Centrality-based Capital Allocations and Bailout Funds*

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Abstract

In this paper we look at the effect of capital rules on a large banking system that is connected through two main sources of systemic risk: correlated credit exposures and interbank lending. We apply capital rules that are based on a combination of individual bank characteristics and network interconnectivity measures based on the interbank lending network. We also define a bailout strategy based on different combinations of observable individual bank and network position measures. Both capital rules and bailout strategies are formulated with the intention of minimizing a measure of system wide losses. We use the German Credit Register, a data base with information on all of the 1,764 active banking groups in Germany. These data include all of banks' bilateral exposures, both those between banks and firms, and those among banks. Capital rules based on the so-called Opsahl centrality turn out to dominate any other centrality measure tested, apart from those based on total assets. Similarly, rules for the bailout fund are optimized based on network centrality measures and bank asset size. Finally, we compare measures of the total system loss across different types of capital allocation and sizes of the bailout fund.

JEL Classification: G21, G28, C15, C81

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1 Introduction

> "The difficult task before market participants, policymakers, and regulators with systemic risk responsibilities such as the Federal Reserve is to find ways to preserve the benefits of interconnectedness in financial markets while managing the potentially harmful side effects." Yellen (2013)

One of the obvious possible venues to avoid the "potentially harmful side effects" of interconnectedness is to tax interconnectivity through a capital charge, or to bailout banks based on their interconnectivity. This paper examines policies of mitigating the harmful side effects of interconnectedness in the context of a model of interbank contagion that gives a large weight to interconnectedness. Although the model is fairly classical in the way it handles contagion, it uses a very rich dataset of credit exposures of a large private domestic banking system, the fifth largest system in the world. What we find is that for both capital requirements and bailout strategies, for all measures of interconnectedness that we tried, the optimal rule that relies on bank asset size dominates any rule based on interconnectivity.

We focus on two main sources of systemic risk: correlated credit exposures and interbank connectivity. First, banks' balance sheets can be simultaneously affected by macro or industry shocks since the credit risk of their borrowers is correlated. Second, these shocks can, on the one hand, trigger the default of certain financial institutions and, on the other hand, capital of the entire system is eroded, making the system less stable. The latter effect is modeled in the interbank market. Since banks are highly connected through interbank exposures, we focus on those negative tail events in which correlated losses of their portfolios trigger contagion in the interbank market. Our model comes close to the framework proposed by Elsinger, Lehar, and Summer (2006) and Gauthier, Lehar, and Souissi (2012), combining common credit losses with interbank network effects and externalities in the form of asset fire sales . The aim of our paper is different. We propose a tractable framework to re-allocate capital for large financial systems in order to minimize contagion effects and costs of public bailout. We contrast two different capital allocations: the benchmark case, in which we allocate capital based on the risks in individual banks' portfolios, and new capital allocations based on some interbank network metrics that capture the potential contagion risk of the entire system.

We use the credit register for the German banking system, an interbank network that incorporates 1,764 different banking groups. The richness of our data set allows us to explore the implications of both the joint credit risk and the interconnected direct claims of the banking system. Because we utilize our access to the individual direct exposures of a bank group to each firm, along with a respectable credit model for firm default, we can derive the joint distribution functions of the likely shocks to the banks within the system. In addition, we have the bilateral exposures within the banking system, so that we can simulate the effect of the shocks as they work through the interbank network.

Thus, we combine correlated credit exposures, interbank contagion, and network analysis to develop rules that improve financial stability. In this sense, we propose two solutions: first a capital re-allocation that accounts for systemic contributions in the interbank market and second a bailout fund mechanism that can rescue certain financial institutions based on their relative systemic importance. Both regimes are optimized via some interconnectedness measures calculated from the interbank network, with the intention of making the financial system more robust.

The idea of tying capital charges to interbank exposures and interconnectedness to improve the stability of the banking system, i.e. to minimize expected social costs (e.g. arising from bailouts, growth effects, unemployment) is in the spirit of the regulatory assessment methodology for systemically important financial institutions (SIFIs) proposed by the Basel Committee on Banking Supervision (2011). In contrast to the latter, our study determines an optimal rule based on several interconnectedness measures and the size of total assets and compares the results under different capital allocations. One advantage of our approach is that interbank network topology builds on real balance sheet information of the German Large-Exposures Database, which covers around 99% of the interbank transacted volume. This allows us to identify interbank contagion channels and compute centrality measures accurately.

The second policy direction highlighted in this paper is estimating a proper mechanism and the size of a bailout fund for the financial sector. We start in a benchmark case where capital is allocated based on Value-at-Risk (VaR hereafter) at a security level that would provide comparably high protection against bankruptcy if interconnectedness were irrelevant. The basic idea of the bailout fund is to require less capital instead and to pool the aggregate capital relief in the fund. In the benchmark case, banks are required to hold capital equal to their VaR($\alpha = 99.9\%$) while the requirement in presence of the bailout fund is the VaR ($\alpha = 99\%$) only. The bailout fund uses its resources to rescue banks. The mechanism of rescuing banks is based on an importance ranking of financial institutions. Banks are ranked based on a trade-off rule between size and centrality measures. We consider several sizes of the bailout fund and compare expected losses with the results obtained from reallocation of capital using centrality measures.

We use a framework to assess the impact of different capital allocations on *financial stability*. We integrate a sound credit risk engine (i.e. CreditMetrics) to generate correlated shocks to credit exposures of the entire German banking system (1764 Monetary financial institutions (MFIs) active in the interbank (IB) market). This engine gives us the opportunity to focus on correlated tail events (endogenously determined by common exposures to the real economy). Our credit risk engine is associated to previous work that uses the *CreditMetrics* framework (see Bluhm, Overbeck, and Wagner (2003)). Based on a multi-factor credit risk model, this framework helps us to deal with risk concentration caused by large exposures to a single sector or correlated sectors. Even explicit common credit exposures are precisely addressed. *CreditMetrics* helps us to generate scenarios with large correlated losses across the entire banking system. These events are our main focus since capital across financial institutions is eroded simultaneously and the banking system becomes more prone to interbank contagion.

Moreover, we model interbank contagion based on Eisenberg and Noe (2001) and extend it to include bankruptcy costs as in Elsinger et al. (2006). This feature allows us to measure expected contagion losses and to observe the propagation process. To empirically exemplify our framework, we use several sources of information: the German central credit register (covering large loans), aggregated credit exposures (small loans), balance sheet data (e.g. total assets), market data (e.g. to compute sector correlations in the real economy or credit spreads), and data on rating transitions. The framework can be applied in any country or group of countries where this type of information are available. The main advantage of this framework is that policymakers can deal with large banking systems, making a regulation of systemic risk more tractable.

We focus on *capital reallocations* and try to minimize a *target function* with the scope of improving financial stability. We used several target functions, including, total system losses, second-round contagion effects (i.e. contagious defaults) or losses from fundamental defaults (i.e. banks that default from real-economy portfolio losses). We report results for *total expected bankruptcy costs* of defaulted banks. This measure of system losses is especially interesting as it represents a deadweight social loss. We determine capital allocations that improve the the stability of the financial system (as defined expected bankruptcy costs) based on *interconnectedness measures* gained from the IB network such as the degree, eigenvector and weighted eigenvector centrality, weighted betweenness, Opsahl centrality, closeness or the clustering coefficient. Balance sheet information such as total assets or total interbank assets and liabilities is applied in the same way as interconnectedness measures.

We also implement a *bailout fund mechanism* that offers a fairly priced insurance against the default of certain entities, however with priorities depending on a ranking that builds on banks' size and centrality in the IB market. We compare measures of the total-system loss across different types of capital allocation and sizes of the bailout fund. Our policy conclusions related to too-interconnected-to-fail versus too-big-to-fail externalities suggest that the latter dominates the former in terms of expected system losses.

This study is related to several strands of the literature including applications of network theory to economics, macro-prudential regulations and interbank contagion. Cont, Moussa, and e Santos (2010) find that not only banks' capitalization and interconnectedness are important for spreading contagion but also the vulnerability of neighbors. Gauthier et al. (2012) use different holdings- based systemic risk measures (e.g. MES, ΔCoVar, Shapley value) to reallocate capital in the banking system and to determine macroprudential capital requirements. Using the Canadian credit register data for a system of six banks, they rely on an "Eisenberg-Noe"type clearing mechanism extended to incorporate asset fire sales externalities. In contrast to their paper, we reallocate capital based on centrality measures extracted directly from the network topology of interbank market. Webber and Willison (2011) assign systemic capital requirements optimizing over the aggregated capital of the system. They find that systemic capital requirements are directly related to bank size and interbank liabilities. Tarashev, Borio, and Tsatsaronis (2010) claim that systemic importance is mainly driven by size and exposure to common risk factors. In order to determine risk contributions they utilize the Shapley value. In the context of network analysis, Battiston, Puliga, Kaushik, and Caldarelli (2012) propose a measure closely related to eigenvector centrality to assign the systemic relevance of financial institutions based on their centrality in financial network. Similarly, Soramäki and Cook (2012) try to identify systemically financial institutions in payment systems by implementing an algorithm based on absorbing Markov chains. Employing simulation techniques they show that the proposed centrality measure, SinkRank, highly correlates with the disruption of the entire system. In contrast to the latter two studies, we find that size ("Too-big-to-fail") dominates centrality measures ("Too-interconnected-to-fail") obtained from the interbank network. This antithesis might arise from utilizing different target functions in the optimization process.

As the subprime crisis has shown, banks do not have to be large to contribute to systemic risk, especially where banks are exposed to correlated risks (e.g. credit, liquidity or funding risk) via portfolios and interbank interconnectedness. Assigning risks to individual banks might be misleading. Some banks might appear healthy when viewed as single entities but they could threaten financial stability when considered jointly. Gai and Kapadia (2010) find that the level of connectivity negatively impacts the likelihood of contagion. Anand, Gai, Kapadia, Brennan, and Willison (2013) extend their model to include asset fire sale externalities and macroeconomic feedback on top of network structures, in order to stress-test financial systems. These studies illustrate the tipping point at which the financial system breaks down based on the severity of macroeconomic shocks that affect corporate probabilities of default or asset liquidity. Battiston, Gatti, Gallegati, Greenwald, and Stiglitz (2012) show that interbank connectivity increases systemic risk, mainly due to a higher contagion risk. Furthermore, Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013) claim that financial network externalities cannot be internalized and thus, in equilibrium, financial networks are inefficient. This creates incentives for regulators to improve welfare by bailing out SIFIs.

The rest of this paper is structured as follows. In Section 2 we describe our methodology and data sources briefly. Section 3 refers to our interconnectedness measures and the network topology of German interbank market. In Section 4 we describe our risk engine that generates common credit losses to banks' portfolios. Section 5 gives an overview of the contagion algorithm and Section 6 describes how capital is optimized. In Section 7 we present our main results and in Section 8 we provide some robustness checks. Section 9 concludes.

2 Data and methodology

2.1 Methodology

Figure 10 in Appendix A offers an overview of our simulation. Our procedure can be summarized in two stages, along with our initial condition.

In the initial state, we use each bank's measured portfolio, which is composed of large and small credit exposures (e.g. loans, credit lines, derivatives) to real economy and interbank (IB) borrowers. On the liability side, banks hold capital, either set to VaR (at $\alpha = 99.9\%$) in the benchmark case, or according to other capital allocations that partly rely on network measures.[1]Depositors and other creditors are senior to interbank creditors. We present in Figure 1 a standard individual bank balance sheet and the benchmark capital representation based on the credit risk model explained in Section 4.

In the first stage, we simulate correlated exogenous shocks to all banks' portfolios that take the form of returns on individual large loans (where loans that are shared among multiple lenders are accounted for) and aggregated small loans. Due to changes in value

[1]VaR at $\alpha = 99.9\%$ is calculated for each individual bank from 1 Mn simulations. When we calculate benchmark capital allocations, where contagion effects are ignored, the credit risk of German interbank (IB) loans (as assets) are treated "in the traditional way", similarly to large credit exposures to foreign banks (Sector 17, Table 2). The PDs of German interbank loans banks are set to the mean PD of this sector. In this way, there is a capital buffer to withstand potential losses from interbank defaults, however determined in a way as if interbank loans were yet another ordinary subportfolio.

of borrowers' assets, their credit ratings migrate (or they default), and banks make profits or losses on their investments in the real-economy sectors. At the end of this stage, in case of portfolio losses, capital deteriorates and some banks experience negative capital, and default. Thus, we are able to generate correlated losses that affect the capital of each bank simultaneously.[2]

In the second stage, we model interbank contagion. We apply an extended version of the fictitious contagion algorithm as introduced by Eisenberg and Noe (2001), augmented with bankruptcy costs and fire sales. Fundamental bank defaults generate losses to other interbank creditors and trigger some new defaults. Hence, bank defaults can induce domino effects in the interbank market. We refer to new bank failures from this stage as *contagious defaults*.

Finally, we repeat the previous stages for different capital allocations and for bailout fund mechanisms. We discuss the optimization procedure in Section 6. Moreover, Section 3 offers an overview of the interconnectedness measures calculated with the help of network analysis and utilized in the optimization process. The bailout fund mechanism, based on a set of assumptions, is detailed in Section 6.4.

2.2 Data sources

Our model builds on several data sources. In order to construct the interbank network, we rely on the Large Exposures Database (LED) of the Deutsche Bundesbank. Furthermore, we infer from the LED the portfolios of credit exposures (including loans, bond holdings, credit lines, derivatives, etc.) to the real economy of each bank domiciled in Germany. Since this data set is not enough to get the entire picture, since especially the smaller German banks hold plenty of assets falling short the reporting threshold of 1.5 million Euros for the LED, we use balance sheet data and the Borrower Statistics. Finally, we rely on stock market indices to construct a sector correlation matrix and we utilize a migration matrix for credit ratings from Standard and Poor's. Rating dependent spreads are taken from the Merill Lynch corporate spread indices.

2.2.1 Large Exposures Database (LED)

The Large Exposures Database represents the German central credit register.[3] Banks report exposures to a single borrower or a borrower unit (e.g., a banking group) which have a notional exceeding a threshold of €1.5 Mn The definition of an exposure includes bonds, loans or the market value of derivatives and off-balance sheet items.[4]In this paper,

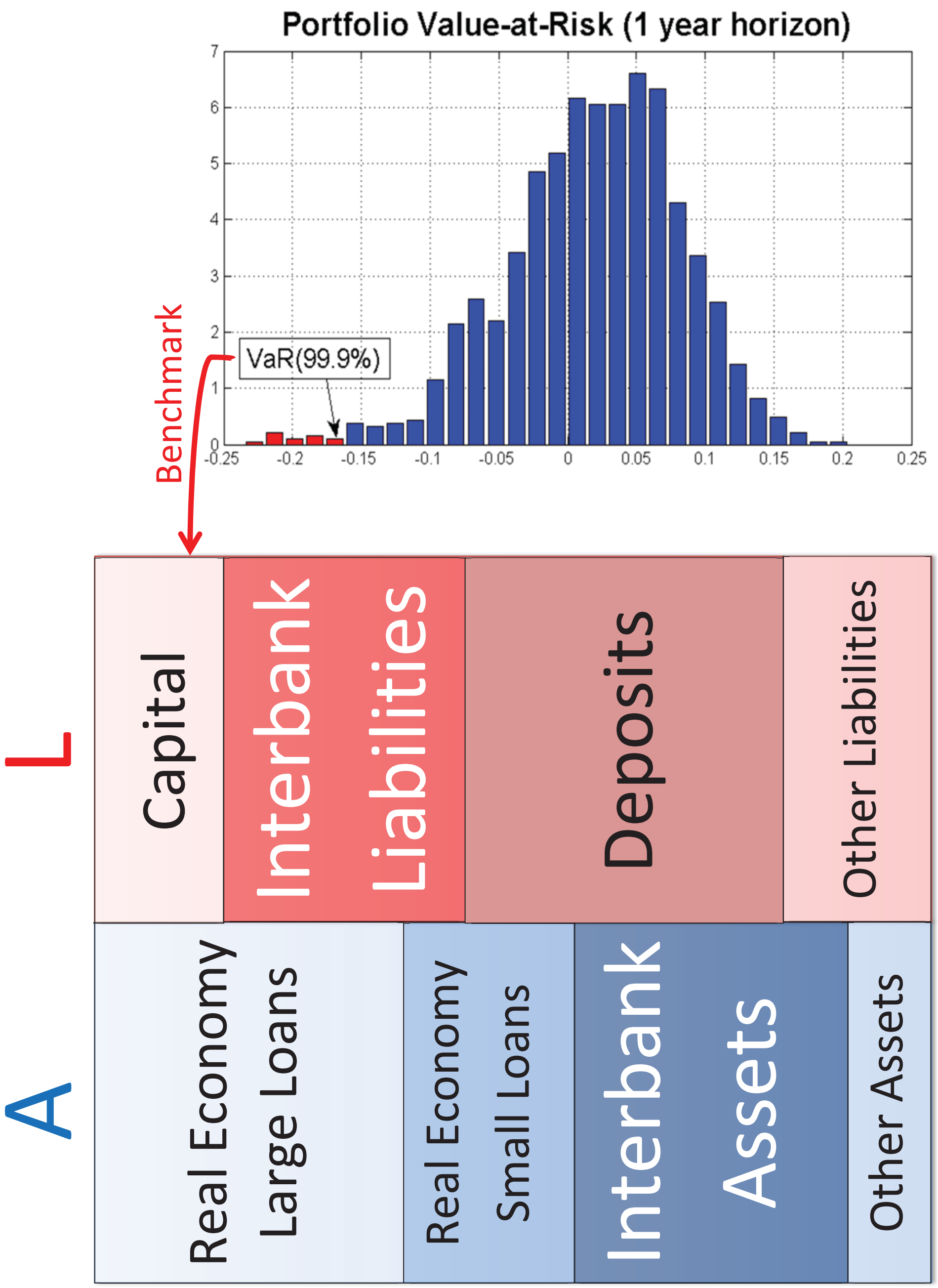
[2]By incorporating credit migrations and correlated exposures, we differ from most of the literature on interbank contagion that usually studies idiosyncratic bank defaults; see Upper (2011). Elsinger et al. (2006) and Gauthier et al. (2012) are remarkable exceptions.

[3]Bundesbank labels this database as *Gross- und Millionenkreditstatistik*. A detailed description of the database is given by *Schmieder (2006)*.

[4]Loan exposures also have to be reported if they are larger than 10% of a bank's total regulatory capital. They are not contained in our dataset of large exposures but represent a very small amount compared to the exposures that have to be reported when exceeding €1.5 Mn. Note that loans reported to the LED but not being part of our set of large loans are captured in the Borrower Statistics though and hence part of our sub-portfolios of "small loans"; see Section 2.2.2.

It is also important to notice that, while the data are quarterly, the loan volume trigger is not strictly related to an effective date. Rather, a loan enters the database once its actual volume has met the criterion

Figure 1: Individual bank balance sheet and benchmark capital



Note: In order to obtain the benchmark capital which equals VaR ($\alpha = 99.9\%$), we use a stylized bank balance sheet. The individual bank portfolio is composed of Large Loans (LL) and Small Loans (SL) to the real economy sectors (this distinction is made for risk modeling purposes). When we calculate the benchmark capital, interbank (IB) assets are treated similarly to large credit exposures (LL) to foreign banks (Sector 17, Table 2). German banks' PDs are set to the mean PD of this sector. In this way, there is a capital buffer to withstand potential losses from interbank defaults. "Other Assets" and "Other Liabilities" are ignored in our model.

we use the information available at the end of Q1 2011. The interbank market consists of 1764 active lenders. Including exposures to the real economy, they have in total around 400,000 credit exposures to more than 163,000 borrower units.[5]

Borrowers in the LED are assigned to 100 fine-grained sectors according to Deutsche Bundesbank (2009). In order to calibrate our credit risk model, we aggregate them to sectors that are more common in risk management (e.g. like those that fit with equity indices, which are the standard source of information used to calibrate asset correlations of the credit risk model). In our credit risk model, we use EUROSTOXX's 19 industry sectors (and later its corresponding equity indices). Table 2 lists risk management sectors and the distribution characteristics of the PDs assigned to them. There are two additional sectors (Households, including NGOs, and Public Sector) that are not linked to equity indices.[6] These 21 sectors represent the *risk model (RM) sectors* of our model.

The information regarding borrowers' PDs is included as well in LED. We report several quantiles, the mean and variance of the sector-specific PD distributions in Table 2. Since only Internal-Ratings-Based (IRB) banks report this kind of information, we draw random PDs from the empirical sector-specific distributions for the subset of borrowers without reported PDs.

2.2.2 Borrower and balance sheet statistics

While LED is an unique database, the threshold of €1.5 Mn of notional is a substantial restriction. Although large loans build the majority of money lent by German banks, the portfolios of most German banks would not be well represented by them. That does not come as a surprise if one takes into account that the German banking system is dominated (in numbers) by rather small S&L and cooperative banks. Many banks hold only few large loans while they are, of course, much better diversified. For 2/3 of banks the LED covers less than 54% of the total exposures. We need to augment the LED by information on smaller loans.

Bundesbank's borrower statistics (BS) dataset reports lending to German borrowers by each bank on a quarterly basis. Focusing on the calculation of money supply, it reports only those loans made by banks and branches situated in Germany; e.g. a loan originated in London office of Deutsche Bank would not enter the BS, even if the borrower is German. Corporate lending is structured in eight main industries: agriculture, basic resources and utilities, manufacturing, construction, wholesale and retail trade, transportation, financial intermediation and insurance, and services. Manufacturing and services are further divided into nine and eight sub-sectors, respectively. Loans to households and non-profit organizations are also reported in the BS database.[7]

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at some time throughout the quarter. Furthermore, the *definition of credit* triggering the obligation to report large loans is broad: besides *on-balance sheet* loans, the database conveys bond holdings as well as *off-balance sheet* debt that may arise from open trading positions, for instance. We use *total exposure* of one entity to another. Master data of borrowers contains its nationality as well as assignments to *borrower units*, when applicable, which is a proxy for joint liability of borrowers. We have no information regarding collateral in this dataset.

[5] Each lender is considered at aggregated level (i.e. as "Konzern"). At single entity level there are more than 4.000 different lending entities reporting data.

[6] We consider exposures to the public sector to be risk-free (and hence exclude them from our risk engine) since the federal government ultimately guarantees for all public bodies in Germany. See Section 4.

[7] A financial institution has to submit BS forms if it is an monetary financial institution (MFI), which

While lending is disaggregated into various sectors, the level of aggregation is higher than in LED, and sectors are different from the sectors in the risk model. Yet, there is a unique mapping from the many LED sectors to the ones of the borrower statistics (BS) sectors. We use this mapping and the one from the LED sectors to the risk model (RM) sectors to establish a compound mapping from BS to RM sectors. They are based on relative weights gained from the LED that are assumed also to hold for small loans. Detailed information on the mapping is available on request.

In addition to borrower statistics, we also use some figures from the monthly balance sheet statistics that is also reported to Bundesbank. These sheets contain lending to domestic insurances, households, non-profit entities, social security funds, and so-called “other financial services” companies. Lending to foreign entities is given by a total figure that covers all lending to non-bank companies and households. The same applies to domestic and foreign bond holdings which, if large enough, are also included in LED.

2.2.3 Market data

The credit ratings migration matrix is presented in Table 5 and is provided by S&P. Market credit spreads are derived from a daily time series of Merill Lynch option-adjusted euro spreads covering all maturities, from April 1999 to June 2011. Correlations are computed based on EUROSTOXX weekly returns of the European sector indices for the period April 2006 - March 2011 covering most of the recent financial crisis.[8]

3 Interbank network

Economic literature related to network analysis has exploded since the beginning of the financial crisis in 2007, with many papers discussing the network properties of interbank exposures throughout the world. For example, focusing on centrality and connectivity measures of the financial network, Minoiu and Reyes (2011) analyze the dynamics of the global banking interconnectedness over a period of three decades. Financial networks are defined by a set of nodes (financial institutions) that are linked through direct edges that represent bilateral exposures between them. Focusing on the German interbank market Craig and von Peter (2010) find that a core-periphery model can be well fitted to the German interbank system. Core banks are a subset of all intermediaries (those banks that act as a lender and as a borrower in the interbank market) that share the property of a complete sub-network (there exist links between any two members of the subset). According to their findings, the German interbank market exhibits a *tiered structure*. As they empirically show, this kind of structure is highly persistent (stable) over time. Sachs (2010) concludes that the distribution of interbank exposures plays a crucial role for the stability of financial networks. She randomly generates interbank liabilities matrices and investigates contagion effects in different setups. She finds support for the “knife-edge” or

does not necessarily coincide with being obliged to report to LED. There is one state-owned bank with substantial lending that is exempt from reporting BS data by German law. Backed by a government guarantee, we consider this bank neutral to interbank contagion.

[8]We present market credit spreads together with the sector correlation matrix in Table 4 in Appendix A.

tipping-point feature (as mentioned by Haldane (2009)), the non-monotonic completeness property of highly interconnected networks.

3.1 The German interbank market

In Q1 2011, there were 1921 MFIs in Germany, holding a total balance sheet of €8233 Bn[9]. The German banking system is composed of three major types of MFIs: 282 commercial banks (including four big banks and 110 branches of foreign banks) that hold approximately 36% of total assets, 439 saving banks (including 10 *Landesbanken)* that hold roughly 30% of system's assets and 1140 credit cooperatives (including 2 regional institutions) that hold around 12%. Other banks (i.e. mortgage banks, building and loan associations and special purpose vehicles) are in total 60 MFIs and represent approx. 21% of system's balance sheet.

Our interbank (IB) network consists of 1764 active banks (i.e. aggregated banking groups). These banks are actively lending and/or borrowing in the interbank market. They hold total assets worth €7791 Bn, from which 77% represent large loans and 23% small loans.

Table 1 presents the descriptive statistics of the main characteristics and network measures of the German banks utilized in our analysis.[10]

The average size of an bank individual IB exposure is around €1 Bn. As figures show, there are few very large total IB exposures, since the mean is between quantiles 90^{th} and 95^{th}, making the distribution highly skewed. Similar properties are observed for total assets, large loans and out degrees, sustaining the idea of a tiered system with few large banks that act as interbank broker-dealers connecting other financial institutions (see Craig and von Peter (2010)).[11]

3.2 Centrality measures

In order to assign the interconnectedness relevance/importance to each bank of the system we rely on several centrality characteristics . The descriptive statistics of our centrality measures are summarized in Table 1. The information content of an interbank network is best summarized by a matrix X in which each cell x_{ij} corresponds to the liability amount of bank i to bank j. As each positive entry represents an edge in the graph of interbank lending, an edge goes from the *borrowing* to the *lending* node. Furthermore, the adjacency matrix (A) is just a mapping of matrix X, in which $a_{ij} = 1$ if $x_{ij} > 0$, and $a_{ij} = 0$ otherwise. In our case, the network is directed, and our matrix is weighted, meaning that we use the full information regarding an interbank relationship, not only its existence. We do not net bilateral exposures. Our network has a *density* of 0.7%

[9] According to Deutsche Bundesbank's Monthly Report (March 2011).

[10] In Section 8, we discuss in detail the properties of interbank liabilities' distribution and compare them over time. Moreover, we provide a discussion over the dynamics of network measures.

[11] One aspect that needs to be mentioned here is that the observed interbank network is not the complete picture, since interbank liabilities of German banks raised outside Germany are not reported to LED. For example, LED does not capture a loan made by Goldman Sachs to Deutsche Bank in London. This aspect might bias downwards exposures and centrality measures of big German banks that might borrow/lend outside the German interbank market.

Table 1: Interbank (IB) market and network properties

	Quantiles							**mean**	**std dev**
	5%	**10%**	**25%**	**50%**	**75%**	**90%**	**95%**		
Total IB Assets*	7591	13,089	35,553	100,450	310,624	868,299	1,647,666	990,433	7,906,565
Total IB Liabilities*	2640	6,053	19,679	61,450	180,771	527,460	1,212,811	990,433	7,782,309
Total Assets*	37,798	63,613	160,741	450,200	1,290,698	3,412,953	7,211,608	4,416,920	37,938,474
Total Large Loans*	8719	17,293	60,692	208,475	675,199	2,054,840	4,208,323	3,424,445	30,221,363
Total Small Loans*	8906	34,932	85,741	217,516	550,855	1,287,390	2,253,149	992,475	8,748,757
Out Degree	1	1	1	2	4	9,1	16	13	82
In Degree	1	1	4	9	14	19	25	13	37
Total Degree	2	3	5	11	18	27	38	26	111
Opsahl Centrality	51.5	80.6	165.8	345.9	791.6	2071.2	4090	3342	24,020
Eigenvector Centrality	0.000003	0.000019	0.000078	0.000264	0.001092	0.004027	0.009742	0.003923	0.023491
Weighted Betweenness	0	0	0	0	0	401.5	6225.8	10,491	82,995
Weighted Eigenvector	0.000004	0.000012	0,000042	0,000142	0.000611	0.002217	0.004839	0.003148	0.023607
Closeness Centrality	253.2	328.8	347.8	391.8	393,6	411.5	427.4	371.22	69.13
Clustering Coefficient	0	0	0	0.00937	0.04166	0.12328	0.16667	0.0379	0.0677
No of obs	88	176	441	882	1323	1587	1675	1764	1764
Number of banks	1764**								
Number of links	22,752								

Note: * in thousand € ; ** no of banks active in the interbank market. Data point: 2011 Q1 given that it includes 1764 nodes and 22,752 links.[12] This sparsity is typical for interbank networks (see for example Soramäki, Bech, Arnolda, Glass, and Beyeler, 2007).

As outlined by Newman (2010), the notion of centrality is associated with several metrics. In economics the most-used measures are: *out degree* (the number of links that originate from each node) and *in degree* (the number of links that end at each node), *the strength* (the aggregated sum of interbank exposures), *betweenness centrality* (the inverse of the number of shortest paths[13] that pass through a certain node), *Eigenvector centrality* (centrality of a node given by the importance of its neighbors) or clustering coefficient (how tightly connected is a node to its neighbors). The strength of a link is defined by the size (volume) of exposure and the direction (ingoing or outgoing) shows whether money has been lent/borrowed (i.e. *out degree* refers to borrowing relationships and *in degree* to lending ones).[14]

Out Degree is one of the basic indicators and it is defined as the total number of direct interbank creditors that a bank borrows from:

$$k_i = \sum_{j}^{N} a_{ij} \tag{1}$$

Similarly, we can count the number of lending relationships from i to j (*in degree*). Since is a directed graph, we distinguish in our network analysis between *out degrees* and *in degrees*, referring to borrowing and lending.. *D*egree is the sum of *out degree* and *in degree*. In economic terms, for example in case of a bank default, a certain number (the *out degree*) of nodes will suffer losses in the interbank market.

Given that our matrix is weighted, we are able to compute each node's *strength*, that is its total amount borrowed from other banks:

$$s_i = \sum_{j}^{N} x_{ij} \tag{2}$$

The *strength* of a node is represented in Table 1 as *total IB liabilities*. Similarly, we construct the strength of the interbank assets.

The degree distribution shows a tiered interbank structure. A few nodes are connected to many banks. For example, 20 banks (around 1%) lend to more than 100 banks each. On the borrowing side, 30 banks have a liability to at least 100 banks. These banks are part of the *core* of the network as defined by Craig and von Peter (2010). In terms of strength of interbank borrowing, 158 banks have a total IB borrowed amount in excess of €1 Bn while only 27 banks have total interbank liabilities in excess of €10 Bn. On the assets side, 103 banks lend more than €1 Bn and 25 financial institutions have German interbank assets in excess of €10 Bn.

[12] The *density* of a network is the ratio of the number of existing connections divided by the total number of possible links. In our case of a directed network, the total number of possible links is $1764 \times 1763 = 3,109,932$

[13] A *path* from node A to B is a consecutive sequence of edges starting from A and ending in B. Its *length* is the number of edges involved. The (directed) *distance* between A and B is the minimum length of all paths between them.

[14] For a detailed description of centrality measures related to interbank markets see Gabrieli (2011) and Minoiu and Reyes (2011).

Opsahl, Agneessens, and Skvoretz (2010) introduce a novel centrality measure that we label *Opsahl centrality*. This measure combines the out degree (eq. 1) with the borrowing strength (total IB liabilities, eq. 2) of each node, using a tuning parameter φ:[15]

$$OC_i = k_i^{(1-\varphi)} \times s_i^{\varphi} \tag{3}$$

The intuition of a node with high Opsahl centrality is that, in the event of default, this node is able to infect many other banks with high severity. This translates into a higher probability of contagion (conditional on the node's default) compared with other nodes. We discuss these network measures of centrality further in an appendix.

4 Credit risk model

Our credit risk engine is a one-period model, and all parameters are calibrated to a 1-year time span. In order to model credit risk, we utilize lending information from two data sources at different levels of aggregation: large loans and small loans. These loans are given to the "real economy". Since borrowers of large loans are explicitly known, along with various parameters such as the loan volume, probability of default and sector, we can model their credit risk with high precision. When simulating defaults and migrations of individual borrowers, we can even account for the fact that loans given by different banks to the same borrower should migrate or default simultaneously.

We cannot keep this level of precision for small loans because we only know their exposures as a lump-sum to each sector. Accordingly, we simulate their credit risk on portfolio level.[16]

4.1 Large loans

In modeling credit portfolio risk we closely follow the ideas of CreditMetrics.[17] We start with a vector $Y \sim N(0, \Sigma)$ of systematic latent factors. Each component of Y corresponds to the systematic part of credit risk in one of the *risk modeling (RM)* sectors. The random vector is normalized such that the covariance matrix Σ is actually a correlation matrix (see Table 4a). In line with industry practice, we estimate correlations from co-movements of stock indices.[18] For each borrower k in RM sector j, the systematic factor Y_j assigned to the sector is coupled with an independent idiosyncratic factor $Z_{j,k} \sim N(0, 1)$. Thus, the stylized asset return of borrower (j, k) can be written as:

$$X_{j,k} = \sqrt{\rho}Y_j + \sqrt{1-\rho}Z_{j,k}.$$

[15]In our analysis we set $\varphi = 0.5$, leading to the geometric mean between strength and degree.

[16]The parameters of our model are presented in Table 3 in Appendix A.

[17]For a detailed description of this model see Gupton, Finger, and Bhatia (1997).

[18]Our correlation estimate is based on weekly time series of 19 EUROSTOXX industry indices from April 2006 to March 2011. The European focus of the time series is a compromise between a sufficiently large number of index constituents and the actual exposure of the banks in our sample, which is concentrated on German borrowers but also partly European wide.

The so-called intra-sector asset correlation ρ is common to all sectors.[19] The latent factor $X_{j,k}$ is mapped into rating migrations via a threshold model. We use 16 S&P rating classes including notches AAA, AA+, AA,... ,B–, plus the aggregated "junk" class CCC–C. Moreover, we treat the default state as a further rating (D) and relabel ratings as numbers from 1 (AAA) to 18 (default). Let R_0 denote the initial rating of a borrower and R_1 the rating one period later. A borrower migrates from R_0 to rating state R_1 whenever

$$X \in [\theta(R_0, R_1),\ \theta(R_0, R_1 - 1)]\,,$$

where θ is a matrix of thresholds associated with migrations between any two ratings. For given migration probabilities $p(R_0, R_1)$ from R_0 to R_1, the thresholds are chosen in a way such that[20]

$$\mathbf{P}\left(\theta(R_0, R_1) < X_{j,k} \leq \theta(R_0, R_1 - 1)\right) = p(R_0, R_1),$$

which is achieved by formally setting $\theta\left(R_0, 18\right) = -\infty$, $\theta\left(R_0, 0\right) = +\infty$ and calculating

$$\theta\left(R_0, R_1\right) = \Phi^{-1}\left(\sum_{R>R_1} p\left(R_0, R\right)\right),\ 1 \leq R_0, R_1 \leq 17.$$

The present value of each non-defaulted loan depends on notional value, rating, loan rate, and time to maturity. In this section we ignore the notional value and focus on D, the *discount factor*. A loan is assumed to pay an annual loan rate C until maturity T, at which all principal is due. We set T equal to a uniform value of 4 years, which is the digit closest to the mean maturity of 3.66 estimated from the borrower statistics.[21] Payments are discounted at a continuous rate $r_f + s\left(R\right)$ where r_f is the default-free interest rate and $s(R)$ is the rating-specific credit spread; see Table 4b. The term structure of spreads is flat. We ignore the risk related to the default-free interest rate and set $r_f = 2\%$ throughout. The discount factor for a non-defaulted, R-rated loan at time t is

$$D\left(C, R, t, T\right) \equiv \sum_{u=t+1}^{T} \left(C + I_{\{u=T\}}\right) e^{-\left(r_f + s(R)\right)(u-t)}. \tag{4}$$

If the loan is not in default at time 1, it is assumed to have just paid a coupon C. The remaining future cash flows are priced according to (4), depending on the rating at time $t = 1$, so that the loan is worth $C + D\left(C, R_1, 1, T\right)$. If the loan has defaulted at time 1, it is worth $(1 + C)\left(1 - LGD\right)$, where LGD is an independent random variable drawn from a beta distribution with expectation 0.39 and standard deviation 0.34.[22] This means, the

[19] This could be relaxed but would require the inclusion of other data sources. In the simulations we use a value of 0.20, which is very close to a value reported by Zeng and Zhang (2001). It is the average over their sub-sample of firms with the lowest number of missing observations.

[20] In our model we use the 1981–2010 average one-year transition matrix for a global set of corporates from Standard and Poor's (2011).

[21] The borrower statistics report exposures in three maturity buckets. Exposure-weighted averages of maturities indicate only small maturity differences between BS sectors. If we wanted to preserve them, the differences would shrink even more in the averaging process involved in the mapping from BS sectors to RM sectors. By setting the maturity to 4 years we simplify loan pricing substantially, mainly since calculating sub-annual migration probabilities is avoided.

[22] Here we have chosen values reported by Davydenko and Franks (2008), who investigate LGDs of loans to German corporates, similar to Grunert and Weber (2009), who find a very similar standard deviation of 0.36 and a somewhat lower mean of 0.275.

same relative loss is incurred on loan rates and principal. The spreads are set such that each loan is priced at par at time 0:

$$C(R_0) \equiv e^{r_f + s(R_0)} - 1, \quad D(C(R_0), R_0, 0, T) = 1.$$

Each loan generates a return equal to

$$ret(R_0, R_1) = -1 + \begin{cases} D(C(R_0), R_1, 1, T) + C(R_0) & \text{if } R_1 < 18 \\ (1 + C(R_0))(1 - LGD) & \text{if } R_1 = 18 \end{cases},$$

which has an expected value of

$$\begin{aligned} \mathbf{E}ret(R_0, R_1) &= -1 + \sum_{R_1 < 18} p(R_0, R_1)\left[D(C(R_0), R_1, 1, T) + C(R_0)\right] \\ &\quad + p(R_0, 18)(1 + C(R_0))(1 - \mathbf{E}LGD). \end{aligned}$$

Besides secure interest, the expected return incorporates credit risk premia that markets require in excess of the compensation for expected losses. We assume that the same premia are required by banks and calibrate them to market spreads, followed by slight manipulations to achieve monotonicity in ratings.[23]

Having specified migrations and revaluation on a single-loan basis, we return to the portfolio perspective. Assuming that k in (j, k) runs through all sector-j loans of all banks, we denote by $R_1^{j,k}$ the rating of loan (j, k), which is the image of asset return $X_{j,k}$ at time 1. If bank i has given a (large) loan to borrower (j, k), the variable $LL_{i,j,k}$ denotes the notional exposure; otherwise, it is zero. Then, the euro return on the large loans of bank i is

$$ret_{\text{large},i} = \sum_{j,k} LL_{i,j,k}\, ret\left(R_0^{j,k}, R_1^{j,k}\right).$$

This model does not only account for common exposures of banks to the same sector but also to individual borrowers. If several banks lend to the same borrower, which may concern a large exposure, they are simultaneously hit by its default or rating migration.

4.2 Small loans

As previously described, for each bank we have further information on the exposure to loans that fall short of the €1.5 Mn reporting threshold of the credit register. However, we know the exposures only as a sum for each RM sector so that we are forced to model its risk portfolio-wise. However, as portfolios of small total volume tend to be less diversified than larger ones, we steer the amount of idiosyncratic risk adding to the systematic risk of each sector's sub-portfolio by its volume.

[23] Market spreads are derived from a daily time series of Merill Lynch euro corporate spreads covering all maturities, from April 1999 to June 2011. The codes are ER10, ER20, ER30, ER40, HE10, HE20, and HE30. Spreads should rise monotonically for deteriorating credit. We observe that the premium does rise in general but has some humps and troughs between BB and CCC. We smooth them out as they might have substantial impact on bank profitability but lack economic reason. To do so, we fit $\mathbf{E}return(R_0)$ by a parabola, which turns out to be monotonous, and calibrate spreads afterwards to make the expected returns fit the parabola perfectly. Spread adjustments have a magnitude of 7bp for A– and better, and 57bp for BBB+ and worse. Final credit spreads are presented in Table 4b.

In this section, we sketch the setup only; details are found in Appendix C. We consider the portfolio of a bank's loans belonging to one sector; they are commonly driven by that sector's systematic factor Y_j, besides idiosyncratic risk. If we knew all individual exposures and all initial ratings, we could just run the same risk model as for the large loans. It is central to notice that the individual returns in portfolio j would be independent, *conditional* on Y_j. Hence, if the exposures were extremely granular, the corresponding returns would get very close to a deterministic function of Y_j, according to the conditional law of large numbers.[24] We do not go that far since small portfolios will not be very granular; instead, we utilize the central limit theorem for conditional measures, which allows us to preserve an appropriate level of idiosyncratic risk. Once Y_j is known, the total of losses on an increasing number of loans converges to a (conditionally!) normal random variable. This conditional randomness accounts for the presence of idiosyncratic risk in the portfolio. Correspondingly, our simulation of losses for small loans involves two steps. First, we draw the systematic factor Y_j. Second, we draw a normal random variable, however with mean and variance being functions of Y_j that match the moments of the exact Y_j-conditional distribution. The Y_j-dependency of the moments is crucial to preserve important features of the exact portfolio distribution, especially its skewness. Also, it preserves the correlation between the losses of different banks in their sector-j portfolios.

An exact fit of moments is not achievable for us as it would require knowledge about individual exposures and ratings of the small loans, but an approximate fit can be achieved based on the portfolio's Hirschman-Herfindahl Index (HHI) of exposures. As we also do not explicitly know the portfolio HHI, we employ an additional large sample of small loans provided by a German commercial bank, to estimate the relationship between portfolio size and HHI. The estimate is sector specific. It provides us with a *forecast* of the actual HHI based on the portfolio's size and the sector. The forecast is the second input to the function that gives us Y_j-conditional variances of the (conditionally normal) portfolio losses. Details are described in Section C.1.

This modeling step ends up with a (euro) return on each bank's small loans, denoted by $ret_{\text{small},i}$ (see eq. 13).

5 Modeling contagion

As introduced in Section 2, we differentiate between *fundamental defaults* and *contagious defaults* (see Elsinger et al. (2006) or Cont et al. (2010), for instance). Fundamental defaults are related to losses from the credit risk of "real economy" exposures, while contagious defaults are related to the interbank credit portfolio (German only). [25]

Moreover, we construct an *interbank clearing mechanism* based on the standard assumptions of interbank contagion (see e.g.Upper, 2011): First, banks have *limited liability.* Next, interbank liabilities are senior to equity but *junior* to non-bank liabilities (e.g. deposits). Losses related to bank defaults are *proportionally* shared among inter-

[24]This idea is the basis of asymptotic credit risk models. The model behind Basel II is an example of this class.

[25]Foreign bank exposures are included in Sector 17 of the "real economy" portfolio, since we have to exclude them from the interbank network. Loans made by foreign banks to German financial entities are not reported to LED.

bank creditors, based on the share of their exposure to total interbank liabilities of the defaulted bank. In other words, its interbank creditors suffer the same loss-given-default. [26] Finally, non-bank assets of a defaulted bank are liquidated at a certain discount. This extra loss is referred to as *fire sales* and is captured by *bankruptcy costs* , defined below. The clearing mechanism closely follows Eisenberg and Noe (2001) and the extension by Elsinger et al. (2006) to bankruptcy costs. It is described in detail in an appendix.

5.1 Losses and bankruptcy costs

In our analysis, we are particularly interested in bankruptcy costs since they represent a dead-weight loss to the economy. We model them as the sum of two parts. the first one is a function of bank's total assets, because there is empirical evidence for a positive relationship between size and bankruptcy costs of financial institutions; see Altman (1984).

The second part incorporates fire sales and their effect on the value of the defaulted bank's assets. For their definition, recall that each bank makes a return on its large and small loans. We switch the sign and define losses

$$L_i^{\text{real}} \equiv -\left(ret_{\text{large},i} + ret_{\text{small},i}\right) \,, \tag{5}$$

highlighting that we deal here with losses related to the real economy. If a loss exceeds the bank's capital K_i, it defaults for *fundamental* reasons, and the bank's creditors suffer a loss the extent of which equals $\max\left(0, L_i^{\text{real}} - K_i\right)$. Note that this is a loss before bankruptcy costs and contagion. In the whole economy, the fundamental losses add up to

$$\overline{L}_{\text{real}} \equiv \sum_i \max\left(0, L_i^{\text{real}} - K_i\right)$$

It is this total fundamental loss in the system by which we want to proxy lump-sum effects of fire sales. The larger $\overline{L}_{\text{real}}$, the more assets will the creditors of defaulted banks try to sell quickly, which puts asset prices under pressure. We proxy this effect by a system-wide relative loss ratio λ being monotonic in $\overline{L}_{\text{real}}$. In total, if bank i defaults, we define bankruptcy costs as the sum of two parts related to total assets and fire sales:

$$BC_i \equiv \phi\left(TotalAssets_i - L_i^{\text{real}}\right) + \lambda\left(\overline{L}_{\text{real}}\right)\max\left(0, L_i^{\text{real}}\right) \tag{6}$$

We consider ϕ the proportion of assets lost due to litigation and other legal costs. In our analysis we set $\phi = 5\%$.[27] It is rather for convenience than for economic reasons that we set the monotonic function λ equal to the cumulative distribution function of $\overline{L}_{\text{real}}$. Given this choice, the more severe total fundamental losses in the system are, the closer λ gets to 1.[28]

[26] We do not have any information related to collateral or the seniority of claims.

[27] Our results remain robust also for other values $\phi \in \{1\%, 3\%, 10\%\}$. Alessandri, Gai, Kapadia, Mora, and Puhr (2009) and Webber and Willison (2011) use contagious bankruptcy costs as a function of total assets, and set ϕ to 10%. Given the second term of our bankruptcy costs function that incorporates fire sales effects, we reach at a stochastic function with values between 5% and 15% of total assets.

[28] We acknowledge that real-world bankruptcy costs would probably be sensitive to the amount of interbank credit losses, which we ignore. This simplification, however, allows us to calculate potential bankruptcy costs before we know which bank exactly will default through contagion, so that we do not have to update bankruptcy costs in the contagion algorithm. If we did, it would be extremely difficult to preserve proportional loss sharing in the Eisenberg-Noe allocation.

6 Optimization

This paper compares losses in the system, according to some loss measure and subject to different capital and bailout rules. The comparison allows us to minimize the loss measure. However, an optimal bailout or capital rule is not the sole focus of this paper. There are two reasons for this.

First, the loss functions that would be optimized are only few of many that could be the optimal outputs of the system. We discuss different target functions (i.e. system losses) in Section 6.2. Our emphasis is on how the different loss functions interact with the different capital or bailout rules, to provide insights into how the rules work, i.e..

Second, the rules themselves are subject to a variety of restrictions. These include the fact that the rules must be simple and easily computed from observable characteristics, and they should preferably be smooth to avoid cliff effects. Simplicity is important not just because of computational concerns. Simple formal rules are necessary to limit discretion on the ultimate outcome of capital rules. Too many model and estimation parameters set strong incentives for banks to lobby for a design in their particular interest. While this is not special to potential systemic risk charges, it is clear that those banks who will most likely be confronted with increased capital requirements are the ones with the most influence on politics. Vice versa, simplicity can also help to avoid arbitrary punitive restrictions imposed upon individual banks. In this sense, the paper cannot offer deliberately fancy first-best solutions for capital requirements.

In our analysis we keep the total amount of capital in the system constant; otherwise, optimization would be simple but silly: more capital for all, ideally 100% equity funding for banks. As a consequence, when we require some banks to hold more capital, we are willing to accept that others may hold less capital as in the benchmark case. Taken literally, there would be no lower limit to capital except zero. However, we also believe that there should be some minimum capital requirement that applies to all banks for reasons of political feasibility, irrespectively of their role in the financial network. Implementing a uniform maximum default probability for all banks might be one choice.

Finally, there are also technical reasons for simplicity. Each evaluation step in the optimization requires a computationally expensive full-fledged simulation of the financial system. We therefore restrict ourselves to an optimization over one parameter, focusing on whether the various network measures analyzed are able to capture aspects actually relevant for total system losses at all.

6.1 Capital allocations

In this subsection, we introduce a range of simple capital rules over which we minimize the total system loss. The range represents intermediate steps between two extremes.

One extreme is our benchmark case, by which we understand capital requirements in the spirit of Basel II, i.e., a system focused on a bank's portfolio risk (and not on network structure). For our analysis we require banks to hold capital equal to its portfolio VaR on a high security level $\alpha = 99.9\%$, in line with the level used in Basel II rules for the banking book. There is one specialty of this VaR, however. In line with Basel II again, the benchmark capital requirement treats interbank loans just as other loans. For the determination of bank i's benchmark capital $K_{\alpha,i}$ (and *only* for this exercise), each bank's

German interbank loans (on the asset side) are merged with loans to foreign banks into portfolio sector 17 where they contribute to losses just as other loans.[29] In the whole system, total required capital adds up to $TK_\alpha \equiv \sum_i K_{\alpha,i}$.

The other extreme has one thing in common with the benchmark case: total required capital in the system must be the same. On bank level, required capital consists of two components in this case. One is given by a comparably low amount $K_{\min,i}$ derived under a "mild" rule, which we set to the portfolio VaR on a moderate security level of 0.99, again treating interbank loans as ordinary loans. This limits bank PDs to values around 1%, which could be a politically acceptable level.[30] The other component is allocated by means of a centrality measure. Given $Centrality_i$ to be one of the measures introduced in Section 3.2, the simple idea is to allocate the system-wide hypothetical capital relief from the "mild" capital rule proportionally to the centrality measure:

$$K_{\text{centr},i} \equiv K_{\min,i} + \left(TK_\alpha - TK_{\min}\right) \frac{Centrality_i}{\sum_i Centrality_i}, \tag{7}$$

where $TK_{\min} \equiv \sum_i K_{\min,i}$. Having defined the two extremes K_α and K_{centr}(now understood as vectors) , the range of regimes we optimize over is given by the straight line between them, which automatically keeps total required capital constant and guarantees that bank individual capital is always positive, since both extremes are so:[31]

$$\tilde{K}(\beta) \equiv \beta K_\alpha + (1-\beta) K_{\text{centr}}, \quad 0 \leq \beta \leq 1.$$

The parameter β is subject to optimization.

In general, the approach is not limited to one centrality measure. We could define multiple centrality based "extreme" (or better corner) regimes that build together with the benchmark allocation a set of M capital vectors and optimize over a (now M-dimesional) vector β under the restrictions $\sum_j \beta_j = 1$ and $0 \leq \beta_j \leq 1$ for all j. This procedure is numerically expensive and not yet carried out. Clearly, *re*-allocating capital implies that some banks will have to hold more capital (implying lower PDs) and some less (higher PDs) than in the benchmark case. This might be politically undesirable, requiring a modification of our approach. For example, regulators could agree on a total amount of additional capital to be held in the system, compared to TK_α, that would be allocated using centrality based rules.

In this section we have dealt with capital *requirements* only. To assess their consequences on actual systemic risk, we also have to specify in which way banks intend to

[29] Default probabilities for these loans are taken from the Large Exposure Database in the same way as for loans to the real economy.

[30] Actual bank PDs after contagion can be below or above 1 percent, depending on whether defaults in a risk model where interbank loans are directly driven by systematic factors, are more frequent than in presence of contagion (but without direct impact of systematic factors). However, the probability of fundamental losses cannot exceed 1 percent; see Section 5.1. (In principle, they could, but this would require a – counterfactual – hedging effect between interbank and normal corporate loans.)

[31] We could also intermediate between the two extremes in other ways, e.g. by flooring at $K_{\min}$ and appropriate subsequent rescaling, to keep total capital constant. However, we would expect more interesting results from a (yet outstanding) optimization over linear combinations of multiple centrality measures, than of such kind of modification. In addition, our approach has the advantage that is is differentiable in β rather than Lipschitz continuous only, as it would be the case with flooring.

obey the requirements. In practice, banks hold a buffer on top of required capital. Otherwise, they could easily get into "regulatory distress" , which would regularly lead to a replacement of most board members, for instance. We abstract from this buffer and assume banks to hold *exactly* the amount of capital they are required to hold.

6.2 Target function(s)

In the optimization process we try to minimize a measure of system losses (i.e. the target function). The mechanism of contagion proposed by us has several sets of agents, each of whom suffers separate kinds of losses. There are many conflicting arguments which agents' losses the regulator should particularly be interested in, for instance those of depositors (as a proxy for "the public" that is likely to be the party that ultimately bails banks out) or even those of bank equity holders (who are at risk while offering a valuable service to the real economy). While all of them may be relevant, our primary target function are the *expected bankruptcy costs*, which is just the sum of the bankruptcy costs of the defaulted banks:

$$EBC = \mathbf{E}\sum_i BC_i D_i \,, \tag{8}$$

where D_i is the default indicator of bank i, as set in (**??**); for the definition of BC_i see (6). Expected bankruptcy costs are a total social deadweight loss that does not include the initial portfolio loss due to the initial shock. While this is a compelling measure of social loss, there are distributional reasons that it might not be the only measure of interest.

6.3 Testing separate capital allocations

Setting the capital allocations as a function of a rule based upon network topology, other measure variables, and a minimum capital requirement for all banks was straightforward and computed on network measures presented in Table 1.In our analysis minimum capital is set at the VaR ($\alpha = 99\%$) where interbank loans are treated as ordinary loans. It (imperfectly) implements a cap of 1% on bank PDs. The difference between benchmark capital (i.e. VaR($\alpha = 99.9\%$)) and minimum capital (i.e. VaR($\alpha = 99\%$)) is then pooled and divided based on a capital allocation rule. In each capital allocation rule, given the centrality measure chosen and given a coefficient, β, we calculate required capital for each bank, which is assumed to be actually held by banks. Typically in our simulations the total amount of capital held by the banks is assumed constant, and divided between the banks according to the rule,β. Then, as described above, we simulate a large sample of losses to "real-economy" bank portfolios; and for each capital allocation we simulate interbank contagion. This gives, for each allocation, a sample of after-contagion losses from which the target function (total expected bankruptcy costs) and further measures are calculated. For each centrality measure, we perform a simple grid search over β to minimize the target function.

6.4 Bailout fund mechanism

Our second direction to improve the stability of a financial system is developing the concept of a bailout fund mechanism. The fund acts a lender of last resort, covering credit losses resulting from losses on loans to the "real economy" and recapitalizing the bank (with an amount related to its interbank assets). In our proposed policy tool, the bailout fund obtains resources directly from banks. Banks receive a capital relief compared to the benchmark case, and these resources are pooled in a bailout fund. The purpose of the bailout mechanism is to provide policymakers with a simplified framework for deciding whom to rescue – or not – in case of distress.The bailout fund implements some basic ad-hoc rules, which could be relaxed or extended and be subject to further optimization:

(i) The bailout fund has limited resources;

(ii) it saves banks based on a ranking obtained from a centrality-based index and other measured bank characteristics; and

(iii) it utilizes funds to rescue and recapitalize banks before interbank contagion takes place.

We now develop the concept of the bailout fund in three steps.

First, and similarly to capital re-allocation as defined in Section 6.1, we choose a parameter $\beta \in [0, 1]$ which now, however, defines a direct tradeoff between a centrality measure and capital in the benchmark case. We compute an index for each financial institution as a β-weighted average of two variables, which are transformed such that they are in a comparable range. More formally, we set

$$index_i = \beta \times \frac{\log(VaR_{\alpha,i})}{\max_i(\log(VaR_{\alpha,i}))} + (1-\beta) \times \frac{Centrality_i}{\max_i(Centrality_i)}. \tag{9}$$

This index is then mapped into a ranking $rank_i$, where low values determine high indices.

As the second step, we define the mechanics of how the fund is used. When a bank is to be saved by the bailout fund, not only the creditors are satisfied but the bank is also recapitalized with a buffer equal to a fixed percentageϵ of total interbank assets. This buffer helps the bank to sustain further losses on its interbank assets generated by contagious defaults.

If we took our model literally, we would let the bailout fund rescue banks in the order down the rank and stop when the fund is exhausted. This would – formally! – be possible since in our model fundamental losses emerge at the same time; right after they have become known, the regulator could look at them collectively and decide on whom to rescue based on previous knowledge AND the combination of losses.

We think this picture is too ideal. Rather, we expect fundamental losses to appear during some non-negligible period of time, and to do so in an exogenous, random sequence the regulator has no influence on. We also think that the regulator must react to a critical fundamental loss *immediately*. This means, while the regulator can make the rescue decision about a certain bank depend on anything known in advance and anything up to the time of the loss, the decision must not depend on the fundamental losses of those banks who default later, even if it would be better to put the money there. In other words, we assume that there an implicit time order in which fundamental losses appear, and that the regulator may condition its decisions on the past and presence, but not on the future.

To keep things simple enough, the regulator makes a plan in advance how much exactly each bank would get from the fund, given it is needed and given the fund still has reserves.

The planned amount to be given to a bank is defined by a function that jointly depends on the rank, the fundamental losses L_i^{real}, and interbank lending A_i^{IB}. For simplicity, we separate the function into two factors, one of which depends on the index only.

$$bail_i^{\text{plan}} \equiv I_{\{rank_i \le C\}} \times I_{\left\{L_i^{\text{real}} > K_i\right\}} \left(L_i^{\text{real}} - K_i + \epsilon A_i^{\text{IB}}\right).$$

In words, the plan targets at a full bailout, plus capital endowment at some percentage ϵ of interbank assets, if the bank belongs to the C most "important" banks, according to the index measure. If a bank is not "important" enough, it gets no subsidy and goes bankrupt. Other rules are possible. For instance, one could remove the strict cutoff at C and make the relative buffer size ϵ depend on the rank. Doing so would eliminate the cliff behavior at the threshold C and possibly mitigate adverse incentives to banks.

In each simulation, after generation of the fundamental losses we draw a random ordering among the banks who have fundamentally defaulted. This is our proxy for the random sequence of loss occurrences. The regulator goes down this ordering and transfers money according to the plan, until the fund is exhausted. Such a strategy could also be implemented if there was an explicit timeline, under the condition that the regulator must decide between bailout and bankruptcy immediately.

We now implement rule (i) formally. Denote the fund's initial endowment by *resources* and assume there are ND fundamentally defaulted banks. The integer valued function $fdidx\,(k)$, $k = 1, \ldots, ND$, runs through the indices of the defaulted banks in a random order; this order is drawn in each simulation and depends on nothing but ND. The amount actually transferred to the banks is defined recursively. Below, the variable $bailcum_k$ denotes cumulative payments; the payment to bank i is denoted by $bail_i$.

$$\begin{aligned}
bailcum_0 &\equiv 0; \\
bail_{fdidx(k)} &\equiv \min\left\{bail_{fdidx(k)}^{\text{plan}}, resources - bailcum_{k-1}\right\}, \quad k = 1, \ldots, ND \\
bailcum_k &\equiv bailcum_{k-1} + bail_{fdidx(k)}, \quad k = 1, \ldots, ND \\
bail_i &\equiv 0 \quad \text{for all other banks.}
\end{aligned}$$

As the third step, it remains to choose the resources of the bailout fund. As in definition (7) of centrality based capital rules, there is a minimum required capital $K_{\min,i}$ under the "mild" rule, but this now remains the ultimate required capital, unlike in Section 6.1. Compared to the benchmark case, banks get a (now actual) capital relief summing up to $TK_\alpha - TK_{\min}$ over the whole system. A fraction $\eta \in [0, 1]$ of it defines the reserves of the bailout fund:

$$resources = \eta\,(TK_\alpha - TK_{\min}).$$

The parameters β and η define a set of possible bailout rules, which can be used to determine the optimal one.

While banks hold less capital than in the benchmark case (or with centrality based capital), the nice feature of the fund is that it works in a *targeted* way. It is able to inject fresh capital into those banks after it is known that their fundamental losses have exceeded their capital. If calibrated well, the bailout fund will thereforedominate a comparable capital allocation mechanism, on average, (assuming total systemic capital equals the fund's resources) because capital allocation takes place before losses are known, whereas the bailout requirement utilizes extra information.

7 Results

In this section we present our framework in both policy directions discussed above: first, Section 6.1 outlines our results for capital re-allocations using centrality measures and, second, Section 7.2 presents an application of the bailout fund mechanism.

7.1 Capital allocations

Our main results from capital re-allocation exercise are depicted in Figure 2. As defined in Equation 8, the target function that we compare across centrality measures and within one measure is the total expected bankruptcy cost. Our benchmark allocation (based only on VaR) is represented by the point where $\beta = 1$. Indeed, some centrality measures help to improve the stability of the banking system in terms of expected total losses. Among them are Total Assets (TA), Opsahl centrality, IB Liabilities, Out Degrees, Weighted and non-weighted Eigenvector centrality. In contrast, capital allocations based on Clustering Coefficient, Closeness, Weighted Betweenness, In Degrees and IB Assets have a higher expected loss than in the benchmark allocation for any $\beta < 1$. Opsahl Centrality (Equation 3) dominates any other network measures, apart from Total Assets.

By setting a weight of 70% on the difference between initial capital allocation (VaR $(\alpha = 99.9\%)$) and minimum capital (VaR$(\alpha = 99\%)$) and the rest according to Opsahl Centrality, we obtain an improvement of around 20% of expected bankruptcy costs, from €1211 Mn to €977 Mn. The allocation based on TA beats any network based measure, improving the expected loss by almost 40%, reducing it from €1211 Mn to €730 Mn.[32]

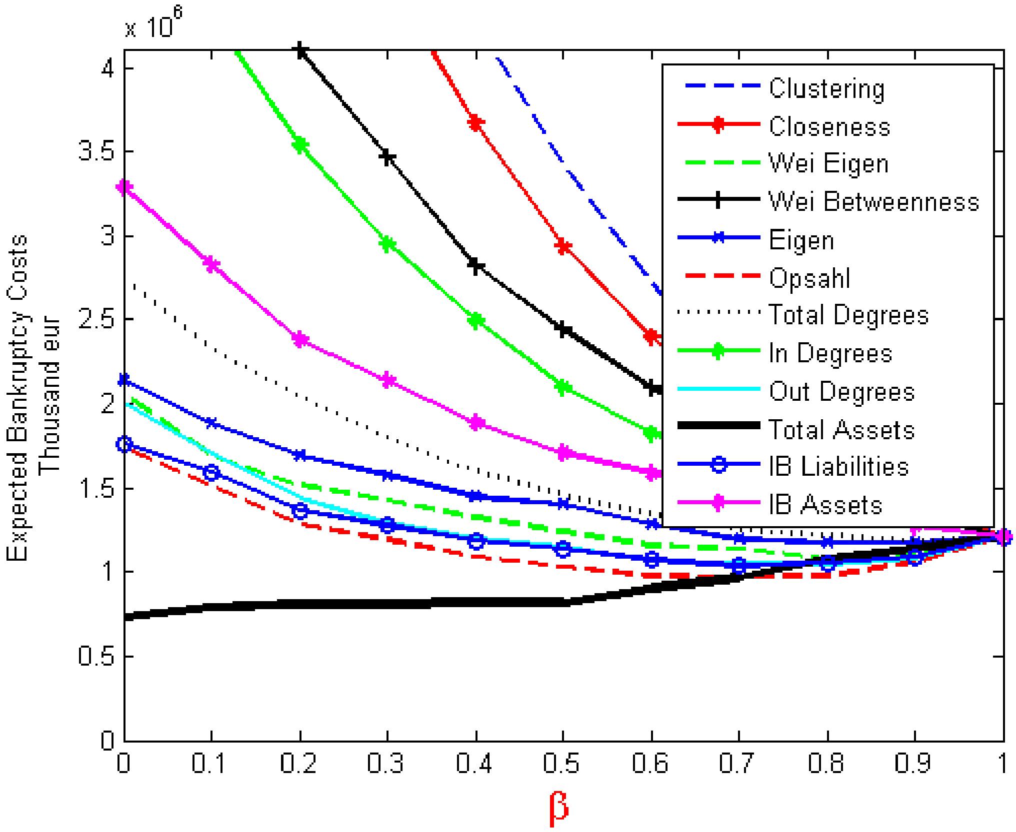
Figure 3a and 3b present expected loss functions under the best-performing centrality measures: Opsahl Centrality and Total Assets. In the case of Opsahl centrality (Figure 3a), on the one hand, expected bankruptcy costs of fundamental defaults are always above the benchmark case and, on the other hand, losses from contagion reach a minimum when $\beta = 0.4$. This allocation does not coincide with the best allocation in terms of total expected bankruptcy costs $(\beta = 0.7)$. The non-linearity observed in the case of Opsahl Centrality is similar to other centrality measures (e.g. Weighted Eigenvector, Total Liabilities, or Out Degrees). In contrast, TA allocations (Figure 3b) have a monotonic effect on total expected bankruptcy costs. Although varying the weight between full TA allocation and the benchmark case has virtually no impact on expected bankruptcy costs from fundamental defaults, it has a strong mitigating effect on expected bankruptcy costs from contagion. In this case, the best allocation is $\beta = 0$, meaning all weight on TA, apart from the minimum capital ($K_{\min}$) that was set based on the VaR at a low quantile, e.g. $\alpha = 99\%$.

Figures 4a and 4b are histograms of all banks' default probabilities before and after contagion based on three allocation rules: the benchmark case (VaR based capital[33]), the best allocation based on the combination between VaR capital and Total Assets (TA) $(\beta = 0)$, and the best allocation based on the combination between VaR measure and Opsahl centrality $(\beta = 0.7)$. On the one hand, results depicted in Figure 4a show that the

[32]Since bankruptcy costs (BC) are directly linked to TA (eq. 6), this might be one factor that explains the good performance of TA in minimizing system losses.

[33]Recall that the VaR used for capital is not identical with the actual loss quantile in the model that includes contagion, be it before or after contagion; see Section 6.1. If it were, the histogram for the

Figure 2: A comparison of different capital allocations across network measures



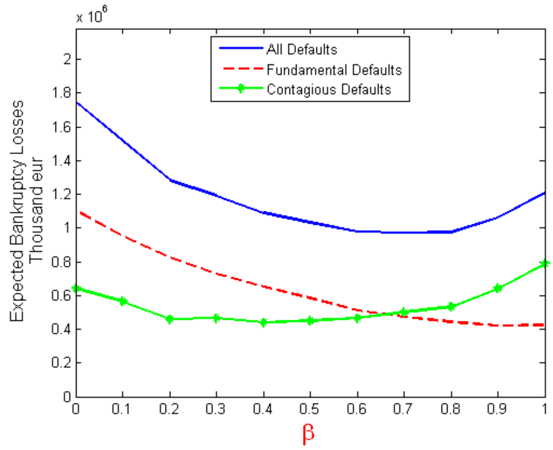
Note: On Y-axis is represented the size of Total Expected System Loss (as measured by Equation 8) from all defaults (both fundamental and contagious) under different capital allocations. On X-axis, β represents the weight on VaR (whereas (1-β) represents the weight on centrality measure).

best TA-based allocation gives a distribution of fundamental defaults (before contagion) with longer and fatter tail than in the case based only on VaR. This is not surprising as the PD before contagion is limited to 0.1% by construction (cf. footnote 30); any exceedance is caused by simulation noise. The best allocation based on Opsahl Centrality is something in between. There are less banks with a PD very close to 0 than in the case of benchmark allocation (VaR), but PDs of several banks increase relative to the benchmark case. On the other hand, the PDs observed in Figure 4b after interbank contagion show a completely different picture. Best allocations based on TA and Opsahl perform much better in terms of the overall distribution of bank PDs than in the benchmark case.

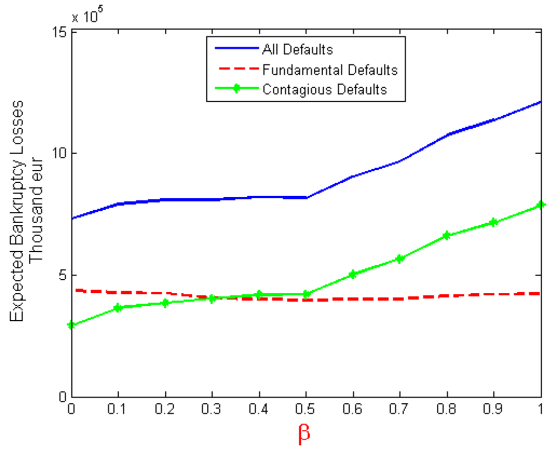
Figure 4b can also be used as a validation of the "traditional" way the risk of interbank loans, that is, by treating them as in the benchmark case; cf. Section 6.1. There they are part of an ordinary industry sector and driven by a common systematic factor. This treatment is very much in line with the approach taken in the Basel III framework. The bank PDs sampled in Figure 4b are default probabilities from the model including contagion. If the bank individual loss distributions generated under the "traditional" treatment were a perfect proxy of the ones after contagion, we should observe the histogram of PDs in the "VaR" case to be strongly concentrated around 0.001. Instead, the PDs are widely

Figure 3: Expected bankruptcy costs: All defaults, fundamental defaults and contagious defaults

(a) Capital allocations under rules based on Opsahl Centrality



(b) Capital allocations under rules based on Total Assets



Note: On Y-axis is represented the size of Expected System Loss (as measured by Equation 8) from fundamental, contagious defaults, and all defaults under different capital allocations. On X-axis, β represents the weight on VaR ($\alpha = 99.9\%$), whereas (1-β) represents the weight on centrality measure.

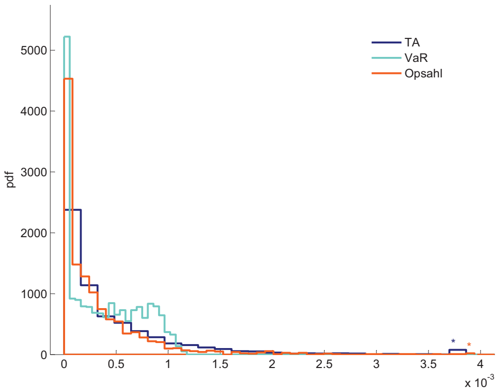
distributed and even on average approximately as doubly as high as possibly concluded from the label "99.9-percent-VaR". While the link to the Basel framework is rather of methodological nature, our analysis clearly documents that interbank loans are special and that correlating their defaults by Gaussian common factors may easily fail to capture the true risk. As there are, however, also good reasons to remain with rather simple "traditional" models, as those behind the Basel rules, our modeling framework lends itself to a validation of the capital rules for interbank credit. This exercise is beyond the scope of this paper.

Figure 5a is based on 20,000 simulations in each of which at least one bank has gone bankrupt. Each observation represents a bank for which we count the number of fundamental defaults (x-axis) and the total number of defaults including contagion (y-axis). In the case of the VaR based benchmark allocation, the relationship is clearly non-linear. Most banks, although their fundamental PD is effectively limited to 0.1 percent, experience much higher rates of default due to contagion. Suggested (but not proven) by the graph, there seems to be a set of some 30 events where the system fully breaks down, leading to a default of most banks, irrespectively of their default propensity for fundamental reasons.

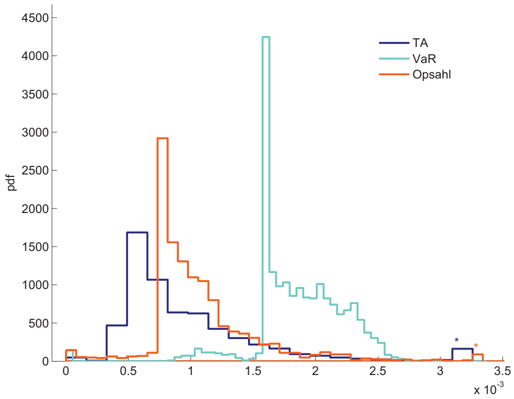
In contrast, when using Opsahl Centrality, some banks have a higher probability of default that can be close to 0.5% in some cases before contagion and 0.7% after contagion, but there is now a different relationship between fundamental and contagious defaults. First, a significant group of banks is now found on the diagonal; they default with – partly substantial – probability for fundamental reasons but never do so because of their interbank exposures. Second, the parallelly raised cloud of banks being involved in contagion is beneath its benchmark counterpart, suggesting that the (asssumed) set of "total

Figure 4: Frequency distributions of individual bank PDs

(a) *Before* IB contagion



(b) *After* IB contagion



Note: On Y-axis is presented the frequency distribution (no of occurrences) of individual bank PDs. On X-axis, are represented PDs (per bank). Sign "*" denotes that the distribution has a longer tail but has been truncated and observations after that threshold are aggregated at that point. Capital allocations are: i) BLUE based on Total Assets (TA) (with min capital VaR ($\alpha = 99\%$)), ii) GREEN based on VaR ($\alpha = 99.9\%$) [*our benchmark allocation*], iii) ORANGE based on Opsahl centrality. Results obtained with 20,000 simulations (using importance sampling).

breakdown" events has become scarcer.We investigate further which bank characteristics can be distinguished between the two groups. Similar results are shown for the best allocation under TA when we compare them with the benchmark case in Figure 5b. Thus, we claim that allocations based on TA and Opsahl centrality shift capital from smaller and less interconnected nodes to bigger and more interconnected and therefore the system becomes less prone to interbank contagion.

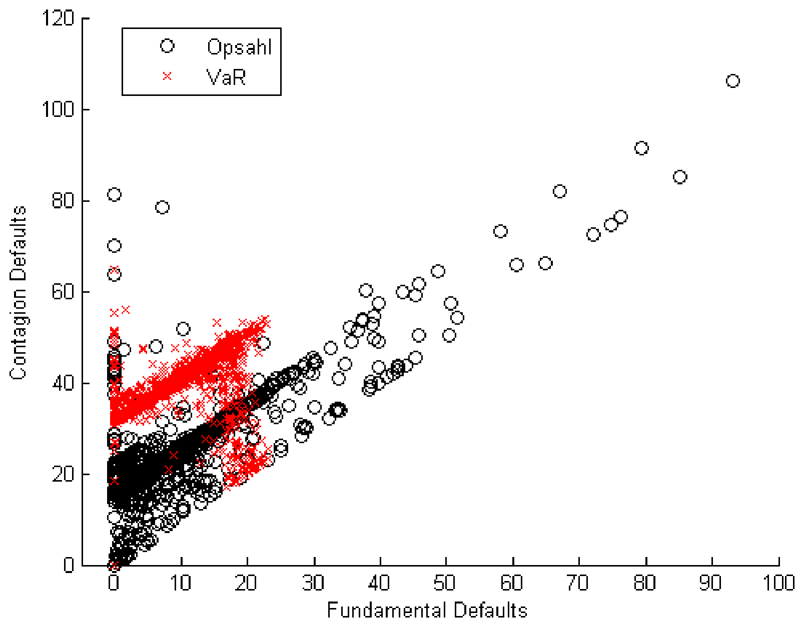
In order to get the full picture, instead of focusing on unconditional expected bankruptcy costs we now switch to tail expectations.[34] First we provide in Figure 6a the 99%-tail of the distribution of total bankruptcy costs for the benchmark case and the optimal TA and Opsahl-Centrality-based allocations. Tail-conditional expected bankruptcy costs under the benchmark allocation are equal to around €115 Bn, while under TA allocation they are almost halved. With Opsahl Centrality allocation, conditional expected losses are around €89 Bn. Total bankruptcy costs reach a maximum around €900 Bn. This amount is equivalent to a total system collapse where approx. 5% of total system assets are lost. In Figure 6b we zoom further into the conditional tail and measure losses in the 99.9%-tail. Our findings reinforce the idea that TA and Opsahl centrality perform better than the benchmark allocation. Top 0.1% losses exceed on average €435 Bn in the case of TA, €589 Bn under the best allocation using Opsahl centrality and around €663 Bn under the benchmark case.

Finally, Figure 7 shows bank individual ratios between best capital allocations under Total Assets (TA), Opsahl centrality, and Weighted Eigenvector centrality, respectively, to

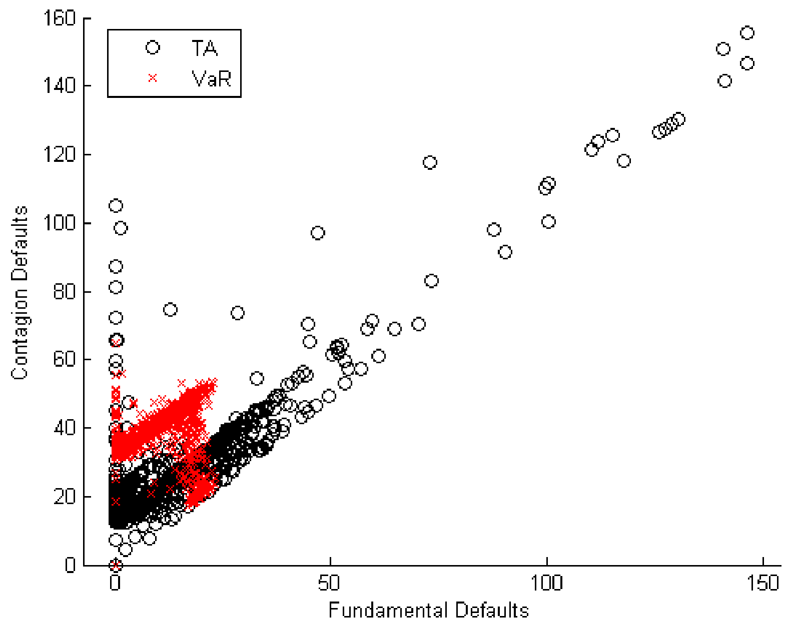
[34] In Figure 12 we present the unconditional distributions of system losses, as defined in eq. 8. The unconditional mean under the benchmark allocation (VaR) equals €1211 Mn, while under best TA allocation it is €730 Mn and €977 Mn under Opsahl Centrality allocation.

Figure 5: Occurrences of individual bank defaults

(a) Opsahl vs VaR

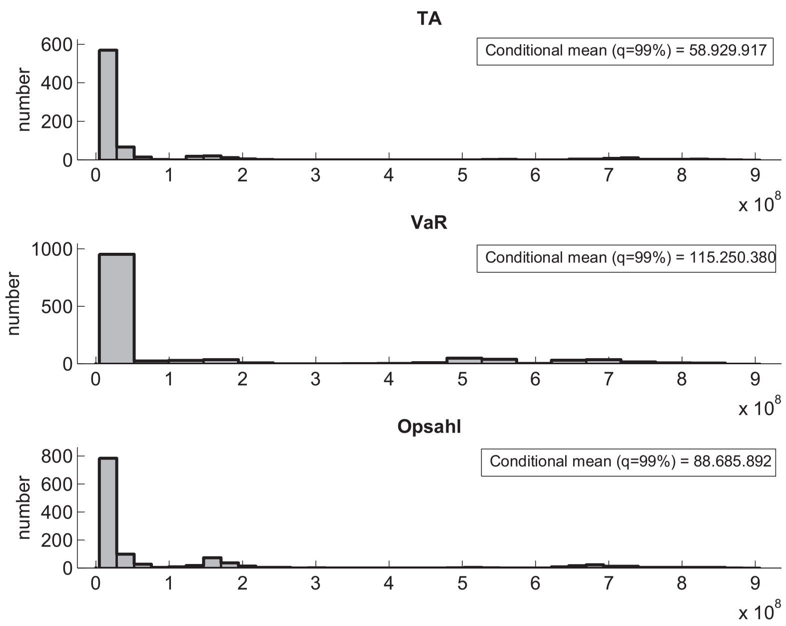


(b) Total Assets (TA) vs VaR

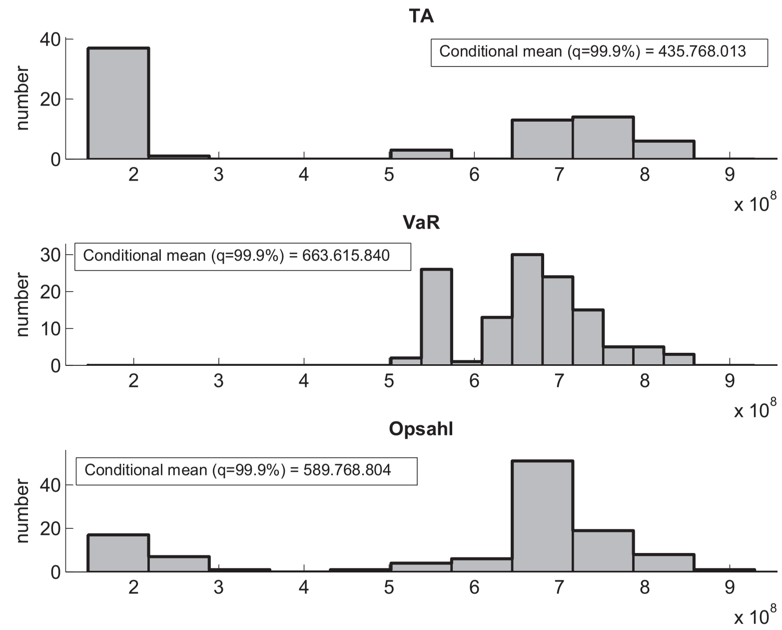


Note: Each point represents a bank. On X-axis – number of a bank's fundamental defaults in a sample of 20,000 simulations where at least one bank has defaulted. On Y-axis – number of defaults, including contagion.

Figure 6: Tail of frequency distribution (q=99%) of total bankruptcy costs for best capital allocation under rule based on: Total Assets (TA), Value-at-Risk (VaR, benchmark case), and Opsahl Centrality (Opsahl)



(a) Frequency distribution of bankruptcy costs conditional on BC exceeding the 99% quantile of the unconditional distribution

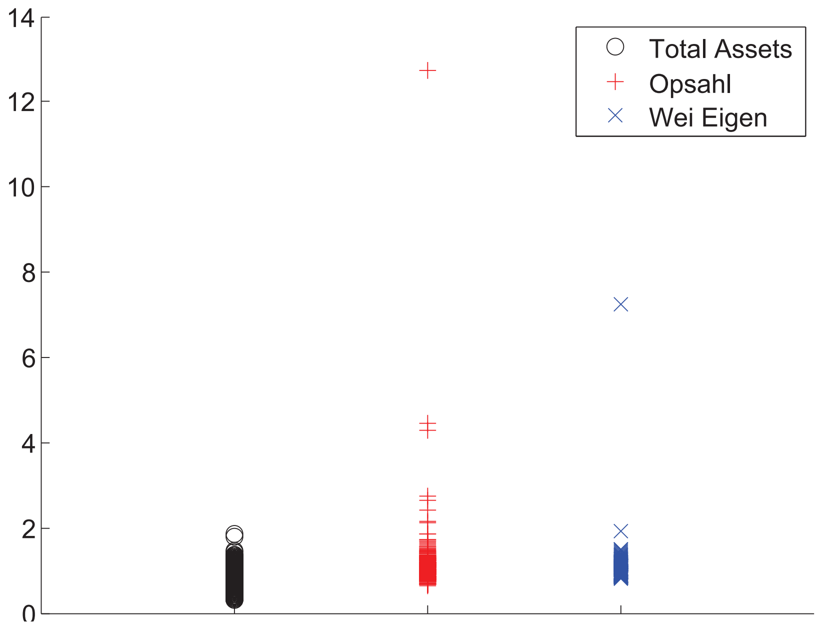


(b) Frequency distribution of bankruptcy costs conditional on BC exceeding the 99.9% quantile of the unconditional distribution

VaR benchmark allocation. The first observation is that ratios of TA to VaR allocations have a maximum around two. That implies that capital doubles for some banks under this allocation. On the lower side, some banks may hold less capital, floored at a minimum of 20% of initial capital. The ratios of Opsahl vs. benchmark allocation look more dispersed. There is one bank that receives 13 times more capital than in the benchmark case, and several that are required to hold around four times more. On the lower side, banks hold around 60% of initial capital. The third allocation, which involves a tradeoff between Weighted Eigenvector centrality and the benchmark, shows one outlier with around seven times more capital than in the benchmark case and another node with twice the initial capital. On the lower side, results are similar to the Opsahl best allocation. To conclude, we infer that the TA based allocation is a comparably "mild" modification of the benchmark case because the extra capital allocated to some banks is not excessive. Of course, we must acknowledge that this finding may largely be driven by our data. [35] Another intuitive inference is that the other centrality measures might mis-reallocate capital. For example, let's say that benchmark capital for a bank represents 8% of the total assets. Forcing this bank to hold 13 times more capital under the new allocation rule would mean it would have to hold more capital than total assets. Due to this misallocation likelihood, we infer that more constraints should be set in order to achieve better performing allocations using centrality measures. We leave this extra mile for future research.

[35] For instance, we could construct a large bank with loans to the real economy being as safe as we like. Lowering their PDs while keeping total assets constant, we could make its VaR converge to zero, and so its benchmark capital. In contrast, TA-based capital would remain almost stable so that the ratio of TA-based vs. benchmark capital would go to infinity.

Figure 7: Relative changes of required capital for different capital allocations



Note: Each observation represents a bank. The graph plots capital under the optimal capital allocations (based on Total Assets, Opsahl Centrality and Weighted Eigenvector)divdided by capital in the benchmark case.

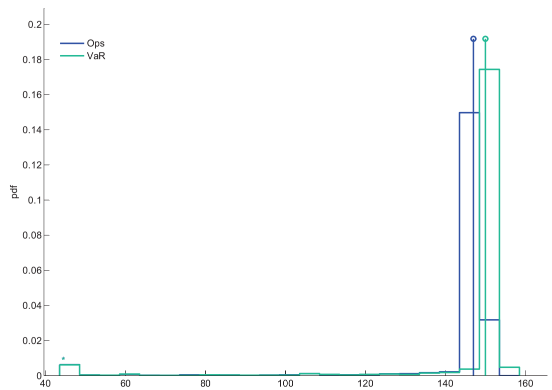
7.2 Bailout fund mechanism

As we discussed in Section 6.4, we design a second policy direction to deal with SIFIs in form of a bailout fund mechanism. Figure 8 depicts the probability distributions of the number of bank defaults at different levels of total available resources of the bailout fund, using a bank ranking (exclusively) based on Opsahl Centrality (top row) and using a ranking (exclusively) based on Total Assets (bottom row), in contrast to a ranking based only on the VaR of the benchmark case. As we increase the size of the bailout fund, the bank ranking looses importance. In the case of a bailout fund with 100% resources, rankings that consider VaR, TA or Opsahl Centrality lead to very similar PD distributions. The difference is made when the bailout fund has less resources. Thus, targeting them plays an important role. With 10% resources of the maximum size, the ranking based on Opsahl centrality combined with the size delivers the best results. Rankings are actually very similar. Compared with capital re-allocations, the bailout fund mechanism targets resources when they are needed (when losses exceed existing capital) and to those who are more systemically important (based on the bank ranking). This method gives less room to misusing resources. Still the difference is made which entities to save or not, given the limited amount of resources. Using the maximum amount of resources, the expected system losses are close to zero. With 40% of the funds, the expected system losses are similar to the optimal capital allocation obtained above.

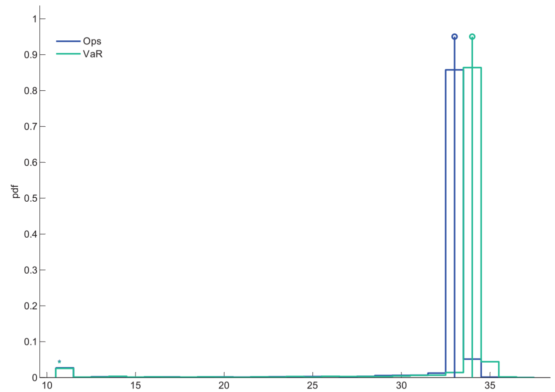
Figure 9 shows us how the expected bankruptcy costs evolve at different levels of

Figure 8: Pdfs of the number of bank defaults after contagion, using a bailout fund mechanism with rules based on: Opsahl Centrality (Ops) versus VaR (upper level); Total Assets (TA) versus VaR (lower level).

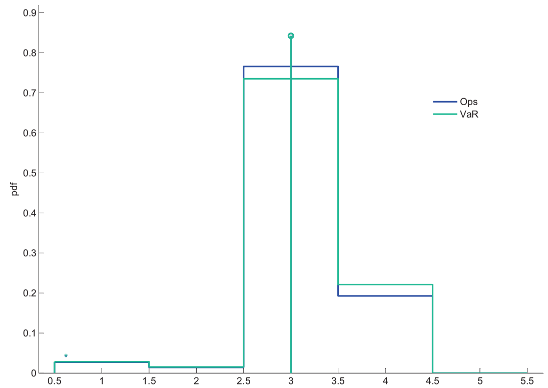
(a) Bailout funds: 10%



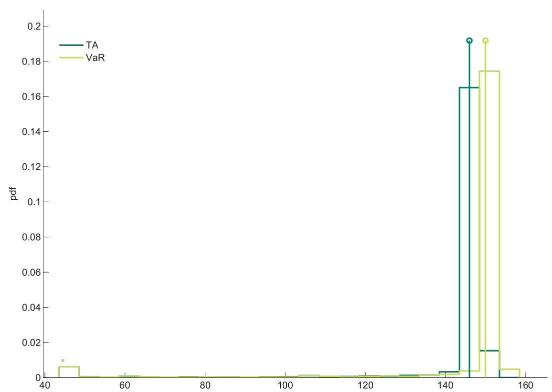
(b) Bailout funds: 50%



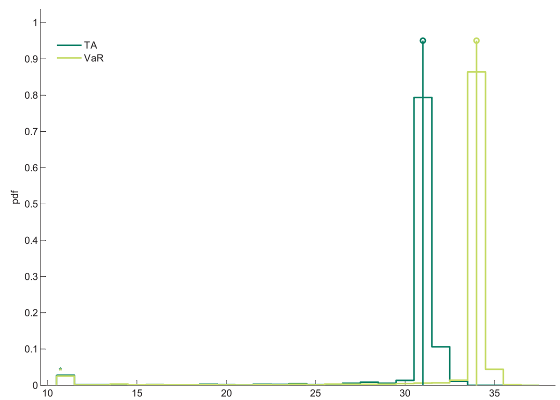
(c) Bailout funds: 100%



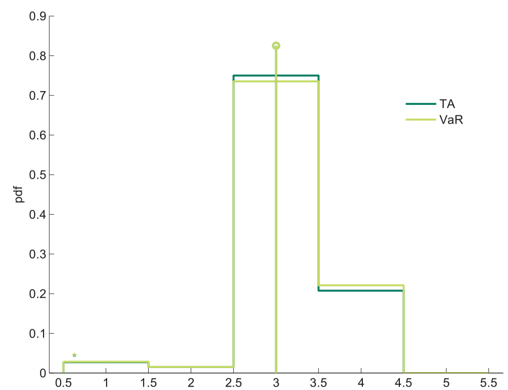
(d) Bailout funds: 10%



(e) Bailout funds: 50%



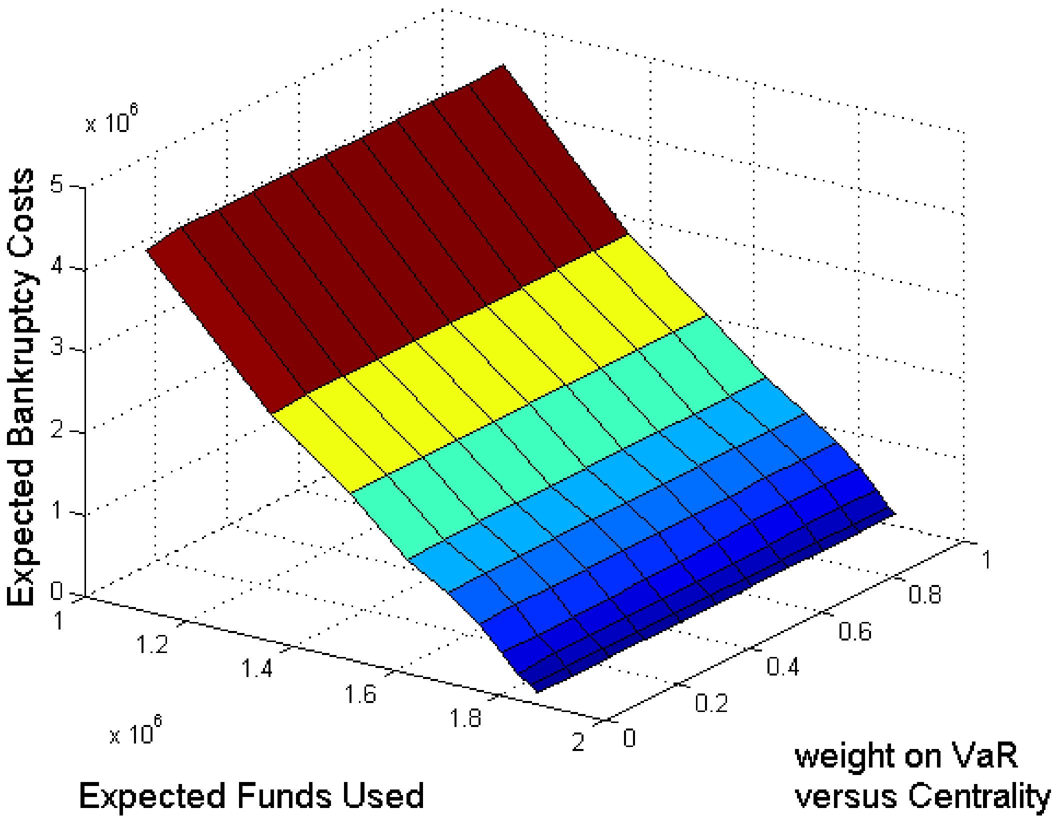
(f) Bailout funds: 100%



Note: Results obtained with 10,000 simulations (with importance sampling).

bailout funds and at different weights on centrality measure as compared to VaR . As resources in the bailout fund increase (X-axis) the expected losses decrease (Y-axis). Fund resources increase from €54 Bn (20% of the maximum bailout fund) to nearly €270 Bn (100% of the maximum bailout fund). This effect is non-linear. On the Z-dimension, in the case of the maximum bailout fund, there is a minimum at the ranking based on 90% weight on VaR and 10% weight on Opsahl centrality. The optimal bailout rule changes as bailout funds decrease, with the weight on Opsahl centrality gaining in importance.

Figure 9: Bailout efficiency surface: Opsahl centrality



Note: Expected Funds Used are the expected value of the utilized resources by the bailout fund, as we increase the capital of Bailout fund from €54 Bn to €270 Bn in steps of €54 Bn (20% of €270bn) up to a maximum amount pooled (i.e. $\sum(-VaR_{99.9\%} + VaR_{99\%})$ over all 1764 banks is €270 Bn). Results obtained with 10,000 simulations (using importance sampling).

8 Robustness checks

In this section we provide an analysis of the stability of centrality measures, liabilities' distributions and robustness checks of the parameters utilized throughout this paper.

8.1 Interbank liabilities

In Table 1 we have shown the properties of the interbank assets and liabilities for our empirical application using the data reported at the end of Q1 2011.

Since the utilized data is highly confidential, we focus on distributional properties of the interbank liabilities. In Figure 13 we attempt to compare the goodness-of-fit for a power-law distribution versus a log-normal one. The coefficient α of the power law is 0.45 while the constant term is around 4. As a standard convention, the minus in front of α corresponds to the negative slope. Figure 14 shows ranked liabilities for the first quarter of 2005, 2007, 2009, and 2011.[36] We find the α-coefficient to change from 0.51 in 2005 to 0.45 in 2011. The constant term decreases from 4.7 in 2005 to 4 in 2011. At the top of exposures we observe an increase in volumes over time. The number of interbank liabilities decreases from 29,000 in 2005 to 22,000 in 2011. This effect is probably also a consequence of mergers and acquisitions in the German banking sector.[37] Most of the interbank lending volume then turns into intra-group transactions, which are neglected in our analysis.

8.2 Network structure

As a robustness check of our results, we compare interbank properties, liabilities and centrality measures over the period 2005 - 2011. Tables 6, 7, and 8 are similar to Table 1. As mentioned before both number of banks and number of interbank exposures decrease by 11% and around 21%, respectively, from 2005 to 2011. The average individual bank total IB exposure increased from €900 Mn to almost €1 Bn. The average number of connections decreases from 29 to 26. The distributions of centrality measures are very similar over this period. For example, looking at different quantiles, mean or standard deviation of Opsahl centrality distributions we observe similar ranges across the four snapshots presented in this paper. At the higher end of the distribution, connectivity as measured by this measure picks in 2007 Q1, meaning that big banks are trading less in the interbank market. Very small banks show a lag in this behavior, with a peak in 2009 Q1. Weighted betweenness and clustering coefficient show a very skewed distribution. Most of the banks have a zero coefficient in this this case. We infer that this feature has an impact on our results when we try to re-allocate capital based on these network measures. Over time, these two indicators show a less clustered network structure and rather a tiered one. This feature is strengthen by the dynamics of closeness centrality, that decreases over the period 2005 - 2011. On average, eigenvector-type centrality measures bottom in 2009 Q1, in the midst of the financial crisis. At the top of the distribution, the most recent results reveal that important banks became even more interconnected compared to the before crisis period.

This comparison over time leaves room for a dynamic re-adjustment process of the capital re-allocations. Since interbank network properties do not change at a daily or weekly frequency, we propose a one year gap between re-estimations as acceptable.

[36] For confidentiality reasons, we have transformed real data into a sequence of averages, using three consecutive data points.

[37] The number of active banks decreases from 1.989 in 2005 to 1.764 in 2011.

8.3 Credit risk parameters

In this subsection we outline the robustness checks that we employed with respect to the credit risk model. We re-run several times the computation of VaR measures at quantile $\alpha = 99.9\%$. For this computation we employed a new set of 1 Mn. simulations, and kept the same generated PDs. Results are very similar, the average variance is under 2%. At lower quantiles the variance decreases considerably, e.g. VaR measures at 99% have a variance of under .5%.

9 Conclusion

In this paper we present a tractable framework to assess the impact of different capital allocations on the financial stability of large banking systems. Furthermore, we attempt to provide some empirical evidence of the usefulness of network-based centrality measures. Combining simulations techniques with confidential bank balance sheet data, we test our framework for different capital re-allocations. Our aim two fold. First, to provide regulators and policymakers with a stylized framework to assess capital for SIFIs. And second, to give a new direction to future research in the financial stability field using network analysis.

Our main results show that there are certain capital allocations that improve financial stability, as defined in this paper. Focusing on the system as a whole and assigning capital allocations based on networks metrics produces outperforming results compared with the benchmark capital allocation, that is based solely on the individual bank balance sheet. Similarities among bank portfolios make the financial system vulnerable to common macro-shocks. Our findings come with no surprise when considering a stylized contagion algorithm. The improvement comes from getting the "big picture" of the entire system with interconnectedness and centrality playing a major role in triggering and amplifying contagious defaults. What is interesting is that capital allocations based on total assets dominate any other centrality measure tested. These results strengthen the findings that systemic capital requirements should depend mainly on total assets as proposed by Tarashev et al. (2010). Combining total assets and network metrics on top of individual bank asset riskiness (given for example by VaR measure), one could improve even further system's stability.

We find our work relevant for the forthcoming micro-prudential mandate of the ECB to supervise the entire European banking system with a focus on SIFIs. In order to apply our methodology, an harmonized European credit register is necessary. Micro-prudential supervision should incorporate macro-prudential implications. The information regarding both interconnectedness, that could fuel interbank contagion, and correlated credit exposures, that show vulnerability to common shocks, are strongly dependent on the availability of this kind of dataset. As shown by Löffler and Raupach (2013), market-based systemic risk measures seem unreliable when willing to assign capital surcharges for systemically important institutions (or other alternatives like for example the systemic risk tax proposed by Acharya, Pedersen, Philippon, and Richardson (2010)). Thus, what we propose in this paper is a novel tractable framework to improve system stability based on network and balance sheet measures. Our study complements the methodology proposed by Gauthier et al. (2012). Since market data for all financial intermediaries does not exist when dealing with large financial systems, we propose a methodology that relies mainly on the information extracted from the central credit register.

We are not providing technical details on how capital re-allocation in the system could be implemented by policymakers. This aspect is complex and the practical application involves legal and political consensus.

Future research could include several directions. First, we would like to refine the allocation rule to combine more measures. Second, the methodology can be extended by including information regarding other systemically important institutions (e.g. insurance companies or other shadow banking institutions). In order to accomplish this direction further reporting requirements are necessary. Last, we want to calculate the insurance premium for each bank based on the expected bailout resources utilized.

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Appendix

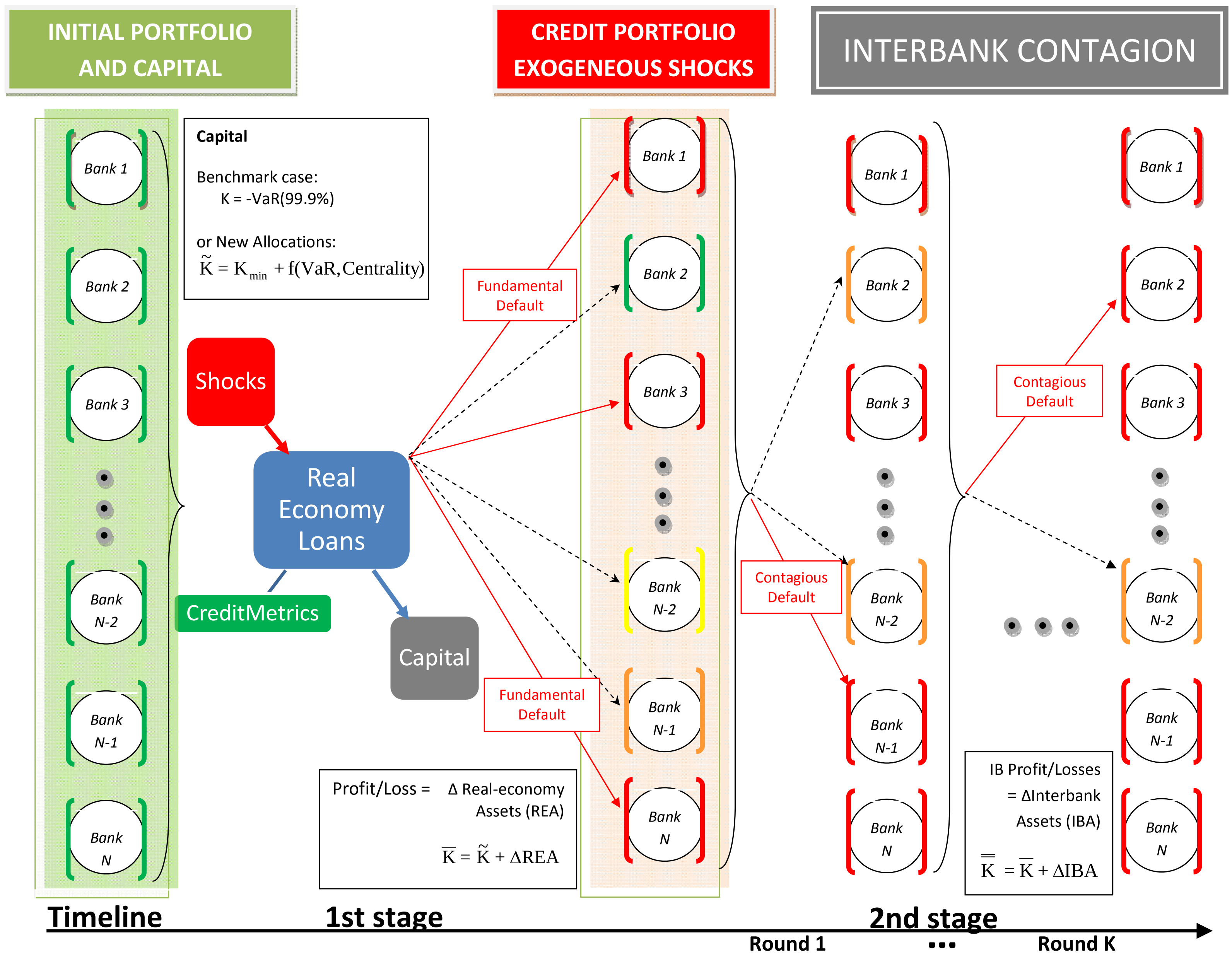
A Portfolio risk model and contagion mechanism

Table 2: Risk model (RM) sectors

No	Risk Model Sector	No of Borrowers	Volume Weight	Default probabilities 5%	25%	50%	75%	95%	mean
1	Chemicals	3200	0.88%	0.00008	0.0019	0.0059	0.0154	0.0666	0.0166
2	Basic Materials	14,419	1.49%	0	0.0027	0.0085	0.0205	0.1000	0.0220
3	Construction and Materials	17,776	1.34%	0	0.0012	0.0066	0.0185	0.0795	0.0199
4	Industrial Goods and Services	73,548	15.06%	0	0.0023	0.0077	0.0207	0.1300	0.0257
5	Automobiles and Parts	1721	0.67%	0.00001	0.0031	0.0105	0.0300	0.1498	0.0291
6	Food and Beverage	13,682	0.76%	0.00001	0.0027	0.0082	0.0185	0.0800	0.0192
7	Personal and Household Goods	21,256	1.26%	0	0.0017	0.0074	0.0199	0.1490	0.0275
8	Health Care	16,460	0.95%	0	0.0003	0.0012	0.0086	0.0384	0.0098
9	Retail	25,052	1.62%	0	0.0017	0.0079	0.0267	0.1140	0.0237
10	Media	2,534	0.24%	0	0.0017	0.0045	0.0171	0.0790	0.0181
11	Travel and Leisure	8,660	0.68%	0	0.0036	0.0117	0.0292	0.2000	0.0316
12	Telecommunications	299	0.75%	0	0.0012	0.0036	0.0232	0.0632	0.0182
13	Utilities	15,679	3.22%	0	0.0009	0.0039	0.0126	0.0677	0.0162
14	Insurance	1392	4.12%	0.00029	0.0003	0.0009	0.0066	0.0482	0.0115
15	Financial Services	23,634	22.48%	0.00021	0.0003	0.0005	0.0057	0.0482	0.0107
16	Technology	2249	0.16%	0	0.0020	0.0050	0.0160	0.0468	0.0131
17	Foreign Banks	3134	22.06%	0.00003	0.0003	0.0009	0.0088	0.0795	0.0140
18	Real Estate	56,451	11.39%	0	0.0010	0.0051	0.0168	0.0831	0.0182
19	Oil and Gas	320	0.48%	0.00038	0.0022	0.0081	0.0296	0.1286	0.0303
20	Households (incl. NGOs)	79,913	1.26%	0	0.0005	0.0035	0.0124	0.0600	0.0134
21	Public Sector	1948	9.15%	0	0	0	0	0	0
	TOTAL	388,327	100%						0.015

Note: Volume weight refers to credit exposure.

Figure 10: Risk model sketch



Note: Capital is identified by the color of parentheses around banks: green (well capitalized), yellow (medium capitalized), orange (low capitalized), red (default status).

Table 3: Model parameters

Parameter	Description	Value	Robustness
Portfolio Credit Risk			
Σ	correlation matrix of systematic factors Y	Table 4a	
ρ	intra-sector asset correlation	0.1995	
e_{LGD}	LGD beta distribution: expectation	0.39	
v_{LGD}	LGD beta distribution: standard deviation	0.34	
r_f	risk free rate	2%	
$s(R)$	rating specific credit spreads	Table 4b	
T	loan maturity	4 yr	
Centrality measures			
φ	Opsahl centrality coefficient	0.5	0.2; 0.8
Bankruptcy costs			
ϕ	potential costs associated with Total Assets	5%	1%; 3%; 10%
λ	fire sales costs, associated with the severity of total losses in the system	$\in [0,1]$	
Capital allocation rule			
β	weight on VaR based benchmark capital	$\in [0,1]$	
$\alpha_{benchmark}$	benchmark capital:$VaR_\alpha$is the euro value at quantile α of the losses distribution	99.9%	
α_{min}	minimum capital:$VaR_\alpha$is the euro value at quantile α of the losses distribution	99%	98%; 99.5%
Bailout mechanism			
β	weight on VaR	$\in [0,1]$	
η	maximum bailout size proportion	$\in [0,1]$	
ϵ	new capital buffer as a proportion of IB assets	20%	10%

B Centrality measures – technical details

In this part, we follow the technical details provided by Newman (2010) to define centrality measures used in our analysis. *Eigenvector centrality* is a recursive concept. A bank is considered an important IB borrower if it borrows from many banks *and* they are themselves considered important (as IB borrowers). *Eigenvector centrality* is defined as the principal eigenvector of the adjacency matrix (see for example Bonacich, 1987). The centrality of each node corresponds to its component in the eigenvector. It tends to be large if the node has a high number of outbound connections (i.e., a high out degree) or has such connections to other important nodes – or both, of course. In our analysis we use both an unweighted and a weighted version of eigenvector centrality. The latter uses

Table 4: Credit risk parameters

(a) Sector correlation matrix

	S1	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**	**S8**	**S9**	**S10**	**S11**	**S12**	**S13**	**S14**	**S15**	**S16**	**S17**	**S18**	**S19**	**S20**	**S21**
S1	1	0.81	0.81	0.87	0.56	0.58	0.78	0.44	0.62	0.64	0.67	0.56	0.68	0.71	0.78	0.71	0.67	0.60	0.74	0	0
S2	0.81	1	0.80	0.86	0.56	0.47	0.69	0.26	0.61	0.60	0.64	0.49	0.58	0.64	0.77	0.69	0.64	0.59	0.80	0	0
S3	0.81	0.80	1	0.91	0.68	0.55	0.79	0.38	0.74	0.74	0.79	0.55	0.60	0.78	0.86	0.77	0.75	0.69	0.70	0	0
S4	0.87	0.86	0.91	1	0.65	0.61	0.86	0.41	0.75	0.76	0.82	0.60	0.66	0.78	0.88	0.84	0.75	0.71	0.77	0	0
S5	0.56	0.56	0.68	0.65	1	0.37	0.60	0.23	0.55	0.49	0.52	0.38	0.36	0.52	0.60	0.51	0.47	0.52	0.42	0	0
S6	0.58	0.47	0.55	0.61	0.37	1	0.76	0.56	0.63	0.65	0.59	0.55	0.59	0.53	0.62	0.52	0.50	0.51	0.54	0	0
S7	0.78	0.69	0.79	0.86	0.60	0.76	1	0.58	0.79	0.81	0.79	0.64	0.69	0.74	0.84	0.77	0.72	0.67	0.71	0	0
S8	0.44	0.26	0.38	0.41	0.23	0.56	0.58	1	0.39	0.51	0.39	0.58	0.54	0.47	0.49	0.46	0.41	0.31	0.41	0	0
S9	0.62	0.61	0.74	0.75	0.55	0.63	0.79	0.39	1	0.70	0.81	0.48	0.52	0.66	0.76	0.68	0.66	0.64	0.56	0	0
S10	0.64	0.60	0.74	0.76	0.49	0.65	0.81	0.51	0.70	1	0.80	0.63	0.66	0.71	0.77	0.72	0.67	0.66	0.61	0	0
S11	0.67	0.64	0.79	0.82	0.52	0.59	0.79	0.39	0.81	0.80	1	0.51	0.54	0.74	0.84	0.75	0.74	0.76	0.58	0	0
S12	0.56	0.49	0.55	0.60	0.38	0.55	0.64	0.58	0.48	0.63	0.51	1	0.67	0.58	0.62	0.58	0.53	0.48	0.56	0	0
S13	0.68	0.58	0.60	0.66	0.36	0.59	0.69	0.54	0.52	0.66	0.54	0.67	1	0.65	0.68	0.62	0.61	0.53	0.72	0	0
S14	0.71	0.64	0.78	0.78	0.52	0.53	0.74	0.47	0.66	0.71	0.74	0.58	0.65	1	0.86	0.70	0.92	0.73	0.66	0	0
S15	0.78	0.77	0.86	0.88	0.60	0.62	0.84	0.49	0.76	0.77	0.84	0.62	0.68	0.86	1	0.79	0.87	0.82	0.74	0	0
S16	0.71	0.69	0.77	0.84	0.51	0.52	0.77	0.46	0.68	0.72	0.75	0.58	0.62	0.70	0.79	1	0.67	0.60	0.68	0	0
S17	0.67	0.64	0.75	0.75	0.47	0.50	0.72	0.41	0.66	0.67	0.74	0.53	0.61	0.92	0.87	0.67	1	0.75	0.67	0	0
S18	0.60	0.59	0.69	0.71	0.52	0.51	0.67	0.31	0.64	0.66	0.76	0.48	0.53	0.73	0.82	0.60	0.75	1	0.52	0	0
S19	0.74	0.80	0.70	0.77	0.42	0.54	0.71	0.41	0.56	0.61	0.58	0.56	0.72	0.66	0.74	0.68	0.67	0.52	1	0	0
S20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
S21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

(b) Credit spreads by rating

Rating	**Credit spread**
AAA	0.4740%
AA+	0.5417%
AA	0.6588%
AA–	0.7901%
A+	0.9802%
A	1.2236%
A–	1.4902%
BBB+	1.8653%
BBB	2.2476%
BBB-	2.8046%
BB+	3.3978%
BB	4.1033%
BB–	4.9980%
B+	6.3089%
B	8.3518%
B–	10.3230%
CCC/C	16.3980%

Note: Sectors are in the same order as in Table 2.

Table 5: S&P's credit ratings transition matrix, in percent

	AAA	**AA+**	**AA**	**AA−**	**A+**	**A**	**A-**	**BBB+**	**BBB**	**BBB-**	**BB+**	**BB**	**BB-**	**B+**	**B**	**B-**	**CCC**	**D**
AAA	90.867	4.879	2.770	0.703	0.165	0.248	0.145	0.000	0.052	0.000	0.031	0.052	0.000	0.000	0.031	0.000	0.052	0.006
AA+	2.654	78.891	12.151	4.075	0.923	0.674	0.311	0.124	0.124	0.062	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010
AA	0.510	1.375	83.962	8.352	2.999	1.479	0.448	0.427	0.146	0.094	0.052	0.042	0.021	0.000	0.000	0.021	0.052	0.021
AA-	0.052	0.136	4.493	80.439	10.428	2.957	0.731	0.282	0.146	0.073	0.042	0.000	0.000	0.042	0.115	0.021	0.000	0.042
A+	0.000	0.115	0.607	4.666	81.021	9.196	2.689	0.743	0.418	0.094	0.094	0.126	0.010	0.094	0.042	0.010	0.000	0.073
A	0.052	0.063	0.294	0.588	5.269	81.528	7.168	2.834	1.207	0.294	0.157	0.157	0.105	0.126	0.031	0.010	0.021	0.094
A-	0.063	0.011	0.116	0.211	0.643	7.092	79.937	7.893	2.487	0.717	0.169	0.158	0.169	0.148	0.042	0.011	0.053	0.084
BBB+	0.000	0.011	0.074	0.096	0.329	1.114	7.344	77.672	9.403	2.122	0.499	0.424	0.180	0.276	0.159	0.021	0.106	0.170
BBB	0.011	0.011	0.064	0.043	0.182	0.515	1.319	7.551	79.631	6.747	1.738	0.890	0.397	0.333	0.182	0.043	0.097	0.247
BBB-	0.011	0.011	0.011	0.075	0.075	0.259	0.431	1.498	9.239	76.639	5.908	2.781	1.110	0.604	0.367	0.237	0.334	0.410
BB+	0.077	0.000	0.000	0.055	0.022	0.154	0.132	0.694	2.502	12.809	69.290	7.044	3.538	1.378	0.904	0.209	0.584	0.606
BB	0.000	0.000	0.066	0.022	0.000	0.111	0.089	0.254	0.819	2.843	9.470	71.092	8.541	2.987	1.516	0.509	0.797	0.885
BB-	0.000	0.000	0.000	0.011	0.011	0.011	0.078	0.145	0.334	0.535	2.295	9.158	71.078	9.369	3.409	1.081	1.036	1.448
B+	0.000	0.011	0.000	0.045	0.000	0.045	0.101	0.056	0.079	0.112	0.382	1.773	7.754	72.966	8.607	2.963	2.188	2.918
B	0.000	0.000	0.023	0.023	0.000	0.102	0.079	0.045	0.125	0.045	0.261	0.443	1.908	9.641	65.478	8.994	6.155	6.677
B-	0.000	0.000	0.000	0.000	0.046	0.081	0.000	0.163	0.081	0.163	0.209	0.244	0.697	3.625	11.921	59.719	12.455	10.596
CCC	0.000	0.000	0.000	0.000	0.058	0.000	0.164	0.105	0.105	0.105	0.058	0.269	0.655	1.625	3.414	10.173	51.239	32.028

full information of the liabilities matrix $\mathbf{X}$, and the former only the existence of links, i.e., the adjacency matrix.[38]

Betweenness centrality is a "path-type" measure that takes into account the number of shortest paths that pass through a certain node. A higher betweenness coefficient is related to a more strategically positioned node that acts as a bridge in connecting other nodes. In terms of lending, a bank with high *betweenness* will cut off a lot of intermediation in case of its default. We are using the weighted centrality version similar to the case of eigenvector centrality.

Closeness centrality summarizes the shortest paths (distances) between any two nodes of the network. A node is regarded as closer to the center of the network if the sum of distances to the other nodes is lower (compared to others), such that information flow spreads fast in the network. So-called "super-spreaders" have the capacity of contaminating the core institutions in relatively short time, causing the collapse of the entire system.

The *clustering coefficient* measures the likelihood of a certain node to be in a complete graph with its neighbors. A complete graph refers to a fully connected set of nodes, i.e. there exists an edge between any two nodes of this set. As discussed by Watts and Strogatz (1998), this measure is useful to infer whether the network has the *small-world* property.

Using these centrality measures, we intend to assign capital buffers to those nodes which create externalities to the banking system by being able to contaminate the entire network. As we will discuss in Section 6 we intend to shift capital from the periphery to more "central" financial institutions – according to alternative definitions of "centrality" – such that contagion is contained and system losses are minimized.

B.1 Eigenvector centrality

Let $\mathbf{A}$ be the adjacency matrix, with $a_{ij} = 1$ if there is a credit exposure of bank j to bank i, and $a_{ij} = 0$ otherwise. And $\kappa_1$be the largest eigenvalue of A. Then eigenvector centrality is given by the corresponding eigenvector $\mathbf{x}$ so that $\mathbf{Ax} = \kappa_1\mathbf{x}$.Eigenvector centralities of all nodes are non-negative. In the case of weighted eigenvector centrality, the adjacency matrix $\mathbf{A}$ is replaced by the weighted liabilities matrix $\overline{\mathbf{X}}$, where each row is normalized to sum up to 1. This measure was firstly proposed by Bonacich (1987).

B.2 Betweenness centrality

The geodesic distance between any two nodes is given by the shortest path. Let g_{ij} be the number of possible geodesic paths from i to j (there might be more than a single shortest path) and n^q_{ij} be the number of geodesic paths from i to j that pass through node q, then the betweenness centrality of node q is defined as

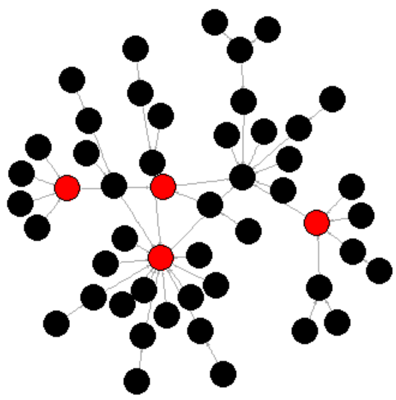
$$B_q = \sum_{\substack{i,j \\ j \neq i}} \frac{n^q_{ij}}{g_{ij}},$$

where by convention $\frac{n^q_{ij}}{g_{ij}} = 0$ in case g_{ij} or n^q_{ij} are zero.

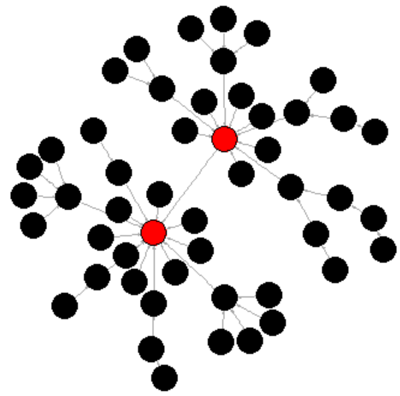
[38]We provide a technical description of centrality measures in Appendix B.

Figure 11: Centrality measures

(a) Betweenness



(b) Eigenvector



Note: The scale-free random graphs were generated with iGraph toolbox in R. The red nodes have the highest centrality values.

B.3 Closeness centrality

For the definition of closeness centrality we follow Dangalchev (2006):

$$C_i = \sum_{\substack{j \\ j \neq i}} 2^{-d_{ij}},$$

where d_{ij} is the lenght of the geodesic path from i to j. This formula is also appropriate for disconnected graphs. Disconnected components have a closeness centrality equal to 0.

B.4 (Local) Clustering coefficient

Watts and Strogatz (1998) show that in real-world networks nodes tend to establish clusters with a high density of edges.

The *global clustering coefficient* refers to the property of the overall network while *local coefficients* refer to individual nodes. This property is related to the mathematical concept of *transitivity*.

$$Cl_i = \frac{\text{number of pairs of neighbors of } i \text{ that are connected}}{\text{number of pairs of neighbors of } i}.$$

The local clustering coefficient can be interpreted as the "probability" that a pair of i's neighbors are connected to each other (i.e. are neighbors as well). The local clustering coefficient of a node with the degree 0 or 1 is equal to zero.

C Credit risk of small loans

Let us consider a portfolio with loans from one sector only. Assume there we have N_r loans in each rating class, which make up N in total. Each one is assigned to a latent "asset return" $X_{i,r}$ (sector index omitted). Conditionally on the single systematic factor Y of the sector, the $X_{i,r}$ are independent and so are corresponding rating migrations

$$R_0^{i,r} = r \to R_1^{i,r}.$$

All loans have the same uniform maturity $T = 4$ yr as the large loans. The notional of loan (i, r) has a weight $w_{i,r}$ relative to the total of all loans in the sector's portfolio. If not in default, its rating specific discount factor is the same as in Section 4.1, i.e., $D(r, r) = 1$ at the beginning of the risk horizon and $D\left(r, R_1^{i,r}\right)$ one period later. As for large loans, the return on loan (i, r) is determined as

$$\Delta_{i,r} = -1 + \begin{cases} D\left(C(r), R_1^{i,r}, 1, T\right) + C(r) & \text{if } R_1^{i,r} < 18 \\ (1 + C(r))(1 - LGD) & \text{if } R_1^{i,r} = 18 \end{cases},$$

where $C(r)$ was the loan rate. The corresponding return in the small-loans portfolio of one sector is then

$$\Delta = \sum_{r=1}^{17} \sum_{i=1}^{N_r} w_{i,r} \Delta_{i,r}.$$

As we want to approximate Δ by a Y-conditionally normal random variable Δ^*, we now determine the conditional expectation $\mathbf{E}(\Delta|\, Y)$ and variance $\operatorname{var}(\Delta|\, Y)$. A further approximation is that these moments are not to be based on all weights $w_{i,r}$ (which we do not know for small loans) but instead on two aggregates that we can estimate: the Herfindahl index, $H = \sum_{r=1}^{17} \sum_{i=1}^{N_r} w_{i,r}^2$, and exposure weights $w_r \equiv \sum_{i=1}^{N_r} w_{i,r}$ of the rating classes. To start with, the conditional migration probability, i.e., the probability of the "asset return" $X_{i,r}$ to fall between two neighbored thresholds, is

$$\begin{aligned} p(r, R|Y) &\equiv \mathbf{P}\left(R_1^{i,r} = R \middle| Y\right) = \mathbf{P}\left(\theta(r, R) < X_{i,r} \leq \theta(r, R-1) |\, Y\right) \\ &= \mathbf{P}\left(\frac{\theta(r, R) - \sqrt{\rho} Y}{\sqrt{1-\rho}} < Z_{i,r} \leq \frac{\theta(r, R-1) - \sqrt{\rho} Y}{\sqrt{1-\rho}} \middle| Y\right) \\ &= \Phi\left(\frac{\theta(r, R-1) - \sqrt{\rho} Y}{\sqrt{1-\rho}}\right) - \Phi\left(\frac{\theta(r, R) - \sqrt{\rho} Y}{\sqrt{1-\rho}}\right) \qquad (10) \end{aligned}$$

with $\theta(r, 18) = -\infty$ and $\theta(r, 0) = +\infty$. The conditional expectation in each rating class immediately calculates as

$$\begin{aligned} \mu_r(Y) &\equiv \mathbf{E}(\Delta_{i,r}|\, Y) \qquad (11) \\ &= -1 + \sum_{R=1}^{17} [D(r, R) + C(r)]\, p(r, R|Y) + (1 + C(r))(1 - \mathbf{E}(LGD))\, p(r, 18|Y) \end{aligned}$$

(which does not depend on i in fact) and, summing up over ratings,

$$\mathbf{E}(\Delta|\, Y) = \sum_{r=1}^{17} w_r \mu_r(Y).$$

The conditional variance of a single $\Delta_{i,r}$ is

$$\begin{aligned} v_r(Y) &\equiv \operatorname{var}(\Delta_{i,r}|\,Y) = \operatorname{var}(\Delta_{i,r}+1|\,Y) = \mathbf{E}\left((\Delta_{i,r}+1)^2\Big|\,Y\right) - (\mu_r(Y)+1)^2 \quad (12) \\ &= \sum_{R=1}^{17} [D(r,R)+C(r)]^2\, p(r,R|Y) \\ &\quad + (1+C(r))^2\, p(r,18|Y)\left[\operatorname{var}(LGD) + (1-\mathbf{E}LGD)^2\right] - (\mu_r(Y)+1)^2 \end{aligned}$$

where we utilize that LGD is independent of the other variables. Recall that migrations are conditionally independent, from which we immediately obtain for the whole sector's portfolio:

$$\operatorname{var}(\Delta|\,Y) = \sum_{r=1}^{17}\sum_{i=1}^{N_r} w_{i,r}^2 v_r(Y)\,.$$

If $v_r(Y)$ were rating independent, we could extract it from the sum and would obtain $H \times v_r(Y)$ as result. Because it is not, we make the weaker assumption that the distribution of – € – loan sizes does not depend on the rating (but well on the sector). If that holds, there is a relationship between H and the Herfindahl indices of the rating buckets that can be used to derive the approximation

$$\operatorname{var}(\Delta|\,Y) \approx H \sum_{r=1}^{17} w_r v_r(Y)\,.$$

To sum up, returns for the small loans are simulated in the following way.

Input data: For each bank b, the exposure to sector j is $SL_{b,j}$. From the $SL_{b,j}$ we infer on the Herfindahl index $H_{b,j}$ of its loans as described in Section C.1. Furthermore, for each sector j the (bank-independent) rating distribution $(w_{r,j})_{r=1\ldots17}$ is gathered from the sample of large loans and assumed to be the same for small loans. The matrix of discount factors $D(r,R)_{r=1\ldots17,R=1\ldots18}$ can be calculated once at the beginning.

Steps of one simulation round:

1. Draw systematic factors Y_j; they affect both large and small loans.
2. Calculate the matrices $(p(r,R\,|Y_j))_{r=1\ldots17,R=1\ldots18}$ according to (10) for all sectors j.
3. Calculate the vectors $(\mu_r(Y_j))_{r=1\ldots17}$ from (11) and $(v_r(Y_j))_{r=1\ldots17}$ from (12) for all j. Based on them, calculate

$$\mu_j^{\text{sect}}(Y_j) \equiv \sum_{r=1}^{17} w_{r,j}\mu_r(Y_j) \ \text{ and } \ \sigma_j^{\text{sect}}(Y_j) \equiv \sqrt{\sum_{r=1}^{17} w_{r,j} v_r(Y_j)} \text{ for all } j.$$

4. For each bank b, the € return on small loans is

$$ret_{\text{small},b} = \sum_{j=1}^{21} SL_{b,j} \times \left(\sqrt{H_{b,j}}\sigma_j^{\text{sect}}(Y_j)\, Z_{b,j}^{\text{small}} + \mu_j^{\text{sect}}(Y_j)\right), \qquad (13)$$

where the $Z_{b,j}^{\text{small}}$ are independent $N(0,1)$ random variables.

C.1 Estimating the granularity of small-loans exposures (Herfindahl index)

In Appendix C we quantify the granularity of a small-loans subportfolio by its Herfindahl index H. We cannot estimate it directly as we know the total exposure SL for each bank and sector only. Although individual exposures cannot exceed €1.5 Mn by construction so that a small-loans portfolio of size SL cannot have an H in excess of 1.5 Mn/SL, this upper bound will often be very imprecise, especially in the case of loans to households.

We therefore perform estimations and a simulation exercise based on a sample of individual loans made by a large German bank. We assign all exposures to risk management sectors and exclude all above €1.5 Mn. This gives us between 86 and 43,000 loans by sector, with a median of 6156 observations. While it would be critical to assume that the loan sizes of our *full* sample, which stems from a large bank, follow the same distribution as those of small banks, we hope that any such difference is not substantial when only small loans are concerned.

Let now U denote the random loan size in a certain sector. If the total exposure SL is large relative to $\mathbf{E}U$ so that probably many loans are in the portfolio, elementary arguments lead to the following approximation; cf. Hart (1975).

$$H \approx {}^{E\left(U^2\right)}/_{(\mathbf{E(U)\times SL})} \tag{14}$$

This relationship is used for large SL. For smaller values of SL, we rely on simulations and subsequent curve fitting: we define a sequence of exposure buckets $[SL_k, SL_{k+1}]$ and, for each of them, randomly collect loans from the sample until the total exposure falls into the bucket; this generates one possible H assigned to SL_{k+1}. Repeating the procedure provides us with a sample of such H for each exposure bucket. The bucket specific average $\bar{H}_k$ is our approximation of the size dependent Herfindahl index. It turns out that (14) works well for $SL > 5.9$ Mn. For smaller values, a sector-specific cubic function (of logarithms) is fitted to the averages $\bar{H}_k$. Some data entries are equal to the minimum reporting unit 1000. We assume they arise from a single loan. To sum up, we define

$$H\left(SL\right) \equiv \begin{cases} 1 & \text{if } SL \leq 1000 \\ \exp\left\{a \log^3 SL + b \log^2 SL + c \log SL + d\right\} & \text{if } 1000 < SL \leq 5,844,325 \\ {}^{\mathbf{E}\left(U^2\right)}/_{(\mathbf{E}(U)\times SL)} & \text{else} \end{cases}$$

where a, b, c, d, $\mathbf{E}\left(U^2\right)$, and $\mathbf{E}\left(U\right)$ are sector specific parameters.

D Other target functions

First, the deadweight loss can be distributed to the banking sector and to the non-banking sector in the sense that those bankruptcy costs not covered by interbank liabilities accrue to the outside sector. Second, the initial losses of the sectors outside of banking are distributed to the banks through the capital losses. Thus, a case can be made that the loss minimized should have the capital losses to the banks subtracted from them:

$$EBC_1 \equiv \mathbf{E} \sum_i I(L_i - K_i > 0)(BC_i + L_i - K_i).$$

The fact that the bankruptcy costs are distributed among the banks only partially, and the rest are absorbed in the non-bank sector indicates a third loss measure which is EBC_1, above, but with the bankruptcy costs modified so reflect the non-banking sector share:

$$EBC_2 \equiv \mathbf{E} \sum_i I(L_i - K_i > 0) \left[I(BC_i > l_i)(BC_i - l_i) + L_i - K_i \right],$$

where l_i are the total interbank liabilities of bank i. In this case, bankruptcy costs in excess of interbank liabilities are counted.

The nice thing about expected bankruptcy costs as a target function is that they are, as a dead-weight social loss, never desirable per se. As well, they are free of distributional assumptions. However, some amount of bankruptcy costs can well be acceptable as an inevitable side-effect of otherwise desirable phenomena (such as the sheer existence of bank business) so that minimizing expected bankruptcy costs does not necessarily lead to a "better" system in a more general sense. Finally, deposits, as a major part of non-bank debt, should always be in the focus of. Therefore, the following losses are at least of interest, if not considered as alternative target functions. It is important to note that we consider banks as institutions only, meaning that any loss hitting a bank must ultimately hit one of its *non-bank* claimants. In our model, these are simply non-bank debtors and bank equity holders, as we split bank debt into non-bank an interbank debt only.

Using the notation of **??**, the *expected total loss to equity holders* (which, as a participation constraint, should be negative) is simply

$$ELE \equiv \mathbf{E} \sum_i \min\left(K_i, L_i\right).$$

Note that L_i includes interbank losses. We do not seriously consider ELE a target function but a figure worth being reported.

The *expected loss to non-bank debt holders* is

$$ENB \equiv \sum_i \max\left(0, L_i + BC_i D_i - K_i - l_i\right)$$

where l_i was total interbank liabilities of bank i. Here again we assume that interbank debt is junior to other debt. The formula would not be correct in presence of a bailout found.

In contrast, counting total losses on banks' balance sheets (or just their credit losses) makes little sense as they involve interbank losses and therefore a danger of double counting. The measures ELE and ENB do not count losses doubly.

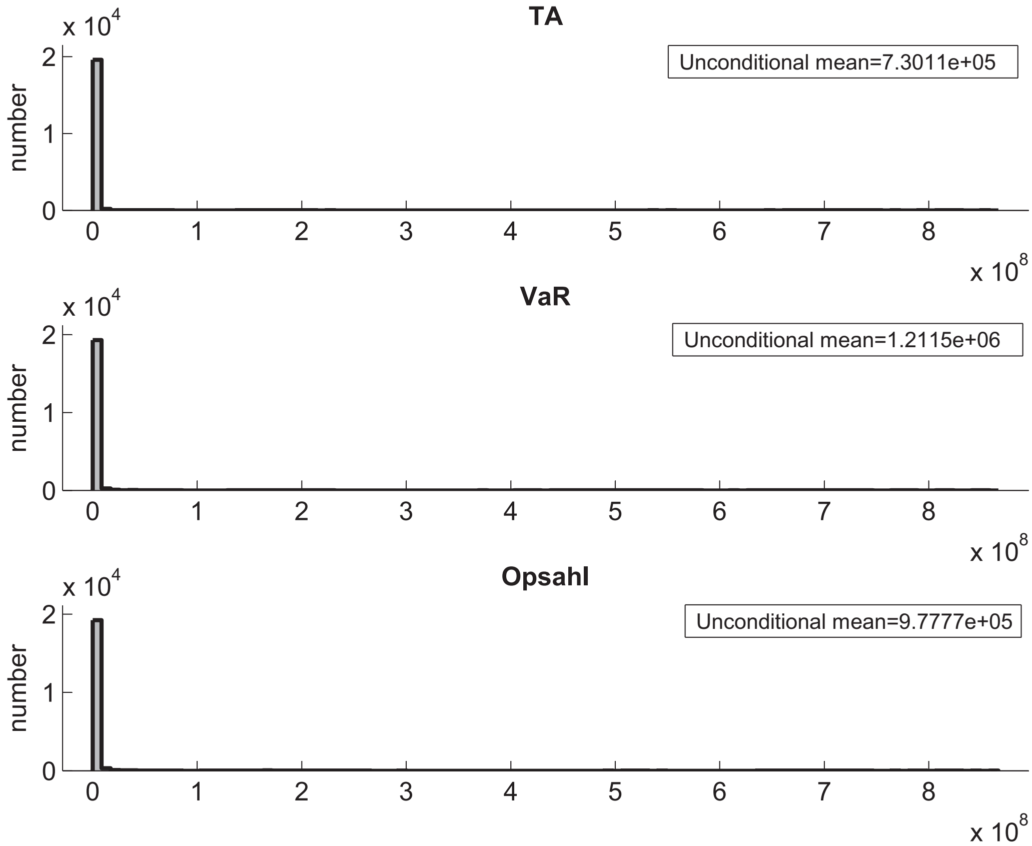
Finally, defining their sum as a target function appears as a natural choice, this way treating the interests of bank owners and (non-bank!) debt holders equally important. This target function is equivalent to ours, the expected bankruptcy costs. To see why, recall that every loss is either generated in the real economy or by bankruptcy and ultimately must end up with any non-bank claimant. This means

$$\begin{aligned} ELE + ENB &= \mathbf{E} \sum_i \left(L_i^{\text{real}} + BC_i D_i \right) \\ &= \mathbf{E} \sum_i L_i^{\text{real}} + EBC. \end{aligned}$$

As the losses L_i^{real} are exclusively driven by portfolio characteristics, $ELE + ENB$ and EBC differ only by a constant when capital is optimized.

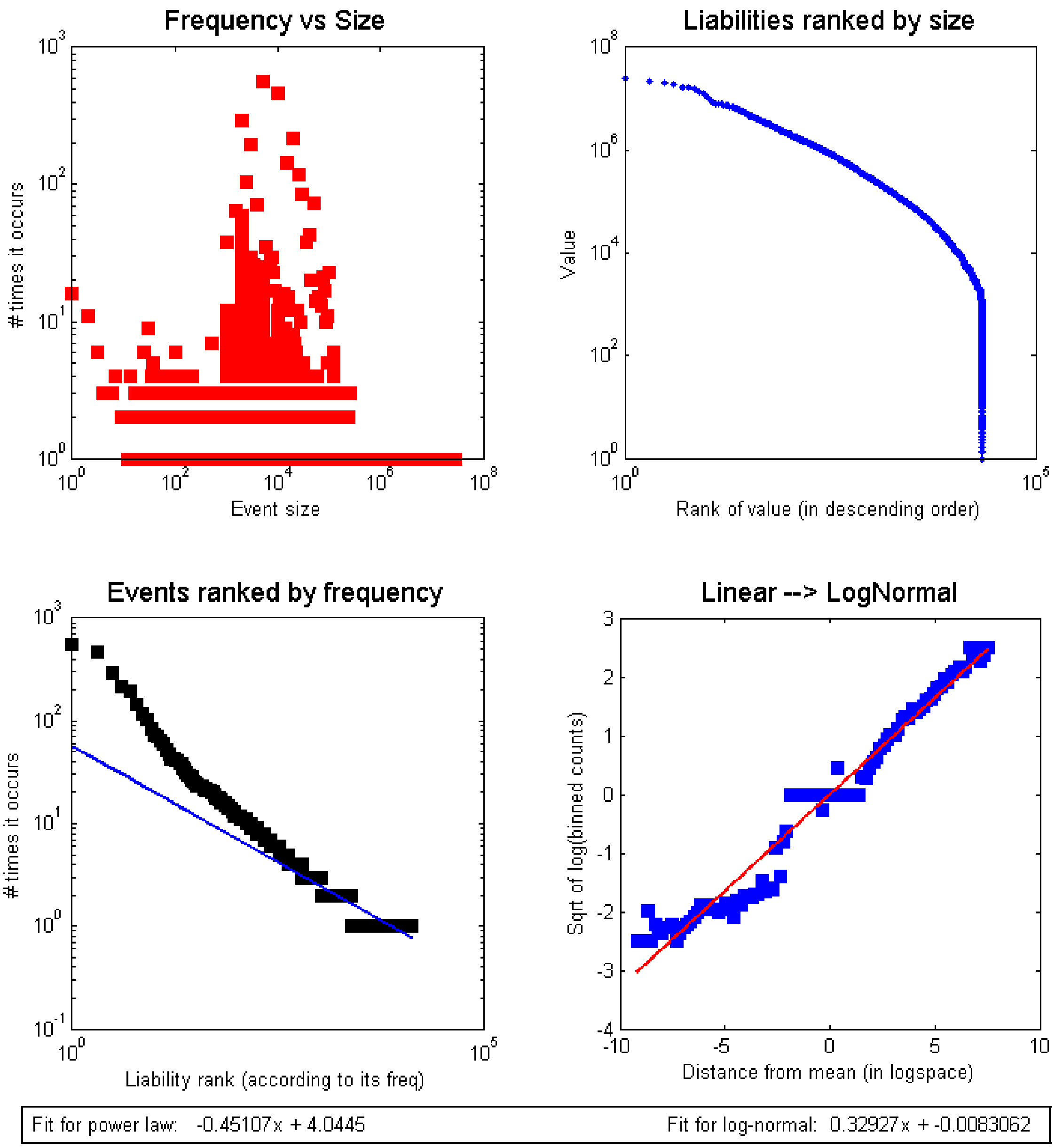
E Other results

Figure 12: Unconditional frequency distribution of total bankruptcy costs



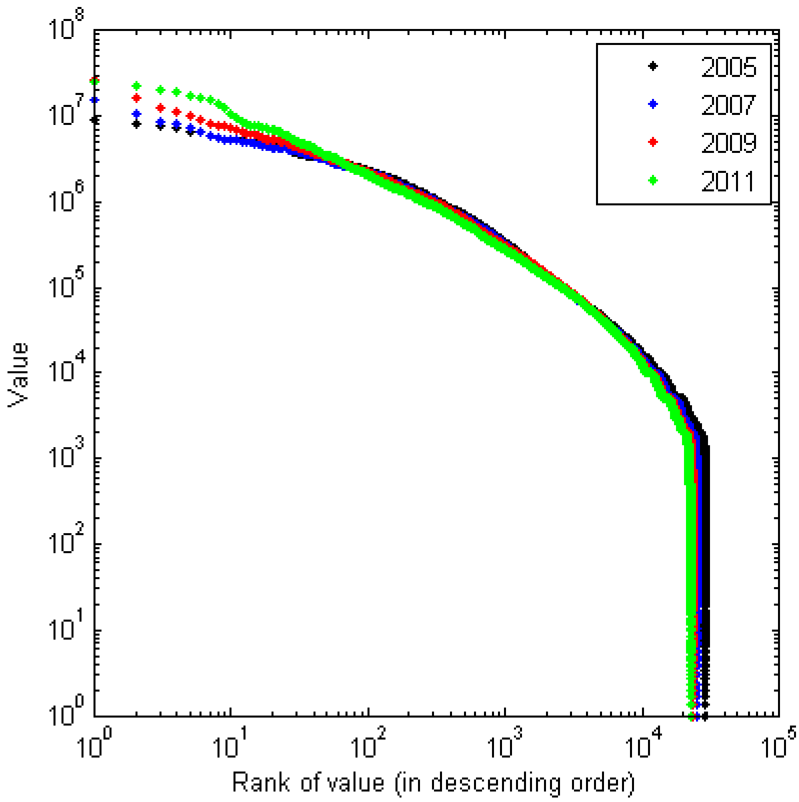
F Liabilities and network properties

Figure 13: Power law vs log-normal diagnostics



Note: For confidentiality reasons liabilities are plotted as averages of three consecutive values (ranked by size).

Figure 14: Comparison between ranked interbank liabilities (by size)



Note: For confidentiality reasons, liabilities are plotted as averages of three consecutive values.

Table 6: Network properties – 2009 Q1

No of banks**		**1780**								
No of links		**23,475**								
		Quantiles							mean	std dev
		5%	10%	25%	50%	75%	90%	95%		
Total IB Assets	*	7779	13,407	35,610	106,684	330,323	930,469	1,861,787	990,894	6,385,872
Total IB Liabilities	*	3310	6264	18,287	56,581	182,849	568,604	1,228,837	990,894	8,170,345
Total Assets	*	46,238	73,476	168,061	440,785	1,233,332	3,157,769	5,984,005	4,763,593	58,325,883
Out Degree		1	1	1	2	4	10	15	13	83
In Degree		1	1	4	9	14	19	24	13	39
Degree		2	3	6	12	19	27	37	26	114
Opsahl centrality		107.5	141.6	246.3	472.9	1088.7	2432.7	4375.7	3484	26,597
Eigenvector centrality		5.99E-06	1.30E-05	4.25E-05	0.000137	0.000699	0.002755	0.006942	0.00306	0.02351
Weighted betweenness		0	0	0	0	0	694.5	7877.75	9522	77,055
Weighted eigenvector		0.000012	0.000017	0.000037	0.000112	0.000523	0.001900	0.003801	0.002913	0.023520
Closeness centrality		317.3125	317.3125	368.4375	402.5	404	422.1875	438.625	387.8771	53.8038
Clustering coefficient		0	0	0	0.0099	0.05	0.1445	0.2008	0.0438	0.0825
No of obs		89	178	445	890	1335	1602	1691	1780	1780

Note: ** number of banks active in the interbank market. * in thousand €

Table 7: Network properties – 2007 Q1

No of banks**		**1822**								
No of links		**25,346**								
		Quantiles							mean	std dev
		5%	10%	25%	50%	75%	90%	95%		
Total IB Assets	*	6235	11.593	29,307	86,765	249,302	735,171	1,621,935	941,918	6,280,025
Total IB Liabilities	*	3526	6020	16,897	50,463	167,960	524,639	1,377,271	941,918	7,005,688
Total Assets	*	43,016	71,051	156,391	407,544	1,146,350	3,037,258	5,635,412	4,867,551	58,588,305
Out Degree		1	1	1	2	5	12	18	14	82
In Degree		1	1	4	9	15	21	27.4	14	42
Degree		2	3	6	12	19	29	43	28	116
Opsahl centrality		104.9	140.1	239.7	475.1	1096.7	2499.4	4546.9	3406	22,514
Eigenvector centrality		0.000012	0.000024	0.000077	0.00024	0.000966	0.003547	0.008665	0.0037	0.0231
Weighted betweenness		0	0	0	0	0	109.6	6009.8	9145	75,048
Weighted eigenvector		0.000021	0.000028	0.000056	0.000149	0.000558	0.001958	0.004772	0.0031	0.0232
Closeness centrality		312.6	327.375	367.375	414.75	417.875	437.45	451.8	398.21	56.93
Clustering coefficient		0	0	0	0.01389	0.05357	0.15486	0.20467	0.0484	0.0868
No of obs		91	182	456	911	1367	1640	1731	1822	1822

Note: ** number of banks active in the interbank market. * in thousand €

Table 8: Network properties – 2005 Q1

No of banks**		**1989**								
No of links		**28,976**								
		Quantiles							mean	std dev
		5%	10%	25%	50%	75%	90%	95%		
Total IB Assets	*	6034	10,662	28,607	82,385	222,942	637,407	1,483,345	904,305	6,252,776
Total IB Liabilities	*	2846	5389	15,751	48,197	161.576	486,450	1,108,158	904,305	6,858,284
Total Assets	*	27,890	50,648	127,479	336,441	929,702	2,538,967	4,483,895	3,145,409	25,946,202
Out Degree		1	1	1	2	5	12	18	15	88
In Degree		1	1	4	9	15	21	27.05	15	45
Degree		2	3	6	13	20	30	43.05	29	123
Opsahl centrality		82.1	115	210.2	442.5	1023.4	2276.5	4080.9	2972	18,568
Eigenvector centrality		0.000012	0.000024	0.000079	0.00025	0.000974	0.0032994	0.007708	0.00356	0.02214
Weighted betweenness		0	0	0	0	0	574.8	8639.9	10,151	82,861
Weighted eigenvector		0.000021	0.000029	0.000061	0.000165	0.000550	0.001851	0.004124	0.003034	0.0223
Closeness centrality		337.325	339.8125	399.1875	452.625	457	475.325	487.275	432.29	62.58
Clustering coefficient		0	0	0	0.01428	0.0585	0.15909	0.2	0.0513	0.093
No of obs		99	199	497	995	1492	1790	1890	1989	1989

Note: ** number of banks active in the interbank market. * in thousand €

G "Eisenberg and Noe" - interbank contagion algorithm

When a bank defaults (or a group of them), it triggers losses in the interbank market. If interbank losses (plus losses on loans to the real economy) exceed the remaining capital of the banks that lent to the defaulted group, this can develop into a domino cascade.

At every simulation when interbank contagion arises, we follow Elsinger et al. (2006), who build on Eisenberg and Noe (2001), to compute losses that take into account the assumptions (1)—(3) at the beginning of Section 5. However, we describe the mechanism in terms of losses rather than of a clearing payment vector.[39]

The following set of definitions and equations describe an equation system. Each bank incurs total portfolio losses L_i, which consists, first, of its fundamental losses made in the real economy[40], and second, of losses on its interbank loans, defined below in (18):

$$L_i = L_i^{\text{real}} + L_i^{\text{IB}}. \tag{15}$$

A bank defaults if its capital K_i cannot absorb the real-economy and interbank credit losses. We define the default indicator as

$$D_i = \begin{cases} 1 & \text{if } K_i < L_i, \\ 0 & \text{otherwise.} \end{cases} \tag{16}$$

We have modeled bankruptcy costs such that their (potential) extent BC_i is known before contagion; i.e., they are just a parameter of the equation system. Yet, whether they become real is captured by D_i.

Total portfolio losses and bankruptcy costs are now distributed to the bank's claimants. If capital is exhausted, further losses are primarily borne by interbank creditors since their claim is junior to other debt, as stated in assumption 5 at the beginning of Section 5. Bank i causes its interbank creditors an aggregate loss of

$$\Lambda_i^{\text{IB}} = \min\left(l_i, \max\left(0, L_i + BC_i D_i - K_i\right)\right), \tag{17}$$

which is zero if the bank does not default. The greek letter signals that Λ_i^{IB} is a loss *on the liability side* of bank i, which *causes* a loss on the assets of its creditors. Recall that x_{ij} denotes interbank liabilities of bank i against bank j. The row sum $l_i = \sum_{j=i}^{N} x_{ij}$ defines total interbank liabilities of bank i. This gives us a *proportionality matrix* π to allocate losses, given by

$$\pi_{ij} = \begin{cases} \frac{x_{ij}}{l_i} & \text{if} \quad l_i > 0; \\ 0 & \text{otherwise.} \end{cases}$$

If the loss amount Λ_i^{IB} is proportionally shared among the creditors, bank j incurs a loss of $\pi_{ij}\Lambda_i^{\text{IB}}$ due to the default of i. Also bank i may have incurred interbank losses; they amount to

$$L_i^{\text{IB}} = \sum_{k=1}^{N} \pi_{ji}\Lambda_j^{\text{IB}}, \tag{18}$$

[39] The clearing payment vector used by Elsinger et al. (2006) and Eisenberg and Noe (2001) is not a sufficient statistic for the payoffs to all claimants. It is so for the interbank market but, for instance, it does not contain full information on the size of losses to non-bank debtors or equity holders.

[40] This loss can also be negative, i.e.,the bank makes a profit on these assets.

which provides the missing definition in (15). This completes the equation system (15)–(18). We can either consider the vector of L_i as a solution or, equivalently, $L_i + BC_iD_i$. The algorithm proposed by Eisenberg and Noe (2001) gives us a unique result.

H Formal treatment of the fixed point problem

$$L_i = L_i^{\text{real}} + L_i^{\text{IB}}. \tag{19}$$

Note that this variable does NOT include bankruptcy costs. Default dummy:

$$D_i \equiv I_{\{L_i > K_i\}}$$

Equivalent vector form :

$$D \equiv I_{\{L>K\}}. \tag{20}$$

Losses in the IB market *caused* by bk i, incurred by the others:

$$\Lambda_i^{\text{IB}} = \min\left(l_i, \max\left(0, L_i + BC_iD_i - K_i\right)\right),$$

or as vector:

$$\Lambda^{IB} = l \wedge \left[L + \tilde{B}D - K\right]^+ \tag{21}$$

where $l = \left(\sum_{j=i}^N x_{ij}\right)_i$ is the vector of interbank liabilities, $\wedge$ is the element-wise minimum, and $\tilde{B}$ is a diagonal matrix with BC on the diagonal. $\tilde{B}D$ are the *actual* bankruptcy costs, the ones that have become real.

Π is the *proportionality matrix* to allocate losses. Interbank losses (on the asset side!) amount to

$$L^{\text{IB}} = \Pi^\top \Lambda^{\text{IB}}, \tag{22}$$

Formulas (19) to (22) make up the fixed point problem. We could consider the total portfolio losses$L$ (not containing BCs) as a solution but prefer to add them, which is why we define $L^{all} \equiv L + \tilde{B}D$. To get the system into a concise form, start with (19) and add BCs on both sides. This gives

$$L^{all} = L^{\text{real}} + \tilde{B}D + L^{IB}$$

Apply (22):

$$L^{all} = L^{\text{real}} + \tilde{B}D + \Pi^\top \Lambda^{IB}$$

Now put (21) in, which gives the 1-line version of the fixed point problem

$$L^{all} = L^{\text{real}} + \tilde{B}D + \Pi^\top \left(l \wedge \left[L^{all} - K\right]^+\right)$$

Notice that BCs are NOT part of the default condition (but a consequence of it) so that, if we want to express the system completely in terms of L^{all}, we utilize the trivial identity $L \equiv L^{all} - \tilde{B}D$ to reach at

$$L^{all} = L^{\text{real}} + \tilde{B}I_{\left\{L^{all} - \tilde{B}D > K\right\}} + \Pi^\top \left(l \wedge \left[L^{all} - K\right]^+\right) \tag{23}$$

Remark. The default condition in (23) looks worse than before, however it can be simplified since each component $L_i^{all} = L_i + BC_i I_{\{L_i > K_i\}}$ is a strictly increasing function of L_i so that it can be inverted, leading to the identity $I_{\left\{L^{all} - \tilde{B}D > K\right\}} = I_{\left\{L^{all} > K\right\}}$. This yields

$$L^{all} = L^{\text{real}} + \tilde{B} I_{\left\{L^{all} > K\right\}} + \Pi^{\top} \left(l \wedge \left[L^{all} - K \right]^{+} \right) \tag{24}$$

but we must add now the restriction

$$L_i^{all} > K_i \;\Rightarrow\; L_i^{all} > K_i + BC_i, \, \forall i$$

to the system (otherwise, an algorithm solving (24) alone might generate values above K that do not, however, exceed K after subtraction of $\tilde{B}$). Taking this complication into account, it finally appears simpler to solve the equivalent equation

$$L = L^{\text{real}} + \Pi^{\top} \left(l \wedge \left[L + \tilde{B} I_{[L > K]} - K \right]^{+} \right)$$

How does this look like in terms of L^{Ben}, the *Losses* in Ben's routine, which are defined as

$$L^{Ben} = (-l) \vee \left(K - L^{all} \right)$$

and initialized by $remEq \equiv K - L^{real}$ in the algorithm? We can rewrite the default condition to $L^{Ben} < 0$

$$\begin{aligned}
L^{Ben} &= (-l) \vee \left(K - L^{all} \right) \\
&= (-l) \vee \left(K - L^{\text{real}} - \tilde{B}D - \Pi^{\top} \left(l \wedge \left[L^{all} - K \right]^{+} \right) \right) \\
&= (-l) \vee \left(remEq - \tilde{B}D + \Pi^{\top} \left((-l) \vee \left(K - L^{all} \right) \wedge 0 \right) \right) \\
&= (-l) \vee \left(remEq - \tilde{B}D + \Pi^{\top} \left(L^{Ben} \wedge 0 \right) \right)
\end{aligned}$$

and finally

$$L^{Ben} = (-l) \vee \left(remEq - \tilde{B} I_{\{L^{Ben} < 0\}} + \Pi^{\top} \left(L^{Ben} \wedge 0 \right) \right).$$

This equation is a *necessary* condition to be checked. To make the system complete, we would somehow have to add the property that L^{Ben} has to jump at 0 by BC. (omitted, since this is already guaranteed by the algorithm).

Remark. L^{Ben} is closely related to (liability) losses in the IB market:

$$\left[-L^{Ben} \right]^{+} = \left[l \wedge \left(-K + L^{all} \right) \right]^{+} = l \wedge \left[-K + L^{all} \right]^{+} = \Lambda^{IB}$$

or

$$-\Lambda^{IB} = 0 \wedge L^{Ben}.$$