_{L,t}$ is the aggregate average lending rate defined in the ex-post balance sheet.

- L_t is the total value of bank lending, i.e. the size of a homogenous portfolio consisting of a large number of individual loans.

- λ is a loss-given-default parameter.

- H_{t+1} is the ex-post portfolio default ratio.

- R_t is the deposit rate, and at the same time the policy rate.

- d_t is the volume of deposits created.

- v is the regulatory penalty for capital shortfalls expressed as a percentage of the value of loans.

- Π_t is the c.d.f. for the portfolio default ratio, whose time dependence comes from the variation in the individual probability of default, p_t, as derived in subsection V.B, see equation (11).

- $\hat{H}_t$ is the cut-off portfolio default ratio, driving ex-post bank capital to the regulatory minimum.

- E_t is the ex-ante value of bank capital (equity). Note that capital requirements only work through ex-post bank capital, and are indifferent to its ex-ante value.

- The term $\frac{1}{2}\xi_E\,\Omega_{E,t}{}^2$ with $\Omega_{E,t} = \log E_t - \log(\bar{R}_{E,t} - \tau_E)\bar{E}_{t-1}$ quantifies the adjustment cost of bank capital, as explained in subsection V.D, equation (15), with last period's retained earnings not internalized by the bank.

Before describing the optimal choice, note first that when designing the optimization problem of banks, we abstract from limited liability. The reason is to keep our analysis significantly simpler by fully exploiting the fact that the *numerical* impact of limited liability is negligible in our simulation experiments presented in the companion paper; see also the discussion of the limited liability and probability of insolvency issues also in Milne (2002) and Peura and Jokivuolle (2004). An alternative interpretation is that we assign the representative bank a charter value that equals exactly the regulatory penalty whenever EE_t becomes negative ex-post. A charter value based analysis of bank capital can be found e.g. in Estrella (2004).

The optimality conditions for the problem (16) are derived as follows. We first substitute for d_t from the balance sheet identity, $d_t = L_t - E_t$. Then, we differentiate (16) w.r.t. L_t and E_t. The first-order condition for L_t is given by

$$L_t: \quad R_{L,t}\Big(1 - \lambda\mathbb{E}_t\left[H_{t+1}\right]\Big) \approx R_t + v\left[1 - \Pi_t(\hat{H}_t) + \frac{\Pi_t'(\hat{H}_t)\,R_t}{\lambda(1-\varphi)R_{L,t}}\cdot\frac{E_t}{L_t}\right]. \tag{17}$$

The $\approx$ sign indicates that we ignore higher-order stochastic interactions between the household's shadow value of wealth, Λ_{t+1}, and the portfolio default ratio, H_{t+1}, approximating the condition as if $\mathbb{E}_t[\Lambda_{t+1}H_{t+1}] = \mathbb{E}_t[\Lambda_{t+1}]\mathbb{E}_t[H_{t+1}]$. The equation says that a hypothetical risk-free lending rate on the left-hand side (i.e. the lending rate adjusted for aggregate credit risk) is a mark-up over the deposit rate, with the spread determined by the expected marginal cost of the regulatory penalty.

The first-order condition w.r.t. E_t can, after some manipulation, be expressed as

$$E_t: \quad \mathbb{E}_t\left[\frac{\hat{\beta}\Lambda_{t+1}}{\Lambda_t}\right] \mathbb{E}_t\left[R_{E,t+1}\right] = 1 + \xi_E\Omega_{E,t} + \tfrac{1}{2}\xi_E\Omega_{E,t}^{\,2} \approx 1 + \xi_E\Omega_{E,t}. \tag{18}$$

The detailed derivation of this equation is provided in Appendix A.3. The equation states that the bank increases the amount of capital up to the point where the return on equity (including the expected cost of the regulatory penalty) equals the household's discount factor corrected for the bank capital adjustment cost. In the extreme case of $\xi_E \to \infty$, the right-hand side of (18) dominates the equation, and the law of motion for bank capital becomes simply

$$E_t = (R_{E,t} - \tau_E)E_{t-1}.$$

2. Individual Lending

Finally, we need to establish the individual lending policy of the bank. The right-hand side of equation (17) determines the expected rate of return on an entire portfolio. For notational convenience, we denote that rate of return by $\hat{R}_t$. The only way for the bank to achieve a given level of return $\hat{R}_t$ in a homogenous portfolio,

$$R_{L,t}\left(1 - \lambda\mathbb{E}_t\left[H_{t+1}\right]\right) = \hat{R}_t, \tag{19}$$

is to make sure that the expected rate of return on each individual exposure is the same and equals the aggregate rate. This follows from the law of large numbers, see equation (9). We can think of this result as follows. The bank first determines the desired return, $\hat{R}_t$, optimal from the point of view of the size of its balance sheet. Then, it gives instructions to its loan officers to offer each applicant-borrower an individual lending supply curve defined by all possible combinations of $\left\{R_{L,t}^i, L_t^i\right\}$ that is consistent with the prescribed rate of return. The borrower is free to choose any point on that curve. The bank is indifferent amongst the points on such a lending supply curve.

Formally, upon loan appliation submission, each household member is presented with
a constraint given by

$$R_{L,t}^i \left(1 - \lambda p_t^i\right) = \hat{R}_t, \tag{20}$$

where p_t^i, the probability of individual default, is itself a function of the loan-to-value ratio, see equation (6). The above equation is, in fact, a simplified version of an individual lending curve derived in the finance literature as early as in Jaffee and Modigliani (1969). Finally, the individual lending supply curves are internalized by household members in their decision-making.

B. Households

The representative household consists of a large number of individual members, indexed by $i = 1,\dots, n$. Within the household, there are two separate sets of decisions: one is made by the household as a whole and the other is made by its individual members. This is a convenient way to introduce idiosyncratic heterogeneity while maintaining full risk sharing ex-post. We first review the preferences and constraints in their entirety, irrespective of who is the decision-maker, and then describe the optimization problems and optimality conditions separately for the household and for its individual members.

The household's lifetime expected utility function derives from consumption and hours worked, and is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\chi)\log\left(C_t - \chi \bar{C}_{t-1}\right) - (\eta+1)^{-1} N_t^{\eta+1}\right].$$

The notation is as follows:

- C_t is the household's real consumption of goods produced locally using both domestic input factors and imports, see the supply sector later in this section.

- $\bar{C}_{t-1}$ is the reference level in consumption with external habit formation.

- N_t is hours worked.

There are a total of four constraints to which the representative household's decision-making is subjected.

1. A dynamic budget constraint:

$$D_t + P_{K,t}\sum K_t^i - \sum L_t^i - R_{t-1}D_{t-1} - R_{K,t}P_{K,t-1}\sum K_{t-1}^i + \sum R_{L,t-1}^i L_{t-1}^i$$

$$- W_t N_t \left(1 - \tfrac{1}{2}\xi_W \,\Omega_{W,t}{}^2\right) + P_t C_t + P_t I_t \left(1 + \tfrac{1}{2}\xi_I \Omega_{I,t}{}^2\right) - P_{K,t}I_t - \bar{\Gamma}_t = 0. \quad (21)$$

with the following notation:

- D_t and $R_{t-1}D_{t-1}$ are bank deposits held by resident households, and the gross earnings on bank deposits from last period, respectively.

- $P_{K,t}\sum K_t^i$ and $R_{K,t}P_{K,t-1}\sum K_{t-1}^i$ are the total value of claims to physical capital held by individual members, and total gross earnings on capital from last period, respectively.

- $R_{K,t}$ is the ex-post return on physical capital, including rentals, Q_t, and capital gains,
$$R_{K,t} = \frac{Q_t + (1-\delta)P_{K,t}}{P_{K,t-1}}.$$

- $\sum L_{t,i}$ and $\sum R_{L,t-1}L_{t-1}$ is the total amount borrowed in bank loans by individual members, and the total gross cost of bank loans from last period, respectively.

- $W_t N_t$ is the household's labor income.

- $\tfrac{1}{2}\xi_W \,\Omega_{W,t}{}^2$ is a wage inflation adjustment cost term, with $\Omega_{W,t} = \log W_t/W_{t-1} - \log \bar{W}_{t-1}/\bar{W}_{t-2}$.

- $P_t C_t$ and $P_t I_t$ are outlays on consumption and investment, respectively.

- $\tfrac{1}{2}\xi_I \Omega_{I,t}{}^2$ is a investment adjustment cost, with $\Omega_{I,t} = \log I_t/I_{t-1}$.

- $P_{K,t}I_t$ is the resale value of new investment installed by the household in this period and sold in the market.

- $\bar{\Gamma}_t$ sums up all flows into or out of the budget constraint that are not internalized by the household or its members; these flows are detailed below.

2. A downward-sloping labor demand curve with monopoly power of μ. The household behaves as a monopolistically competitive supplier of labor, see the explanatory note on how we model monopolistic competition in section IV. The constraint is given by

$$N_t = \left(\frac{W_t}{\bar{W}_t}\right)^{-\mu/(\mu-1)} \bar{N}_t.$$

This constraint is not imposed through the Lagrangian, but rather used to define N_t as a function of W_t, $\bar{W}_t$, and $\bar{N}_t$. Note that the first derivative of N_t w.r.t. W_t is

$$\frac{\mathrm{d}N_t}{\mathrm{d}W_t} = \frac{\mu}{1-\mu}\left(\frac{W_t}{\bar{W}_t}\right)^{-\mu/(\mu-1)}\frac{\bar{N}_t}{W_t}.$$

3. A financing constraint, whereby the household is required to hold a certain amount of bank deposits at the beginning of each period to complete its planned transactions: specifically, purchases of consumption and investment goods, together with trade in claims to physical capital). The amount of bank deposits, including interest, carried over from the previous period is $R_{t-1}D_t$. This amount is increased by new loans from banks by the individual members, $\sum L_t^i - \sum R_{L,t-1}^i L_{t-1}^i$. The total amount of deposits, DD_t, available at the beginning of period t is therefore

$$DD_t = R_{D,t-1}D_{t-1} + \sum L_t^i - \sum R_{L,t-1}^i L_{t-1}^i.$$

This amount differs, in general, from that held at the end of the same period, $DD_t \neq D_t$, where the latter is used in the budget constraint, (21). The financing constraint is then given by

$$DD_t = \underbrace{\phi_C P_t C_t + \phi_I P_t I_t + \phi_K P_{K,t}\sum K_t^i}_{\text{Financing of household outlays}}. \tag{22}$$

4. An upward-sloping individual bank lending supply curve, derived in VI.A.2,

$$R_{L,t}^i\left(1 - \lambda p_t^i\right) = \hat{R}_t, \qquad \forall i = 1,\dots,n,$$

where p_t^i and $\hat{R}_t$ are given by equations (6) and (19), respectively. Recall that p_t^i is a function of both $R_{L,t}^i$ and L_t^i. To make the analysis easier to follow, we therefore do not use the constraint to express L_t^i as a function of $R_{L,t}^i$, and do not substitute for it in the optimization problem. Instead, assigning its own multiplier to the constraint, we add it to the overall Lagrangian.

The term $\bar{\Gamma}_t$ in the budget constraint, (21), includes two types of flows not internalized by the household. First, flows of equity between households and banks. These are designed in the same way as the flow of funds between households and "foreign exchange dealers" in Devereux and Engel (2002). Banking activities are delegated by households to banks; the banks give households net transfers of equity (positive or negative), which the households take as given.

Second, all types of adjustment costs. The adjustment costs are private but not social costs; they are ultimately returned to the representative household's budget constraint and do not affect the real resources of the economy, see e.g. Edwards and Végh (1997). Note that this assumption has no first-order effect on the model solution or simulations. The full list of the terms included in $\bar{\Gamma}_t$ is shown in subsection VI.G.

The complete Lagrangian describing the representative household's problem is given by

$$
\mathscr{L}_0 = \sum_{t=0}^{\infty} \beta^t \Bigg\{ \underbrace{\left[(1-\chi)\log\left(C_t - \chi \bar{C}_{t-1}\right) - (\eta+1)^{-1} N_t^{\eta+1} \right]}_{\text{Utility function}}
$$

$$
+ \underbrace{\Lambda_t \Big[D_t + P_{K,t}\textstyle\sum K_t^i - \sum L_t^i - R_{t-1}D_{t-1} - R_{K,t}P_{K,t-1}\sum K_{t-1}^i + \sum R_{L,t-1}^i L_{t-1}^i }
$$

$$
\underbrace{- W_t N_t \left(1 - \tfrac{1}{2}\xi_W \Omega_{W,t}^2\right) + P_t C_t + P_t I_t \left(1 + \tfrac{1}{2}\xi_I \Omega_{I,t}^2\right) - P_{K,t}I_t - \bar{\Gamma}_t \Big]}_{\text{Budget constraint}}
$$

$$
+ \underbrace{\Lambda_t \Xi_t \Big[\phi_C P_t C_t + \phi_I P_t I_t + \phi_K P_{K,t}\textstyle\sum K_t^i - DD_t \Big]}_{\text{Financing constraint}}
$$

$$
+ \underbrace{\Lambda_t \sum_{i=1}^{n} \Psi_t^i \Big[\hat{R}_t - R_{L,t}^i \left(1 - \lambda p_t^i\right) \Big]}_{\text{Individual bank lending supply curves}} \Bigg\}. \quad (23)
$$

1. Individual Members

Household member i chooses a combination of a bank loan, L_t^i, and a lending rate, $R_{L,t}^i$, from the individual lending supply curve offered by the bank, and the holdings of physical capital, K_t^i, to maximize the expected value of the Lagrangian (23), taking the other choices made by the household as a whole as given:

$$
\max_{\left\{ L_t^i, R_{L,t}^i, K_t^i \right\}} \mathbb{E}_0\big[\mathscr{L}\big].
$$

After differentiating the relevant part of the Lagrangian w.r.t. to the three choice variables, we use the first-order condition w.r.t. $R_{L,t}^i$ to substitute for the multiplier associated with the lending supply curve, Ψ_t^i, in the other conditions. The first-order conditions w.r.t. L_t^i and K_t^i then become, respectively,

$$
L_t^i: \quad \Lambda_t \left(1 + \Xi_t\right) = \beta \mathbb{E}_t \left[\Lambda_{t+1} \left(1 + \Xi_{t+1}\right) \left(1 + V_t^i\right) \right] R_{L,t}^i, \quad (24)
$$

$$
K_t^i: \quad \Lambda_t \left(1 + \phi_K \Xi_t\right) = \beta \mathbb{E}_t \left\{ \Lambda_{t+1} \left[R_{K,t+1} + \left(1 + \Xi_{t+1}\right) R_{L,t}^i k_t^i V_t^i \right] \right\}, \quad (25)
$$

where $k_t^i = L_t^i / P_{K,t} K_t^i$ is an individual loan-to-value ratio. The term V_t^i is a premium occuring in the first-order conditions because of an upward sloping lending curve internalized by each household member. The premium is given by

$$V_t^i = \frac{\lambda \, p_t^{i\prime}}{\varsigma - \varsigma \lambda \, p_t^i - \lambda \, p_t^{i\prime}}.$$

The details of these equations are provided in Appendix A.4. Note that absent the premium, V_t^i, and the financing constraint, the first-order conditions (24) and (25) would reduce to their more common forms found in standard macroeconomic models.

2. The Household as a Whole

The household as a whole chooses consumption, C_t, investment, I_t, bank deposits, D_t, and the wage rate, W_t, to maximize the expected value of the Lagrangian (23), taking the choices of individual members as given:

$$\max\nolimits_{\{C_t, I_t, D_t, W_t\}} \mathbb{E}_t [\mathscr{L}_0].$$

The first-order conditions w.r.t. C_t, I_t, W_t, and D_t are standard for the household's optimization problem. Below, we state the aggregate (not representative) forms of the conditions after using symmetric equilibrium assumptions $\bar{C}_t = C_t$, $\bar{W}_t = W_t$, and $\bar{N}_t = N_t$:

$$C_t: \quad \frac{1}{C_t - \chi C_{t-1}} = \Lambda_t P_t \left(1 + \phi_C \Xi_t\right), \tag{26}$$

$$I_t: \quad P_{K,t} \approx P_t + \xi_I P_t \left(\Omega_{I,t} - \beta \mathbb{E}_t[\Omega_{I,t+1}] + \phi_I \Xi_t\right), \tag{27}$$

$$W_t: \quad \mu \frac{N_t^\eta}{\Lambda_t W_t} - 1 \approx (1 - \mu) \, \xi_W \left(\Omega_{W,t} - \beta \mathbb{E}_t \left[\Omega_{W,t+1}\right]\right), \tag{28}$$

$$D_t: \quad \Lambda_t = \beta \mathbb{E}_t \left[\Lambda_{t+1} \left(1 + \Xi_{t+1}\right)\right] R_t, \tag{29}$$

where the $\approx$ signs indicate the omission of second- or higher-order terms from the equation.

C. Local Supply

The local supply chain consists of local producers, local retailers, and exporters.

1. Local Production

Competitive local producers combine domestic labor, $N_{Y,t}$, and capital, k_t,[9] with imported intermediate inputs, $M_{Y,t}$, to manufacture local goods using a Cobb-Douglas production function with overhead labor,

$$Y_t = M_{Y,t}{}^{\gamma_M} \left[A_t \left(N_{Y,t} - n \right) \right]^{\gamma_N} k_t{}^{1-\gamma_M-\gamma_N},$$

where n is the level of overhead required to conduct production, k_t is capital demanded, and A_t is exogenous technology. The producers maximize the present value of pay-offs,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left[P_{Y,t} Y_t \left(1 - \tfrac{1}{2} \xi_Y \, \Omega_{Y,t}{}^2 \right) - P_{M,t} M_{Y,t} - W_t N_{Y,t} - Q_t k_t \right],$$

where $\Omega_{Y,t} = \log n_t / M_{Y,t} - \log n_{t-1} / M_{Y,t-1}$ is a cost adjustment term associated with changing the proportion of the two input factors, and $n_t = N_{Y,t} - n$ denotes effective labor input.

The inclusion of overhead labor in the production function allows us to match the observed co-movements in output and hours worked. Without overhead labor, the percent fluctuations in hours worked predicted by the model would greatly exceed a typical percent cycle in output, depending mainly on the parameter γ_N, whereas empirically they tend to co-move on a rather one-to-one basis most of the time. With overhead labor, we can calibrate the steady-state ratio of overhead to total labor so as to achieve cycles with realistic orders of magnitude.

The optimal behavior of the local producer is described by the demand equations for labor, intermediate imports, and capital, i.e. first-order conditions w.r.t. $N_{Y,t}$, $M_{Y,t}$, and k_t,

---

[9]We notationally distinguish between capital supplied by households, K_t, and capital demanded by local producers, k_t. This is because for production at time t, only physical capital built up to time $t-1$ is available in the market. In other words, the respective aggregate market clearing condition is $k_t = K_{t-1}$. To comply with the notational convention that time t choice variables are denoted by a time t subscript, we need to introduce two quantities.

respectively:

$$N_t: \quad \frac{W_t\left(N_{Y,t} - n\right)}{\gamma_N P_{Y,t} Y_t} \approx 1 - \frac{\xi_Y}{\gamma_N}\left(\Omega_{Y,t} - \beta\mathbb{E}_t\left[\Omega_{Y,t+1}\right]\right), \tag{30}$$

$$Y_{M,t}: \quad \frac{P_{M,t} M_{Y,t}}{\gamma_M P_{Y,t} Y_t} \approx 1 - \frac{\xi_Y}{\gamma_M}\left(\Omega_{Y,t} - \beta\mathbb{E}_t\left[\Omega_{Y,t+1}\right]\right), \tag{31}$$

$$k_t: \quad \frac{Q_t k_t}{(1 - \gamma_M - \gamma_N)P_{Y,t} Y_t} = 1. \tag{32}$$

2. Local Distribution

A representative distributor (a retailer) resells the locally produced goods to households, both for consumption and investment. Total domestic demand for the retail output is $Z_t = C_t + I_t$. The retailer operates with monopoly power of $\mu = \epsilon/(\epsilon - 1)$ in its output market, and chooses his output Z_t and the final price $P_{Z,t}$ to maximize the present value of the pay-offs

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left[P_{Z,t} Z_t \left(1 - \tfrac{1}{2}\xi_P \Omega_{P,t}^2\right) - P_{Y,t} Z_t\right],$$

subject to a downward-sloping demand curve for its final output,

$$Z_t = \left(\frac{P_{Z,t}}{\bar{P}_{Z,t}}\right)^{-\epsilon} \bar{Z}_t,$$

where $\Omega_{P,t} = \log P_{Z,t}/P_{Z,t-1} - \log \bar{P}_{Z,t-1}/\bar{P}_{Z,t-2}$ is a price inflation adjustment cost term. We solve for the optimality conditions by first using the demand curve to express Z_t as a function of $P_{Z,t}$ where

$$\frac{\mathrm{d}Z_t}{\mathrm{d}P_{Z,t}} = -\epsilon \left(\frac{P_{Z,t}}{\bar{P}_{Z,t}}\right)^{-\epsilon-1} \frac{\bar{Z}_t}{\bar{P}_{Z,t}}.$$

and then differentiating an unconstrained objective function w.r.t. $P_{Z,t}$. The aggregate form of the first order condition after using symmetric equilibrium assumptions $\bar{P}_{Z,t} = P_{Z,t}$ and $\bar{Z}_t = Z_t$ is given by

$$P_{Z,t}: \quad \mu\frac{P_{Y,t}}{P_{Z,t}} - 1 \approx (1 - \mu)\,\xi_P\left(\Omega_{P,t} - \beta\mathbb{E}_t\left[\Omega_{P,t+1}\right]\right). \tag{33}$$

3. Exporting Industries

Competitive local exporters combine domestic labor, $N_{X,t}$, and re-exports purchased from abroad, $M_{X,t}$, to produce export goods using a Leontief technology,

$$X_t = \min\left\{\frac{A_{X,t}N_{X,t}}{\alpha}, \frac{M_{X,t}}{1-\alpha}\right\},$$

where $A_{X,t}$ is an exogenous process describing labor productivity in the exporting industries. The exporters choose $Y_{X,t}$ and $M_{X,t}$ to maximize the present value of current and future pay-offs,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left[P_{X,t}X_t\left(1 - \tfrac{1}{2}\xi_X \Omega_{X,t}{}^2\right) - W_t N_{X,t} - P_{M,t}M_{X,t}\right],$$

where $\Omega_{X,t} = \log X_t - \log X_{t-1}$ is a cost adjustment term associated with changes in the level of output of exporters. Note that exporters take the final price, $P_{X,t}$, as given.

For any given level of output, exporters optimally choose to combine the two inputs in fixed proportions,

$$N_{X,t} = \alpha X_t / A_{X,t}, \tag{34}$$

$$M_{X,t} = (1-\alpha)X_t. \tag{35}$$

The marginal cost of production net of adjustment costs is $\Phi_{X,t} = \alpha W_t / A_{X,t} + (1-\alpha)P_{M,t}$, and hence the total cost is $W_t N_{X,t} + P_{M,t}M_{X,t} = \Phi_{X,t}X_t$. After substituting the total production cost into the exporters' pay-offs, we can derive the following first-order condition for the optimal level of output:

$$\frac{\Phi_{X,t}}{P_{X,t}} \approx 1 + \xi_X \left(\Omega_{X,t} - \beta \mathbb{E}_t \left[\Omega_{X,t+1}\right]\right).$$

D. Monetary and Macroprudential Policy

The monetary authority implements inflation targeting based on a simple interest rate rule,

$$\log R_t = \theta_1 \log R_{t-1} + (1-\theta_1)\left\{\log R + \theta_2 \mathbb{E}_t \left[\log\left(P_{t+3}/P_{t-1}\right) - \pi\right]\right\},$$

where $\mathbb{E}_t \left[\log P_{t+3}/P_{t-1}\right]$ is the 3-quarters ahead expected year-on-year rate of inflation in final demand prices, R is the steady-state levels of the nominal rate of interest, and π is the inflation target.

The prudential regulator implements macroprudential policy based on a rule for the minimum capital adequacy ratio, ϕ. In the basic specification, this ratio is kept constant. In some of the simulation experiments, we introduce time-varying macroprudential policy, and explain the details of that particular policy in the simulation assumptions.

E. Exchange Rate

In the basic specification, bank loans, L_t, and resident deposits, D_t, are both denominated in local currency. However, because of international payments (such as purchases of imports in excess of receivables for exports), some amount of bank deposits end up held by non-residents,

$$F_t = d_t - D_t.$$

The non-resident deposits, F_t, can be denominated in either currency: local or foreign. We denote the proportion of foreign currency denominations in *total* deposits, d_t, by $\zeta_F \in [0, 1]$, and the respective foreign currency interest rate, which is exogenous, by R_t^*. We now first describe the foreign exchange market, and then explain how banks deal with foreign exchange exposure. Both spot and forward markets are assumed to exist. However, only banks can trade in forwards. The one-period-ahead forward rate, $\hat{S}_{t+1,t}$, is (by our assumption) simply determined by the expectation of the future spot rate,

$$\hat{S}_{t+1,t} = \mathbb{E}_t[S_{t+1}].$$

Furthermore, a *covered* interest parity holds between the local currency and foreign currency rates on bank liabilities, and implies a particular form of an *uncovered* parity (UIP),

$$R_t = R_t^* \frac{\hat{S}_{t+1,t}}{S_t} = R_t^* \frac{\mathbb{E}_t[S_{t+1}]}{S_t}. \tag{36}$$

Banks have open (short) foreign exchange positions on their balance sheets whenever $\zeta_F > 0$, and are exposed to direct foreign exchange risk in that case. To see this, note that the total cost of non-equity liabilities is

$$(1 - \zeta_F)R_t d_t + \zeta_F R_t^* d_t \frac{S_{t+1}}{S_t} = R_t d_t \left(1 - \zeta_F + \zeta_F \frac{S_{t+1}}{\mathbb{E}_t[S_{t+1}]}\right) = R_t d_t J_{F,t+1}, \tag{37}$$

where $J_{F,t+1}$ is the unexpected exchange rate valuation effect.

Direct exchange rate exposures on bank balance sheets are, however, rarely a serious issue in most real-world financial systems. This is because prudential regulations are usually

in place that limit open positions, and effectively force banks to hedge the associated risk using off-balance-sheet instruments. Accordingly, we also impose such regulation in the model, and show how outright forward contracts can be used to comply with it. The results will closely mimic what banks actually do in practice. The regulation prohibits foreign exchange exposures in a way that is formally defined in subsection VII.B[10] where we extend the model to allow for foreign exchange indexed loans. Here, in the basic specification, banks comply with the regulation by simply fully hedging their short positions. Specifically, at time t, they long (buy) foreign exchange forwards from the international market in the amount of

$$\zeta_F R_t^* d_t \left(\frac{\hat{S}_{t+1,t}}{S_t} \right)$$

expressed in local currency. The actual ex-post gross cost of non-equity liabilities is then

$$(1 - \zeta_F) R_t d_t + \zeta_F R_t^* d_t \frac{\hat{S}_{t+1,t}}{S_t} = R_t d_t,$$

and the foreign exchange risk is removed. Neither the bank profits and losses, nor the occurrence of a shortfall in regulatory capital are affected by the currency structure of (non-equity) liabilities, and all optimality conditions derived in subsection VI.A continue to hold.

F. The Rest of the Model

The model is closed by specifying processes for the three external variables, the world interest rate, R_t^*, the import price (expressed in foreign currency), $P_{M,t}^*$, and the terms of trade, T_t, and also for the two exogenous technology processes, local production technology, A_t, and export-specific technology, $A_{X,t}$.

The baseline assumption in our simulations is that these variables are constant at their respective steady-state levels. Any departures from the baseline assumption are described in the design of each simulation experiment.

---

[10]In practice, such regulations typically allow open positions up to a certain percentage of bank capital, for instance 10 %.

Based on $P^*_{M,t}$ and T_t, we can define the import and export prices expressed in local currency, as respectively

$$P_{M,t} = P^*_{M,t}S_t,$$
$$P_{X,t} = T_t P_{M,t}.$$

G. Symmetric Equilibrium and Aggregation

We impose the following market-clearing and symmetric-equilibrium conditions:

- The market for physical capital clears, $k_t = K_{t-1}$ (see also footnote 9 on page 34).

- The market for domestic labor clears, $N_t = N_{Y,t} + N_{X,t}$.

- The market for local goods clears, $Y_t = Z_t$.

- The market for imports clears, $M_t = M_{Y,t} + M_{X,t}$.

- The market for bank loans clears, $L_t = \sum L^i_t$.

- The law of motion for the aggregate supply of physical capital is given by $K_t = (1-\delta)K_{t-1} + I_t$, where $K_t = \sum K^i_t$.

- The aggregate quantities externalized in optimization equal the respective individual quantities: $\bar{R}_{E,t} = R_{E,t}$, $\bar{E}_t = E_t$, $\bar{C}_t = C_t$, $\bar{W}_t = W_t$, $\bar{N}_t = N_t$, $\bar{P}_{Z,t} = P_{Z,t}$, and $\bar{Z}_t = Z_t$, in symmetric equilibrium.

Next, we itemize the term $\bar{\Gamma}_t$ used in the budget constraint to collect all terms not internalized by the household or its individual members. This term consists of two types of budget flows: (i) net pay-offs (equity) transfers received from local producers, local retailers, exporters, and banks, and (ii) adjustment costs returned bo the household budget constraint

(since they are considered private but not social costs in the model):

$$\bar{\Gamma}_t = \underbrace{P_{Y,t}Y_t\left(1 - \tfrac{1}{2}\xi_Y\,\Omega_{Y,t}{}^2\right) - P_{M,t}M_{Y,t} - W_t N_{Y,t} - Q_t k_t}_{\text{Pay-offs from local producers}} + \underbrace{P_{Z,t}Z_t\left(1 - \tfrac{1}{2}\xi_P\Omega_{P,t}{}^2\right) - P_{Y,t}Z_t}_{\text{Pay-offs from local distributors}}$$

$$+ \underbrace{P_{X,t}X_t\left(1 - \tfrac{1}{2}\xi_X\,\Omega_{X,t}{}^2\right) - W_t N_{X,t} - P_{M,t}M_{X,t}}_{\text{Pay-offs from exporters}} + \underbrace{R_{E,t}E_t - E_{t-1}}_{\text{Pay-offs from banks}}$$

$$+ \underbrace{W_t N_t \cdot \tfrac{1}{2}\xi_W\Omega_{W,t}{}^2}_{\text{Wage adjustment cost}} + \underbrace{P_{I,t}I_t \cdot \tfrac{1}{2}\xi_I\Omega_{I,t}{}^2}_{\text{Investment adjustment cost}} + \underbrace{P_{Y,t}Y_t \cdot \tfrac{1}{2}\xi_Y\Omega_{Y,t}{}^2}_{\text{Input factor adjustment cost}}$$

$$+ \underbrace{P_{Z,t}Z_t \cdot \tfrac{1}{2}\xi_P\Omega_{P,t}{}^2}_{\text{Price adjustment cost}} + \underbrace{P_{X,t}X_t \cdot \tfrac{1}{2}\xi_X\Omega_{X,t}{}^2}_{\text{Export adjustment cost}} + \underbrace{E_t \cdot \tfrac{1}{2}\xi_E\Omega_{E,t}{}^2}_{\text{Bank capital adjustment cost}}.$$

Combining the household budget constraint with the definition of $\bar{\Gamma}_t$, we obtain the following aggregate law of motion for the investment position of the country as a whole:

$$\underbrace{F_t - F_{t-1}}_{\text{Change in investment position}} = \underbrace{(R_{t-1} - 1)F_{t-1}}_{\text{Investment income}} - \underbrace{\left(P_{X,t}X_t - P_{M,t}M_t\right)}_{\text{Trade balance}} + \underbrace{\lambda H_t \cdot R_{L,t-1}L_{t-1}}_{\text{Loss from defaults}},$$

where the last term, $\lambda H_t \cdot R_{L,t-1}L_{t-1}$, is the total loss from defaulted loans. This loss is assumed to be a social cost reducing the real external resources of the economy. There are two important assumptions behind the simple form of the investment position equation above. First, banks are only owned locally, by domestic households. Second, there are no financial positions between the country and the rest of the world other than non-resident bank deposits, F_t. By relaxing these two assumptions in section VII, we will observe more complex behavior.

VII. PRACTICAL EXTENSIONS AND MODIFICATIONS

The previous section provided a complete description of the basic specification of the model. In this section, we explain several extensions and modifications that are introduced to make some of the dynamic properties of the model more realistic, especially when scenarios of large distress are simulated. The extensions are not always derived from strict first principles, and some of them rather rely upon some ad-hoc mechanisms.

A. Non-Price Terms of Bank Lending

Facing a variety of information (and other types of) problems, banks in the real world combine both price and non-price terms and conditions to manage credit risk; the latter may include non-price rationing as well as other kinds of contractual restrictions. The incentives for engaging in non-price lending policies are stronger still if non-diversifiable risk exists in the loan portfolio, as shown e.g. by Arnold, Reeder, and Trepl (2010). There are two main implications for the behavior of credit market participants. First, the observed lending rates do not fully reflect the availability of bank credit. In other words, they do not clear the market: from the point of view of borrowers, the true cost of credit comprises also the shadow price of the non-price conditions. Second, when banks use non-price conditions to cut back on the volume of loans extended, as opposed to increasing the lending rates to the full extent, affected will be also the actual ex-post bank earnings, and hence the ability of banks to recapitalize themselves and keep distance from insolvency after a period of major unexpected losses.

We do not provide explicit theoretical foundations for the existence of non-price conditions in the present model; instead, we only use a simple ad-hoc mechanism to capture the essence thereof, and more importantly, to mimic the implications. The mechanism is designed the following way. At time t, when applying for loans, borrowers are presented with individual supply curves as described in equation (20). Ex-post at time $t + 1$, however, the borrowers are charged a rate that differs from the ex-ante one, a fact not internalized by the borrowers but accounted for by the banks. We denote the rate that borrowers choose at time t by $\hat{R}_{L,t}$, and keep denoting the actual rate applied at time $t + 1$ by $R_{L,t}$. The relationship between the two is given by

$$\hat{R}^i_{L,t} = R_{L,t} + \iota\left(R^i_{L,t} - R_t - \tau_R\right), \tag{38}$$

where τ_R is a technical constant set to the steady-state level of the lending spread, $\tau_R = R_L - R$, and ι is a parameter to control the degree of significance of non-price terms relative to the price of credit. The meaning of equation (38) is as follows. In steady state, the lending rates correspond to the true cost of borrowing, and clear the market; in other words, this model is based on the *temporary* theory of non-price terms, as termed by Harris (1974). In times of regulatory capital distress, when lending spreads rise above their normal levels, banks also start tightening their non-price conditions, and $\hat{R}_{L,t} > R_t$. On the other hand, in times of cheap credit expansion and low spreads, the non-price conditions become laxer, and $\hat{R}_{L,t} < R_{L,t}$.

The fact that borrowers base their decisions on a different rate they are actually charged, is modeled the following way. We replace $R_{L,t}$ with $\hat{R}_{L,t}$ in the lending supply curve and the household budget constraint. At the same time, we add $\left(R_{L,t-1} - \hat{R}_{L,t-1}\right)L_{t-1}$ to the external flows in the budget constraint, Γ_t; this extra term creates effectively a wedge between the rate the household members base their decisions upon ex-ante, and the cost they pay ex-post.

Finally, the decision of banks on the size of their balance sheets, including the determination of the required rate of return, $\hat{R}_t$, and the actual probability of default, p_t^i, are all based on the true interest rate, $R_{L,t}$, as in the basic specification.

B. Foreign Exchange Indexation

We relax the assumption of local denomination of bank loans, made in subsection VI.E, and allow a fixed proportion, $\zeta_L \in [0,1]$, of the loans to be indexed to foreign currency. The idea is to introduce currency mismatches in the non-financial sector (on the household balance sheets) while keeping banks hedged. This is the most typical situation encountered in economies where indexed loans or loans denominated directly in foreign currency exist.

Technically, our indexation scheme requires that the repayment of each individual loan be inclusive of some proportion of ex-post unexpected nominal exchange rate depreciation or appreciation.[11] At time t, the ex-post actual repayment schedule for loans made at time $t-1$ is given by

$$R_{L,t-1}L_{t-1}\left(1 - \zeta_L + \zeta_L \frac{S_t}{\mathbb{E}_{t-1}[S_t]}\right),\tag{39}$$

where ζ_L is the percentage of foreign exchange indexation of bank loans.

We now formally introduce the prudential regulation of foreign exchange positions, and review the bank balance sheets as well as the hedging strategies in response to these new exposures. In our model, the regulation requires that the cut-off portfolio default ratio, $\hat{H}_t$, be independent of the exchange rate. After including the indexation of loans, see equation (39), and the mixed currency structure of deposits, see equation (37), the cut-off port-

---

[11]Note that the *expected* depreciation or appreciation is already reflected in the interest rate differential.

folio default ratio becomes

$$\hat{H}_t = \frac{1}{\lambda} \left[1 - \frac{R_t d_t \left(1 - \zeta_F + \zeta_F \frac{S_{t+1}}{\mathbb{E}_t[S_{t+1}]} \right)}{(1-\varphi) R_{L,t} L_t \left(1 - \zeta_L + \zeta_L \frac{S_{t+1}}{\mathbb{E}_t[S_{t+1}]} \right)} \right]. \tag{40}$$

To devise hedging strategies aimed to offset the exchange rate valuation effect in equation (40), we need to consider three basic cases:

- $\zeta_F = \zeta_L$: the two valuation effects in the numerator and denominator cancel each other, and there is no further trade in off-balance-sheet instruments need to comply with the foreign exchange regulation.

- $\zeta_F > \zeta_L$: banks are short in foreign exchange on the balance sheets, a situation similar to the basic specification where $\zeta_L = 0$. In this case, the banks will hedge the open positions by buying (longing) forwards worth of

$$(\zeta_F - \zeta_L) \, R_t d_t \frac{\hat{S}_{t+1,t}}{S_t}$$

expressed in local currency.

- $\zeta_L > \zeta_F$: banks are long in foreign exchange on the balance sheets. In this case, the banks will hedge the open positions by selling (shorting) forwards worth of

$$(\zeta_L - \zeta_F) \, R_{L,t} L_t \left(1 - \hat{H}_t \right) \frac{1}{\hat{S}_{t+1,t}}$$

expressed in foreign currency. The forward trade will be, at the respective future date, accompanied by a spot exchange to complete the hedge. Albeit this strategy leaves an ex-post open exposure worth of

$$R_{L,t} L_t \lambda \left(\hat{H}_t - H_{t+1} \right) \left(1 - \zeta_F + \zeta_F \frac{S_{t+1}}{\mathbb{E}_t[S_{t+1}]} \right)$$

expressed in local currency, it keeps $\hat{H}$ unaffected by the exchange rate nonetheless.

Irrespective of the situation, the result will always be a cut-off portfolio default ratio given by

$$\hat{H}_t = \frac{1}{\lambda} \left[1 - \frac{R_t d_t \left(1 - \zeta + \zeta \frac{S_{t+1}}{\mathbb{E}_t[S_{t+1}]} \right)}{(1-\varphi) R_{L,t} L_t \left(1 - \zeta + \zeta \frac{S_{t+1}}{\mathbb{E}_t[S_{t+1}]} \right)} \right] = \frac{1}{\lambda} \left[1 - \frac{R_t d_t}{(1-\varphi) R_{L,t} L_t} \right],$$

where $\zeta = \min\{\zeta_F, \zeta_L\}$, with the valuation terms canceling each other.

The ex-post value of bank capital, EE_t, and hence also the return on bank capital, $R_{E,t}$, remains though still subjected to unexpected movements in the exchange rate,

$$EE_t = \left[R_{L,t-1}L_{t-1}\left(1 - \lambda\hat{H}_t\right) - R_{t-1}d_{t-1}\right]\left(1 - \zeta + \zeta\frac{S_t}{\mathbb{E}_{t-1}[S_t]}\right)$$
$$+ R_{L,t-1}L_{t-1}\lambda\left(\hat{H}_{t-1} - H_t\right)\left(1 - \zeta_L + \zeta_L\frac{S_t}{\mathbb{E}_{t-1}[S_t]}\right). \quad (41)$$

C. Foreign Ownership of Banks

In this extension, we allow for a fixed proportion, $1 - v$, of banks being owned internationally. The remaining ownership, v, is local as in the basic specification. The inflows and outflows of bank capital need to be adjusted accordingly in the household budget constraint,

$$\bar{\Gamma}_t = P_{Y,t}Y_t\left(1 - \tfrac{1}{2}\xi_Y\,\Omega_{Y,t}^2\right) - P_{M,t}M_{Y,t} - W_tN_{Y,t} - Q_tk_t + P_{Z,t}Z_t\left(1 - \tfrac{1}{2}\xi_P\Omega_{P,t}^2\right) - P_{Y,t}Z_t$$
$$+ P_{X,t}X_t\left(1 - \tfrac{1}{2}\xi_X\,\Omega_{X,t}^2\right) - W_tN_{X,t} - P_{M,t}M_{X,t} + R_{E,t}E_t - E_{t-1}$$
$$+ W_tN_t\cdot\tfrac{1}{2}\xi_W\Omega_{W,t}^2 + P_{I,t}I_t\cdot\tfrac{1}{2}\xi_I\Omega_{I,t}^2 + P_{Y,t}Y_t\cdot\tfrac{1}{2}\xi_Y\Omega_{Y,t}^2$$
$$+ P_{Z,t}Z_t\cdot\tfrac{1}{2}\xi_P\Omega_{P,t}^2 + P_{X,t}X_t\cdot\tfrac{1}{2}\xi_X\Omega_{X,t}^2 + \underbrace{vE_t\cdot\tfrac{1}{2}\xi_E\Omega_{E,t}^2}_{\text{Mixed ownership of banks}}.$$

The law of motion for the net investment position under mixed ownership becomes

$$F_t - F_{t-1} = (R_{t-1}-1)F_{t-1} - \left(P_{X,t}X_t - P_{M,t}M_t\right) + \lambda H_t\cdot R_{L,t-1}L_{t-1} + \underbrace{(1-v)\left(R_{E,t}E_{t-1} - E_t\right)}_{\text{Tranfer of foreign owned bank capital}}.$$

Note that in the polar case $v = 0$, the above equation reduces to

$$L_t - D_t = R_{L,t-1}L_{t-1} - R_tD_{t-1} - \left(P_{X,t}X_t - P_{M,t}M_t\right).$$

D. Direct Exchange Rate Pass-through

In the basic specification of the model, imports and import prices affect final demand indirectly, through imported intermediate goods, $M_{Y,t}$. The pass-through of the exchange rate and foreign price shocks is therefore gradual and distributed over time, owing to the price adjustment cost (which gives rise to the Phillips curve). In many economies, though, we often observe much direct influence of the exchange rate and import prices on the final price of some goods (and sometimes also services). In this extension, we show how to introduce such direct pass-through in a simple way.

Final consumption, C_t, and final investment, I_t, will now consist of both locally produced goods and directly imported goods combined in fixed proportion. We can think of this structure as arising from a Leontief production or utility function. Denoting by $C_{Y,t}$ and $I_{Y,t}$ the demand for the respective locally produced goods, and by $C_{M,t}$ and $I_{M,t}$ the demand for the respective imported goods, we impose the following conditions for real quantities without deriving them explicitly from first principles here:

$$C_{M,t} = \omega\, C_t,$$
$$C_{Y,t} = (1 - \omega)\, C_t,$$
$$I_{M,t} = \omega\, I_t,$$
$$I_{Y,t} = (1 - \omega)\, I_t,$$

The final demand price index is then given by

$$P_t = \omega P_{M,t} + (1 - \omega) P_{Z,t}, \tag{42}$$

where $\omega \in [0, 1)$ is the direct import intensity of final goods, respectively, and $P_{Y,t}$ and $P_{M,t}$ are the prices of locally produced goods and imported goods, respectively.

Finally, the market clearing conditions for final demand goods and total imports need to be modified as follows

$$Z_t = (1 - \omega)\,(C_t + I_t), \tag{43}$$
$$M_t = M_{Y,t} + M_{X,t} + \omega\,(C_t + I_t). \tag{44}$$

E. Consumption and Current Income

A large body of empirical studies, most notably Campbell and Mankiw (1990), document considerable departures of consumption from predictions based on a permanent income hypothesis, an assumption underlying the representative household's optimization problem in the basic specification of the model.

We introduce dependence of households on current income, suppressing thus the dominance of the permanent income effect to some extent. Instead of the usual modeling strategy of having two types of households (say, optimizers and rule-of-thumbers as in Galí, López-Salído, and Vallés, 2004), we keep the overall structure of the model unchanged, and instead make it costly for consumption to depart from current income. The cost term oc-

curs in the budget constraint,

$$D_t + P_{K,t}\sum K_t^i - \sum L_t^i - R_{t-1}D_{t-1} - R_{K,t}P_{K,t-1}\sum K_{t-1}^i + \sum R_{L,t-1}^i L_{t-1}^i$$

$$-W_t N_t \left(1 - \tfrac{1}{2}\xi_W \Omega_{W,t}^2\right) + \underbrace{P_{C.t} C_t \left(1 + \tfrac{1}{2}\xi_C \Omega_{C,t}^2\right)}_{\text{Consumption with current income effect}} + P_t I_t \left(1 + \tfrac{1}{2}\xi_I \Omega_{I,t}^2\right) - P_{K,t}I_t - \bar{\Gamma}_t = 0,$$

(45)

where $\Omega_{C,t}$ is given by the log difference between nominal consumption expenditures and a measure of current income consisting of labor income diminished by the interest paid on existing loans, including possible valuation from unexpected exchange rate movements,

$$\Omega_{C,t} = \log C_t - \log \bar{O}_t - \tau_C,$$

$$O_t = \frac{W_t N_t - (R_{L,t-1} - 1)L_{t-1}J_{L,t}}{P_t}$$

and τ_C is a technical constant set to the steady-state difference $\log PC - \log [WN - (R_L - 1)L]$, so that the adjustment cost disappears in the long run. Note that the current income term in the adjustment cost is not internalized by the household. This assumption is for simplicity: the effect on the optimal consumption decision alone is sufficient to generate the desired patterns. With the current income dependence, the first-order optimality condition for consumption, eq. (26), becomes approximately

$$\frac{1}{C_t - \chi C_{t-1}} \approx \Lambda_t P_t \left(1 + \phi_C \Xi_t + \xi_C \Omega_{C,t}\right).$$

F. Asset Price Bubbles

Households and banks expect, ex-ante, the asset prices to follow their fundamental path in the basic specification. This assumption is incorporated in equation 5, where the ex-ante mean of the distribution of the overall risk factor is determined by the model-consistent expectation of the log of the return on physical capital,

$$\mathbb{E}_t[r_{t+1}] = \mathbb{E}_t\left[\log R_{K,t+1}\right].$$

To simulate asset prices bubbles displaying systematic deviations from the fundametals (consistent with the current state of believes and expectations),[12] we introduce an exoge-

---

[12] This situation is often referred to as an *irrational* asset price bubble, see Bernanke and Gertler (1999)

nous process B_t, and modify the ex-ante mean to include this bubble term,

$$\mathbb{E}_t[r_{t+1}] = B_t \mathbb{E}_t \left[\log R_{K,t+1} \right].$$

and define a simply autoregressive process for B_t to describe its expected values,

$$\log B_t = \rho_B \log B_{t-1} + \epsilon_{B,t}.$$

Note that only the ex-ante expectations are affected by B_t. The actual ex-post portofilo default ratio, H_{t+1}, is always based on the actual performance of assets, independent of the ex-ante bubble term. The baseline specification assumes $B_t = 1$; in simulation experiments where the its path deviates from 1, we show the specific numerical values for B_t and the sequence of unexpected shocks to it, $\epsilon_{B,t}$, in the assumptions table.

VIII. Conclusion

In this paper we have presented and discussed the theoretical structure of MAPMOD, a new IMF model that has been designed to study vulnerabilities associated with excessive credit expansions, and to support macroprudential policy analysis. The critical feature of the model is that banks play a far more active role in the macroeconomic transmission mechanism than in the traditional loanable funds model. Banks in MAPMOD, and in the real world, do not have to wait for deposits to arrive before using those deposits to fund loans. Rather, they create new deposits in the process of making new loans, and these deposits serve as the economy's principal medium of exchange. In other words, so long as banks are adequately capitalized, and expect lending to be sufficiently profitable, they can quickly expand (or, in downturns, contract) their balance sheets. This can be beneficial if banks' assessment of economic conditions is accurate. But if their assessment is too optimistic, the growth of bank and borrower balance sheets can build up large vulnerabilities that, as soon as the economy experiences negative shocks, can be revealed in a deep financial crisis that has severe and highly nonlinear effects on the real economy. A distinguishing feature of such crises is that banks' response to deteriorating economic conditions does not come mainly in the form of higher lending spreads, although this does play a role, but rather in the form of severe cutbacks in lending.

A major strength of the model is its ability to simulate a wide variety of policy-relevant scenarios that have an important financial-sector dimension, and that take into account the critical nonlinearities associated with balance sheet problems. In the companion paper,

Benes, Kumhof, and Laxton (2014), we show simulations of actual and expected productivity growth scenarios, changes in the riskiness of bank borrowers, deviations of asset prices from their fundamental values (bubbles), shocks to bank equity as well as to foreign interest rates. We also simulate changes in macroprudential policies, including permanent increases in minimum capital adequacy ratios and countercyclical bank capital policies.

It is important to emphasize that MAPMOD is a prototype simulation model whose parameters have been calibrated to match the basic facts of financial cycles. The existence of nonlinearities, and of evolving financial sector policies to guard against financial crises, poses some very difficult estimation issues. It is well known that the estimation of nonlinear models can require much larger sample sizes to identify functional forms and to detect the very existence of nonlinearities. Such a small sample size problem is particularly challenging for models designed for macroprudential policy analysis, for two reasons. First, as we demonstrate in the companion paper, nonlinearities can be especially severe when modeling the financial sector. Second, to the extent that macroprudential policies based on models such as MAPMOD end up being successful at preventing large boom-and-bust financial cycles, this will severely limit the number of empirical observations that are available for estimation.[13] In choosing the best structure and parameterization of the model, we are therefore likely to have to continue to rely heavily on judgment informed by a reading of existing empirical evidence covering many economies, rather than on formal estimation procedures applied to a particular country.

---

[13]See Committee on the Global Financial System (2010).

REFERENCES

Adrian, Tobias, Paolo Colla, and Hyun Song Shin, 2013, "Which Financial Frictions? Parsing the Evidence from the Financial Crisis of 2007 to 2009." *NBER/Macroeconomics Annual (University of Chicago Press)*, Vol. 27, pp. 159 – 214.

Arnold, Lutz G., Johannes Reeder, and Stefanie Trepl, 2010, "Single-name credit risk, portfolio risk, and credit rationing," University of Regensburg Working Paper, 448.

Benes, Jaromir, Michael Kumhof, and Douglas Laxton, 2014, "Financial Crises in DSGE Models: Selected Applications of MAPMOD," IMF Working Paper.

Bernanke, Ben S., and Mark Gertler, 1999, "Monetary Policy and Asset Price Volatility," FRB of Kansas City Economic Review, Fourth Quarter.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist, 1999, "The Financial Accelerator in a Quantitative Business Cycle Framework," in John B. Taylor and Michael Woodford (eds.), *Handbook of Macroeconomics*, Vol. 1C (North-Holland).

Borio, Claudio, 2012, "The financial cycle and macroeconomics: What have we learnt?" BIS Working Paper, 395.

Campbell, John Y., and Gregory N. Mankiw, 1990, "Permanent Income, Current Income, and Consumption," *Journal of Business and Economic Statistics*, Vol. 8, No. 3, pp. 269–279.

Claessens, Stijn, M. Ayhan, and Macro Terrones, 2011, "How Do Business and Financial Cycles Interact?" IMF Working Paper, 11/88.

Committee on the Global Financial System, 2010, "Macroprudential instruments and frameworks: a stocktaking of issues and experiences," CGFS Paper, 38.

Devereux, Michael B., and Charles Engel, 2002, "Exchange Rate Pass-through, Exchange Rate Volatility, and Exchange Rate Disconnect," *Journal of Monetary Economics*, Vol. 49, No. 5, pp. 913–40.

Disyatat, Piti, and Claudio Borio, 2011, "Global imbalances and the financial crisis: Link or no link?" BIS Working Paper, 346.

Edwards, Sebastian, and Carlos A. Végh, 1997, "Banks and Macroeconomics Disturbances under Predetermined Exchange Rates," *Journal of Monetary Economics*, Vol. 40, No. 2, pp. 239–78.

Estrella, Arturo, 2004, "The cyclical behavior of optimal bank capital," *Journal of Banking and Finance*, Vol. 28, No. 6, pp. 1469–1498.

Galí, Jordi, J. David López-Salído, and Javier Vallés, 2004, "Rule-of-Thumb Consumers And The Design Of Interest Rate Rules," *Journal of Money, Credit and Banking*, Vol. 36, No. 4, pp. 739–763.

Gordy, Michael B., 2000, "A comparative anatomy of credit risk models," *Journal of Banking and Finance*, Vol. 24, No. 1/2, pp. 119–149.

———, 2003, "A risk-factor model foundations for ratings-based bank capital rules," *Journal of Financial Intermediation*, Vol. 12, No. 3, pp. 199–232.

Harris, Duane G., 1974, "Interest Rates, Nonprice Terms, and the Allocation of Bank Credit," *Southern Economic Journal*, Vol. 40, No. 3, pp. 428–433.

Jaffee, Dwight M., and Franco Modigliani, 1969, "A Theory and Test of Credit Rationing," *American Economic Review*, Vol. 59, No. 5, pp. 850–872.

Jakab, Zoltan, and Michael Kumhof, 2014, "Banks Are Not Intermediaries," IMF Working Paper (Forthcoming).

Jokipii, Terhi, and Alistair Milne, 2008, "The cyclical behaviour of European bank capital buffers," *Journal of Banking and Finance*, Vol. 32, No. 8, pp. 1140–1451.

Lorenzoni, Guido, 2008, "Inefficient Credit Booms," *Review of Economic Studies*, Vol. 75, No. 3, pp. 809–833.

Macroeconomic Assessment Group, 2010, "Assessing the macroeconomic impact of the transition to stronger capital and liquidity requirements," Macroeconomic Assessment Group Final Report, Basel Committe on Banking Supervision.

Merton, Robert C., 1974, "On the Pricing of Corporate Debt: The Risk Structure of Interest RAtes," *Journal of Finance*, Vol. 19, No. 2, pp. 449–470.

Milne, Alistair, 2002, "Bank capital regulation as an incentive mechanism: Implications for portfolio choice," *Journal of Banking and Finance*, Vol. 26, No. 1, pp. 1–23.

———, 2009, "Macroprudential policy: what can it achieve?" *Oxford Review of Economic Policy*, Vol. 25, No. 4, pp. 608–629.

Milne, Alistair, and Geoffrey Wood, 2009, "The bank lending channel reconsidered," Bank of Finland Research Discussion Papers 2/2009.

Peura, Samu, and Esa Jokivuolle, 2004, "Simulation based stress tests of banks' regulatory capital adequacy," *Journal of Banking and Finance*, Vol. 28, No. 8, pp. 1801–1824.

Peura, Samu, and Jussi Keppo, 2006, "Optimal Bank Capital with Costly Recapitalisation," *Journal of Business*, Vol. 79, No. 4, pp. 2163–2201.

Schmitt-Grohé, Stephanie, and Martín Uribe, 2003, "Closing Small Open Economy Models," *Journal of International Economics*, Vol. 61, No. 1, pp. 163–85.

Tarashev, Nikola, and Haibin Zhu, 2007, "Modelling and calibration errors in measures of portfolio risk," BIS Working Paper, 230.

Van den Heuvel, Skander J., 2002, "Does Bank Capital Matter for Monetary Transmission?" FRB New York Economic Policy Review, May 2002.

———, 2006, "The Bank Capital Channel of Monetary Policy," 2006 Meeting Papers 512, Society for Economic Dynamics.

———, 2008, "The welfare cost of bank capital requirements," *Journal of Monetary Economics,* Vol. 55, No. 2, pp. 298–320.

Vasicek, Oldrich, 2002, "Loan portfolio value," Risk, December 2002.

APPENDIX A. DETAILS OF SOME EQUATIONS

A.1. Conditional Probability of Individual Default

The conditional probability of individual default is defined as

$$\mathbb{E}\left[H_{t+1}^i\big|r_{t+1}\right] = \Pr\left(r_{t+1}^i < \hat{r}_t^i\big|r_{t+1}\right).$$

Because $r_{t+1}^i = r_{t+1} + u_{t+1}^i$, where the aggregate risk factor, r_{t+1}, is conditioned upon, we can re-write the probability as an unconditional one as follows:

$$\Pr\left(r_{t+1}^i < \hat{r}_t^i\big|r_{t+1}\right) = \Pr\left(u_t^i < \hat{r}_t^i - r_{t+1}\right).$$

The distribution function for u_{t+1}^i is given by assumption (4b). Therefore,

$$\Pr\left(u_t^i < \hat{r}_t^i - r_{t+1}\right) = \Phi\left(\frac{\hat{r}_t^i - r_{t+1}}{\varsigma\sqrt{1-\varrho}}\right).$$

We now further rearrange the argument inside the distribution function:

$$\frac{\hat{r}_t^i - r_{t+1}}{\varsigma\sqrt{1-\varrho}} = \frac{\hat{r}_t^i - \mathbb{E}_t[r_{t+1}] - \left(r_{t+1} - \mathbb{E}_t[r_{t+1}]\right)}{\varsigma\sqrt{1-\varrho}} =$$

$$= \frac{1}{\sqrt{1-\varrho}}\left(\frac{\hat{r}_t^i - \mathbb{E}_t[r_{t+1}]}{\varsigma} - \frac{r_{t+1} - \mathbb{E}_t[r_{t+1}]}{\varsigma\sqrt{\varrho}}\sqrt{\varrho}\right) = \frac{\Phi^{-1}(p_t^i) - \Phi^{-1}\left(q_{t+1}\right)\sqrt{\varrho}}{\sqrt{1-\varrho}}, \quad (46)$$

where we make use of definition (7) in the last term, and denote the percentile of the aggregate risk factor by

$$q_{t+1} = \Phi\left(\frac{r_{t+1} - \mathbb{E}_t[r_{t+1}]}{\varsigma\sqrt{\varrho}}\right).$$

A.2. Distribution of Portfolio Default Ratio

We first derive the cumulative distribution function for the portfolio default ratio, H_{t+1}. We then differentiate the c.d.f. to calculate the probability density function. To that end, we combine two facts: first, that we can establish a one-to-one mapping between H_{t+1} and the aggregate risk factor, r_{t+1}, and second, that we already know the distribution of that aggregate risk factor.

The portfolio default ratio, H_{t+1}, is a monotonically decreasing function of the aggregate risk factor, r_{t+1}. We denote the function by h; the function is given by equations (8) and (10):

$$H_{t+1} = h(r_{t+1}) = \Phi\left(\frac{\hat{r}_t^i - r_{t+1}}{\varsigma\sqrt{1-\varrho}}\right).$$

From this, we can express the inverse function, $r_{t+1} = h^{-1}(H_{t+1})$:

$$h^{-1}(H_{t+1}) = \hat{r}_t^i - \Phi^{-1}(H_{t+1})\varsigma\sqrt{1-\varrho}.$$

The cumulative distribution function for the portfolio default ratio, $\Pi_t(x)$, is then defined by

$$\Pi_t(x) = \Pr(H_{t+1} < x) = \Pr\left[r_{t+1} > h^{-1}(x)\right],$$

where the switch in the inequality sign in the last term is because h is a decreasing function. The c.d.f. for r_{t+1} is known, see assumption (4a), and hence we can express the rightmost probability as follows:

$$\Pi_t(x) = \Pr\left[r_{t+1} > h^{-1}(x)\right] = 1 - \Phi\left(\frac{h^{-1}(x) - \mathbb{E}_t[r_{t+1}]}{\varsigma\sqrt{\varrho}}\right) = \Phi\left(\frac{\mathbb{E}_t[r_{t+1}] - h^{-1}(x)}{\varsigma\sqrt{\varrho}}\right),$$

where the last equality follows from the properties of the normal distribution. Upon substituting for h^{-1}, we obtain

$$\Pi_t(x) = \Phi\left(\frac{\mathbb{E}_t[r_{t+1}] - \hat{r}_t^i + \Phi^{-1}(x)\varsigma\sqrt{1-\varrho}}{\varsigma\sqrt{\varrho}}\right) = \Phi\left(\frac{\Phi^{-1}(x)\sqrt{1-\varrho} - \frac{\hat{r}_t^i - \mathbb{E}_t[r_{t+1}]}{\varsigma}}{\sqrt{\varrho}}\right) =$$

$$= \Phi\left(\frac{\Phi^{-1}(x)\sqrt{1-\varrho} - \Phi^{-1}(p_t)}{\sqrt{\varrho}}\right).$$

where the substitution in the last equality comes from definition (10). The rightmost expression for the c.d.f. in the last equation is particularly convenient as the value of the c.d.f. is determined by only two model-specific input arguments, the individual probability of default, p_t, and the cross-correlation of risk in the portfolio, ϱ.

The probability density function, $\Pi_t'(x)$, is obtained by differentiating $\Pi_t(x)$ w.r.t. x:

$$\Pi_t'(x) = \sqrt{\frac{1-\varrho}{\varrho}}\ \Phi'\left(\frac{\Phi^{-1}(x)\sqrt{1-\varrho} - \Phi^{-1}(p_t)}{\sqrt{\varrho}}\right)\frac{1}{\Phi'\left(\Phi^{-1}(x)\right)},$$

where we use the inverse function differentiation rule,

$$\left[\Phi^{-1}(x)\right]' = \frac{1}{\Phi'\left(\Phi^{-1}(x)\right)}.$$

A.3. Optimal Choice of Bank Capital

We first establish an intermediate result, the expected return on bank capital under optimal behavior. The formula for the expected return will be then used to simplify the first-order condition w.r.t. bank capital to the form provided in the main text in equation (18). Recall from (14) that

$$R_{E,t} = \frac{EE_t - G_t \cdot \upsilon L_{t-1}}{E_{t-1}}.$$

Taking the expectation of the return at $t + 1$, and acknowledging the fact that the probability of regulatory capital shortfall is $\mathbb{E}_t[G_{t+1}] = 1 - \Pi_t(\hat{H}_t)$, we can write

$$\mathbb{E}_t[R_{E,t+1}] = \frac{R_{L,t}(1 - \lambda \mathbb{E}_t[H_{t+1}]) - R_t d_t - \upsilon\left[1 - \Pi_t(\hat{H}_t)\right] L_t}{E_t}.$$

Upon substitution for $d_t = L_t - E_t$ from the balance sheet identity, the expression becomes

$$\mathbb{E}_t[R_{E,t+1}] = R_t + \left\{R_{L,t}(1 - \lambda \mathbb{E}_t[H_{t+1}]) - R_t - \upsilon\left[1 - \Pi_t(\hat{H}_t)\right]\right\}\frac{L_t}{E_t}.$$

We now use the approximate first-order condition (17) to replace the regulatory component of the lending spread, $R_{L,t}(1 - \lambda \mathbb{E}_t[H_{t+1}]) - R_t$, with its optimal value. The expected return on bank capital under optimal behavior is thus given by

$$\mathbb{E}_t[R_{E,t+1}] \approx R_t + \upsilon\left\{1 - \Pi_t(\hat{H}_t) + \frac{\Pi_t'(\hat{H}_t)R_t}{\lambda(1-\varphi)R_{L,t}} \cdot \frac{E_t}{L_t} - \left[1 - \Pi_t(\hat{H}_t)\right]\right\}\frac{L_t}{E_t} \approx R_t + \upsilon\frac{\Pi_t'(\hat{H}_t)R_t}{\lambda(1-\varphi)R_{L,t}}.$$

We can now turn to the first-order condition (18) proper, and differentiate the following expected shareholder value (with deposits, $d_t = L_t - E_t$, substituted for again from the balance sheet identity):

$$\mathbb{E}_t\left\{\frac{\hat{\beta}\Lambda_{t+1}}{\Lambda_t}\left[R_{L,t}(1 - \lambda H_{t+1}) - R_t\right]L_t + R_t E_t - \upsilon L_t\left[1 - \Pi_t\left(\hat{H}_t\right)\right]\right\} - E_t - E_t \cdot \tfrac{1}{2}\xi_E \Omega_{E,t}^2,$$

w.r.t. bank capital, E_t. Bear in mind that both $\hat{H}_t$ and $\Omega_{E,t}$ are both functions of E_t:

$$\frac{\mathrm{d}\hat{H}_t}{\mathrm{d}E_t} = \frac{R_t}{\lambda(1-\varphi)R_{L,t}}\frac{1}{L_t},$$

$$\frac{\mathrm{d}\Omega_{E,t}}{\mathrm{d}E_t} = \frac{1}{E_t}.$$

The first-order condition is then

$$E_t: \quad \mathbb{E}_t\left\{\frac{\hat{\beta}\Lambda_{t+1}}{\Lambda_t}\left[R_t + v\frac{\Pi'_t(\hat{H}_t)R_t}{\lambda(1-\varphi)R_{L,t}}\right]\right\} = 1 + \xi_E\Omega_{E,t} + \tfrac{1}{2}\xi_E\Omega_{E,t}{}^2.$$

Since the adjustment cost term is zero in steady state, $\Omega_E = 0$, the quadratic term $\tfrac{1}{2}\xi_E\Omega_{E,t}{}^2$ has no first-order effect, and we drop it from the equation. Furthermore, we observe that the term inside the brackets on the left-hand side equals the optimal expected return on bank capital derived above. The approximate first-order condition thus becomes

$$E_t: \quad \mathbb{E}_t\left[\frac{\hat{\beta}\Lambda_{t+1}}{\Lambda_t}\right]\mathbb{E}_t\left[R_{E,t+1}\right] \approx 1 + \xi_E\Omega_{E,t},$$

which is the form reported in the main text.

A.4. Optimal Choices by Individual Borrowers

Bear in mind that the individual probability of default, p_t^i, defined in (7), which enters the the overall Lagrangian, (23), through the individual lending supply curves, is a function of the three choice variables, $R_{L,t}^i$, L_t^i, and K_t^i:

$$\frac{\partial p_t^i}{\partial R_{L,t}^i} = \frac{p_t^{i\prime}}{\varsigma R_{L,t}^i}, \qquad \frac{\partial p_t^i}{\partial L_t^i} = \frac{p_t^{i\prime}}{\varsigma L_t^i}, \qquad \frac{\partial p_t^i}{\partial K_t^i} = -\frac{p_t^{i\prime}}{\varsigma K_t^i}.$$

We start with the first-order condition w.r.t. $R_{L,t}^i$. This result will be subsequently used to substitute the multiplier on the lending supply curve, Ψ_t^i, in the other two conditions.

$$R_{L,t}^i: \quad \beta\mathbb{E}_t\left[\Lambda_{t+1}L_t^i(1+\Xi_{t+1})\right] - \Lambda_t\Psi_t^i\left(1 - \lambda p_t^i - \lambda p_t^{i\prime}/\varsigma\right) = 0.$$

From here, we concentrate out the term $\Lambda_t\Psi_t^i/(\varsigma L_t^i)$ on the left-hand side,

$$\frac{\Lambda_t\Psi_t^i}{\varsigma L_t^i} = \frac{\beta\mathbb{E}_t\left[\Lambda_{t+1}(1+\Xi_{t+1})\right]}{\varsigma - \varsigma\lambda p_t^i - \lambda p_t^{i\prime}}. \tag{47}$$

Next, we calculate the first-order conditions w.r.t. L_t^i and K_t^i,

$$L_t^i: \quad -\Lambda_t(1 + \Xi_t) + \beta \mathbb{E}_t \left[\Lambda_{t+1} R_{L,t}^i (1 + \Xi_{t+1}) \right] + \frac{\Lambda_t \Psi_t^i}{\varsigma L_t^i} R_{L,t}^i \lambda p_t^{i\prime} = 0,$$

$$K_t^i: \quad \Lambda_t \left(1 + \phi_K \Xi_t \right) - \beta \mathbb{E}_t \left[\Lambda_{t+1} R_{K,t+1} \right] - \frac{\Lambda_t \Psi_t^i}{\varsigma P_{K,t} K_t^i} R_{L,t} \lambda p_t^{i\prime} = 0,$$

and substitute for $\Lambda_t \Psi_t^i / (\varsigma L_t^i)$ using (47) in each:

$$L_t^i: \quad \Lambda_t(1 + \Xi_t) = \beta \mathbb{E}_t \left[\Lambda_{t+1} R_{L,t}^i (1 + \Xi_{t+1}) \left(1 + \frac{\lambda p_t^{i\prime}}{\varsigma - \varsigma \lambda p_t^i - \lambda p_t^{i\prime}} \right) \right],$$

$$K_t^i: \quad \Lambda_t \left(1 + \phi_K \Xi_t \right) = \beta \mathbb{E}_t \left[\Lambda_{t+1} R_{K,t+1} + R_{L,t} \frac{L_t^i}{P_{K,t} K_t^i} \Lambda_{t+1} (1 + \Xi_{t+1}) \frac{\lambda p_t^{i\prime}}{\varsigma - \varsigma \lambda p_t^i - \lambda p_t^{i\prime}} \right].$$

Finally, by introducing

$$k_t^i = \frac{L_t^i}{P_{K,t} K_t^i}, \qquad V_t^i = \frac{\lambda p_t^{i\prime}}{\varsigma - \varsigma \lambda p_t^i - \lambda p_t^{i\prime}},$$

we arrive at the first-order conditions reported in the main text:

$$L_t^i: \quad \Lambda_t (1 + \Xi_t) = \beta \mathbb{E}_t \left[\Lambda_{t+1} (1 + \Xi_{t+1}) \left(1 + V_t^i \right) \right] R_{L,t}^i, \tag{48}$$

$$K_t^i: \quad \Lambda_t \left(1 + \phi_K \Xi_t \right) = \beta \mathbb{E}_t \left\{ \Lambda_{t+1} \left[R_{K,t+1} + (1 + \Xi_{t+1}) R_{L,t}^i k_t^i V_t^i \right] \right\}. \tag{49}$$

APPENDIX B. GLOSSARY OF VARIABLES

A	Labor-augmenting productivity in local production sector
A_X	Labor productivity in exporting industries
C	Consumption
C_M	Direct import intensity of consumption
C_Y	Local content of consumption
D	Resident deposits
DD	Resident deposits at the beginning of period
d	Total deposits
dd	Ex-post realized value of deposits
E	Bank capital
EE	Ex-post realized value of bank capital
F	Non-resident deposits
G	Occurrence of shortfall in regulatory capital
H^i	Occurrence of individual default
H	Portfolio default ratio
$\hat{H}$	Cut-off portfolio default ratio
I	Investment
I_M	Direct import intensity of investment
I_Y	Local content of investment
J_F	Unexpected exchange rate valuation in non-resident bank deposits
J_L	Unexpected exchange rate valuation in bank loans
K	Physical capital in production
L^i	Individual loan
LL	Ex-post realized value of loan portfolio
L	Loan portfolio
M	Imports
M_X	Import intensity of exports (re-exports)
M_Y	Import intensity of local production
N	Hours worked
N_Y	Hours worked in local production
N_X	Hours worked in exporting industries
P_C	Price of final consumption
P_I	Price of final investment
P_K	Price of claims on physical capital

P_M	Price of imports (expressed in local currency)
P_X	Price of exports (expressed in local currency)
P_Y	Price of local intermediate production
P_Z	Price of local final goods
p	Probability of default
Q	Rental price of capital
q	Percentile of aggregate risk factor
R	Policy rate, bank deposit rate
R^*	Foreign currency bank deposit rate
$\hat{R}$	Required return on individual loan
R_E	Return on bank capital
R_K	Return on physical capital
R_L	Lending rate
R_W^*	World risk-free rate
r	Aggregate risk factor
S	Spot exchange rate
$\hat{S}$	Forward exchange rate
T	Terms of trade
U	Country risk premium
u	Idiosyncratic risk factor
V	Premium in household intertemporal conditions
W	Nominal wage rate
X	Exports
Y	Local production
Z	Demand for final locally produced goods
Λ	Lagrange multiplier on budget constraint
Ξ	Lagrange multiplier on financing constraint
Ω_E	Bank capital adjustment term
Ω_I	Investment adjustment term
Ω_P	Price adjustment term
Ω_W	Wage adjustment term
Ω_X	Export adjustment term
Ω_Y	Input factor adjustment term